JOURNAL

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXVIII.

$$9\,2\,\,6\,2\,\,4\,\,6$$
$$2\,9\,\,|\,0\,\,|\,0\,\,8$$

CONTENTS OF VOL. XXVIII.

Contents of Vol. XXVIII.

v

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXVIII.—PART I.

JULY 1889.

CONTENTS OF NO. CLIII.

NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

INSTITUTE OF ACTUARIES.

The Problem of Copyhold Fine Enfranchisement. By GEORGE S. MATHEWS, M.A., *Fellow of the Surveyors' Institution.*

[Read before the Institute, 25 February 1889.]

INTRODUCTION.

THE problem of estimating the lord's interest in a copyhold property which is subject to a fine on death and alienation, in other words, the enfranchisement value of such a property, has been a constant source of perplexity to Surveyors.

Its chief difficulty arises from the fact that the contingency of alienation has to be taken into account, in addition to the contingency of death, which is alone involved in ordinary actuarial problems.

These latter, upon which depend material interests of vast magnitude, have been scientifically and exhaustively examined by Actuaries of great mathematical skill, who have expended enormous labour in accumulating the statistics upon which their calculations depend.

The practical problem of enfranchisement is similar in kind but has not been subject to similar treatment, for the ages of admission to copyholds and the durations of copyhold tenancy of

inheritors and purchasers respectively (statistics necessary for its exact solution) have never been recorded.

Several reasons may be assigned for this failure.

In the first place, the material interests involved are of comparatively small importance. Again, Actuaries have not been practically concerned in the solution of this special problem; and again, Surveyors, who are chiefly responsible for enfranchisement valuations, have scarcely understood it.

Enfranchisement tables have from time to time been issued from the office of the Land Commission for the guidance of Surveyors, and other tables have been published with explanations and methods of solution of the problem, notably, by Mr. Rouse in his *Copyhold Enfranchisement Manual,* and by Mr. Ed. Smyth in various papers read before the Institute of Actuaries and the Institution of Surveyors.

But the explanations are insufficient and the solutions unsound, and all the published tables, without exception, have at least one common serious defect.

The enfranchisement value of the fines expectant on death and alienation are not separately stated, and the tables cannot therefore be properly applied to those cases, probably a majority in number, in which these fines are unequal.

An attempt is made in the following pages to set out the enfranchisement problem in a precise actuarial form, and to give a practical solution of it, founded upon certain assumptions which, in the absence of proper statistical information, must necessarily be made.

The Problem Stated and Explained.

The enfranchisement problem in its most general form may be thus stated:

If a copy be held on the condition of a fine being paid to the lord whenever a tenant dies or alienates, what is the present value of all the successive fines payable?

In order that the investigation of this problem may be made as clear as possible, it will be well to start with its simplest form, and introduce gradually the various conditions which complicate it.

Let alienation be for the present excluded, and suppose a fine of £1 to be paid and a new life admitted at the end of the year in which any life becomes extinct.

Let x_1, x_2, x_3, &c., be the ages of the successive lives admitted,

and let the value of enfranchisement when no life is in possession and a fine is immediately due be denoted by F. In this case

$$F = 1 + A_{x_1} + A_{x_1}A_{x_2} + A_{x_1}A_{x_2}A_{x_3} + \&c., \textit{ad inf.}$$

If a fine be not immediately due, but a life of the age y be in possession, the value of enfranchisement will be the assurance of a sum F on the death of y. If this value be denoted by F_y,

$$F_y = F \cdot A_y.$$

If $x_1 = x_2 = x_3 = \&c. = x$, then

$$F = \frac{1}{1 - A_x}, \text{ and } F_y = \frac{A_y}{1 - A_x}.$$

The value of F_y may also be expressed in another form, for $A_x = v - (1-v)a_x$, and therefore

$$F_y = \frac{1 - (1 - A_y)}{1 - A_x} = \frac{1}{1 - A_x} - \frac{1 - A_y}{1 - A_x}$$

$$= F - \frac{1 + a_y}{1 + a_x}.$$

In the case now under consideration, where the lives are ordinary lives and x, the constant age of admission, is known, the value of enfranchisement for any age y in possession can be found by means of any selected standard table of assurances or annuities.

If there were no such table of reference the problem could still be solved theoretically by a comprehensive observation of the actual fine intervals of the manor, from which a copyhold mortality table could be formed, and annuity or assurance tables constructed therefrom at the rates of interest required.

For a fine interval is the interval between two successive payments of fines in any particular copy. If, therefore, all the fine intervals in every copy in a manor are recorded, this record is, in fact, the record of mortality for that particular manor, from the constant age of admission to the limit of life.

Suppose the observed fine intervals, n in number, to be

$$t_1, t_2, t_3, \ldots t_n.$$

Then, since the fine is supposed to be due and paid at the end of the year in which any life becomes extinct, the actual number of years which the several observed lives complete after admission are

$$t_1 - 1, t_2 - 1, t_3 - 1, \ldots t_n - 1.$$

Now e_x, the curtate expectation of life, is

$$= \frac{t_1 - 1 + t_2 - 1 + \&c. + t_n - 1}{n}$$

$$= \frac{t_1 + t_2 + \ldots t_n}{n} - 1.$$

Hence, the mean fine interval of the manor, which may be defined as the manorial interval, $= 1 + e_x$.

Let the problem under consideration be now extended by the introduction of the disturbing element of alienation, the assumption of a constant age of admission being still retained.

The copyhold lives, or, now more strictly speaking, copyhold tenancies, will be subject to the contingency of alienation in addition to the ordinary contingency of death; but it is obvious that this can make no difference in the principle or method of solution.

The difficulty only arises from the fact that the copyhold tenancy extinction rate is no longer the same as the ordinary death rate, and the standard tables are therefore no longer applicable.

But the observed fine intervals, though affected by alienation, still furnish all the materials required for the construction of the copyhold assurance and annuity tables, provided the observations have been sufficiently numerous and extensive to represent fairly the durations of copyhold tenure; and the expressions for the value of enfranchisement will be exactly the same as those already found.

Thus, if $A_{(x)}$, $a_{(x)}$ be the symbols for the values of the assurance and annuity found from the copyhold tables so specially constructed, the value of F will still be denoted by the formula $\frac{1}{1 - A_{(x)}}$, and of F_y, the enfranchisement value for any age y in possession, by $\frac{A_{(y)}}{1 - A_{(x)}}$ or $F - \frac{1 + a_{(y)}}{1 + a_{(x)}}$.

Moreover, if $e_{(x)}$ be the symbol for the curtate expectation of copyhold existence, which, for convenience, may be called the copyhold expectation, $1 + e_{(x)}$ will still represent the manorial interval.

The general effect of alienation must be to increase the copyhold death, or tenancy extinction, rate; and the greater the force of alienation, in other words, the greater the proportion of purchasers to inheritors, the greater will be this increase.

This proportion will be different in different manors. It will be greatest where there is greatest commercial activity, and conversely. Consequently, every manor or group of manors similarly conditioned in this respect must have a separate copyhold mortality, or tenancy extinction, table, and separate copyhold annuity and assurance tables for the calculation of enfranchisement values.

The problem may now be further extended by abandoning the assumption of a single constant age of admission.

Instead of a single age let a single set of ages be assumed, applicable alike to inheritors and purchasers.

The mortality table being constructed as before from the observed fine intervals, the enfranchisement formula will be found as follows.

For the sake of simplicity let two ages only be first taken, x_1 and x_2, and let it be assumed that the succession of either is, in all cases, equally probable.

The value of the first fine immediately due is $\qquad$ 1

The present value of the second fine will be either A_{x_1} or A_{x_2}; and as either is equally probable the value of this expectation is $\qquad$ $\dfrac{A_{x_1} + A_{x_2}}{2}$

The present value of the third fine will be either $A_{x_1} A_{x_1}$, $A_{x_1} A_{x_2}$, $A_{x_2} A_{x_1}$, $A_{x_2} A_{x_2}$; and as either is equally probable the value of this expectation is $\qquad$ $\left(\dfrac{A_{x_1} + A_{x_2}}{2}\right)^2$

The present value of the fourth fine may similarly be shown to be $\qquad$ $\left(\dfrac{A_{x_1} + A_{x_2}}{2}\right)^3$

And so on.

Adding together all these successive values

$$F = \dfrac{1}{1 - \dfrac{A_{x_1} + A_{x_2}}{2}}$$

In the same way it may be shown that, if there are m possible ages of admission, $x_1, x_2, x_3, \ldots x_m$, and these ages recur in the proportions $n_1, n_2, n_3, \ldots n_m$,

$$F = \cfrac{1}{1 - \cfrac{n_1 A_{x_1} + n_2 A_{x_2} + \ldots + n_m A_{x_m}}{n_1 + n_2 + \ldots + n_m}}$$

Moreover, as in the single life problem the manorial interval was shown to be one year more than the expectation of the age of admission, so, in this more general problem, it may be shown that the same relation exists between the manorial interval and the mean of the expectations of the separate ages of admission —*i.e.*:

$$\text{Manorial interval} = 1 + \frac{n_1 e_{x_1} + n_2 e_{x_2} + \ldots + n_m e_{x_m}}{n_1 + n_2 + \ldots + n_m}.$$

If, then, there is one single set of ages of admission from which inheritors and purchasers are indifferently drawn, these ages may be replaced by a single age, the value of the assurance, and therefore also of the annuity, on which is equal to the mean of the values of the assurances, or annuities, on the separate ages, repeated in proportion to their respective probabilities of recurrence.

This age may be called the age of the mean annuity. If it be denoted by x, the value of enfranchisement may always be expressed by the formula

$$F_y = F . A_y = F - \frac{1 + a_y}{1 + a_x}.$$

When y, the age in possession, is equal to x, this value is $F - 1$.

Hence x, the age of the mean annuity, is also the age for which the value of enfranchisement differs from F by a single fine.

It is also obvious that x, the single age which replaces the various ages of admission, cannot be the mean age, and that it is different for every different manorial interval and every different rate of interest.

If the average of age of admission of inheritors is materially different from that of purchasers, so that a distinct age or set of ages must be assumed for each, the enfranchisement formula will be more complicated. This further extension of the problem will be hereafter considered. It is, for the present, sufficient to point out that in this case also the various ages of admission can be replaced by a single age, the age of the general mean expectation for the determination of the manorial interval.

Another and final extension of the problem can now be made.

It has been hitherto assumed that the same fine of £1 is paid on death and alienation.

This, however, is by no means the case in practice. In some manors there are two classes of inheritors, whose fine on admission is different, the customary heir being admitted on payment of a nominal fine only. In some, the fine on death is different from that on alienation. A heriot paid on death only may be included in this class. In some, supposed to be subject to arbitrary fines, which are two years' net annual value of the hereditament, a fine of one and a half years' value is accepted by the lord on alienation.

Observations of the fine intervals properly recorded in the court rolls should distinguish those terminated by death and alienation respectively. The copyhold mortality table could then be separated into two parts, from each of which separate assurance and annuity tables could be constructed. When this is done, the values of the fines expectant on death and alienation could be separately calculated, and the total enfranchisement value thus determined.

The method, however, suggested for finding the copyhold mortality table, though theoretically sound, is not practical. The field of observation is far too limited, and the number of fine intervals observed must, under the most favourable conditions, be far too few for the construction of a trustworthy table, one which, for example, would at all compare in accuracy with our modern standard tables of ordinary mortality.

Moreover, as a matter of fact, no records have been made on the court rolls of the ages either of admission, death, or alienation.

This raises the two peculiar difficulties of the problem, namely, the determination of the age or ages of admission and the construction of the copyhold mortality, or tenancy extinction, table.

These difficulties can only be met by assumptions—first, as to the ages of admission; and, secondly, as to the effect or rate of alienation which, together with the known death rate, will determine the construction of the copyhold tables.

Any valuation, however made, must involve these necessary assumptions, and, to make it consistent and intelligible, it is essential that they should be precisely defined at the outset. The assumptions must be reasonable in themselves, consistent with any generally accepted or recorded facts, and carried to their

legitimate conclusions regardless of any preconceived idea of what the result ought or ought not to be.

If the result is obviously unreasonable, it will be easy to review the calculation and modify any assumption that may be considered erroneous.

Assumption as to the Ages of Admission.

In the absence of any recorded information, the ages of admission must be determined from the opinions of experts.

On this point Mr. Rouse says (*Rouse's Enfranchisement Manual, division iii, part* 1): " Persons are rarely admitted after " a death till somewhat approaching the middle of life, and with " respect to those who purchase, they seldom are in circumstances " enabling them to do so till they arrive at middle age, and, as the " worth which a man attaches to his own life most frequently " counterbalances the desire which would otherwise exist of " having nominees admitted, the owners are generally admitted " themselves. Considering all these circumstances, it may be " just to assume 44 as the fair average age on which to " calculate."

Mr. Ed. Smyth says (*J.I.A.*, xxi, 392): " By way of some- " what testing the correctness of such estimate (Mr. Rouse's), I " turn to the *General Report*, published 1873, on the Census of " 1871, and find that for the years 1861–1870 the mean age of " men at the time of their first marriage was 25·7 years; and the " mean aftertime for this age is nearly 36 years, according to the " English Life Table No. III (Males), upon which I have " grounded my own Enfranchisement Table. Remembering the " uncertainty as to which child will both survive the father " and become his heir, these figures point to the heir being, " at the death of the father, about 32 years old as an average. " Now, if tenants admitted after deaths be regarded as about " equal in number to those admitted after purchases (and this is in " harmony with the commissioner's opinion that the enfranchise- " ment charge for heriots, payable upon death and alienation, " should be double of that for heriots payable upon death only), " Mr. Rouse's estimate of 44 being the average age of admission " of tenants generally, will point to the average age of those " admitted in consequence of purchase as being about 56; and for " his reasons above quoted respecting purchasers this is possibly " not too old."

Mr. E. J. Smith says (*Transactions of Surveyors' Institution,* ii, 138): " Sufficient importance has not been given to the fact " that ordinarily it is the older persons who purchase estates " and the younger who dispose of them. I have found that the " average age of persons purchasing over those selling estates is " fully half as much again. It is not until persons have attained " a certain age that they have much money for the purpose of " investment. In copyhold cases this has a twofold action. The " junior persons, in a large number of cases, pay the fine and sell " the estates soon after they come into possession of them ; the " purchasers, generally much older men, pay a second fine and " hasten the time for a third one also."

These three authorities appear to agree in assuming the general mean age of admission to be about the middle period of life; but while Mr. Rouse assumes the mean age of inheritors and purchasers to be the same, Mr. Smyth and Mr. Smith assume the mean age of purchasers to be materially greater than that of inheritors. It will be safer to adopt the opinion of these latter, founded as it must be on actual and wide experience.

But it is not sufficient to determine the mean age. The calculation requires the ages of mean expectation and mean annuity, which may differ materially from the mean age.

If the ages of admission were properly recorded, the ages of mean expectation and mean annuity would be found from a standard set of ages carefully selected for this purpose.

Such a standard set or sets of ages must now be assumed.

If it is considered sufficient, as it would certainly be simpler, to adopt the opinion of Mr. Rouse, that the ages of inheritors and purchasers are practically the same, the following set, the mean of which is 44 years, may be reasonably assumed to represent the possible ages of admission.

$$14, 24, 34, 44, 54, 64, 74.$$

Again, every age in the set must be repeated in number proportional to the probability of its recurrence, which will be greater the nearer it is to the mean age.

It is proposed, therefore, to make this further assumption, that the numbers

$$5, 10, 20, 30, 20, 10, 5.$$

will sufficiently represent these respective probabilities.

It is not pretended that these figures are exact, or arrived at

by any scientific method. They are intended only to give practical effect in the most reasonable manner to the principle already explained, that the single age which will replace the various ages of admission must be different for every manorial interval and every rate of interest.

On this assumption, if x be the age of the mean annuity,

$$a_{(x)} = \frac{1}{100}\{5a_{(14)} + 10a_{(24)} + 20a_{(34)} + 30a_{(44)} + 20a_{(54)} + 10a_{(64)} + 5a_{(74)}\}.$$

A more accurate representation of actual facts, and therefore a closer approximation to the real value of enfranchisement, can be made by assuming separate sets of ages for inheritors and purchasers.

Let the possible ages of admission for inheritors be represented by the set 14, 24, 34, 44, 54, and for purchasers by 34, 44, 54, 64, 74; and let the probabilities of recurrence for the several ages in each set be represented by the numbers 10, 25, 30, 25, 10.

On this assumption the general mean age of admission is 44 years, and the mean age of inheritors and purchasers 34 and 54 respectively. The general mean age is consistent with the opinion of all the experts above quoted, and the difference of 20 years in the mean ages of inheritors and purchasers agrees as nearly as possible with the opinions of Mr. Ed. Smyth and Mr. E. J. Smith.

Then, if x_1 be the age of mean annuity for inheritors, and x_2 for purchasers,

$$a_{(x_1)} = \frac{1}{100}\{10a_{(14)} + 25a_{(24)} + 30a_{(34)} + 25a_{(44)} + 10a_{(54)}\}$$

$$a_{(x_2)} = \frac{1}{100}\{10a_{(34)} + 25a_{(44)} + 30a_{(54)} + 25a_{(64)} + 10a_{(74)}\}.$$

Moreover, if the number of inheritors and purchasers in any manor is in the proportion of n_1 to n_2, and x be the age of the general mean annuity, then

$$a_{(x)} = \frac{n_1 a_{(x_1)} + n_2 a_{(x_2)}}{n_1 + n_2}.$$

The manorial interval is one year more than the mean of the expectations of the various ages of admission.

On the first assumption of a single set of ages, from which inheritors and purchasers are indifferently drawn,

$$\text{Manorial interval} = 1 + \frac{n_1 e_{(x)} + n_2 e_{(x)}}{n_1 + n_2} = 1 + e_{(x)},$$

and is therefore independent of n_1 and n_2.

On the second assumption of two sets of ages of admission,

$$\text{Manorial interval} = 1 + \frac{n_1 e_{(x_1)} + n_2 e_{(x_2)}}{n_1 + n_2}.$$

In this case the manorial interval cannot be expressed in terms of the expectations unless the relation of n_1 to n_2 in any manor is first found. A method of finding this relation will be subsequently explained.

ASSUMPTION OF THE EFFECT OF ALIENATION.

The effect on the copyhold mortality, or extinction, table, due to the contingency of alienation, has now to be considered.

Alienation must affect every year of copyhold existence, for, whatever the age in possession may be, there must always be some probability of the extinction of the tenancy by alienation within one year.

The question for consideration is, whether this probability is the same for every age, or whether it increases or diminishes with the age, or varies in any other way. A general law of alienation, which will be the same for every manor, has to be determined, and the force of alienation in any particular manor can then be found from the manorial interval.

Looking at the question in itself, without any recorded facts for guidance, there seems to be no reason for supposing that this probability will be at all affected by the age of the copyholder.

If a purchaser must be admitted tenant, and pay a fine on alienation, it will be a matter of indifference to him what the age of the vendor copyholder may be; and, as to the vendor, he may find it convenient to sell, or avail himself of any favourable opportunity of selling, indifferently at any period of his existence.

Turning to the opinions of experts, that of Mr. E. J. Smith will be found in his words before quoted. If his opinion is correct, that sellers are chiefly found among the young, then, the probability of alienation, speaking generally, will diminish with the age.

Mr. Ed. Smyth (*J.I.A.*, xxi, 393, *and Transactions of Surveyors' Institution*, xviii, 56), while agreeing with Mr. E. J. Smith, suggests that this probability may increase with the age of the tenant, because the lord cannot compel the purchaser to come into Court to be admitted and pay the customary fine. If, therefore, the purchaser is old, and the vendor young, the former is tempted to evade the fine by retaining the vendor on the rolls as nominal tenant.

If this were customary and right, the probability of the extinction of the tenancy by alienation might not be greater in the case of young than of old lives.

But the purchaser can only be tempted to do this in the small minority of cases where the value of his own life is materially less than that of the vendor, and, as already pointed out, this is to a great extent counterbalanced by the worth which a man generally attaches to his own life, and by the greater feeling of personal possession when his property is enrolled in his own name.

Moreover, this possible evasion of a fine justly due to the lord ought scarcely to be taken into account in a valuation intended to do even justice between the lord and tenant. In fact, throughout this investigation it is always assumed, as is usual in similar actuarial problems, that the fine is due and paid at the end of the year in which any tenancy becomes extinct, either by death or alienation.

However this may be, Mr. Smyth is apparently of opinion that the probability of alienation may be taken as the same for every age, and this is the direct opinion of the late Mr. G. Pownall, a surveyor of great practical experience, who says: "Alienation is likely to be just as frequent in the case of young as of old lives" (*Transactions of Surveyors' Institution*, ii, 99).

This is the assumption here adopted for the construction of the copyhold tables. Reason and the balance of experience appear to justify it, and it will be found to lead to a comparatively simple calculation of enfranchisement values for different manorial intervals and different rates of interest.

It may be urged that, as the very young have usually no power to sell, and the very old have little disposition for change of any kind, the probabilities of alienation at the two extremes of life are less than on this assumption.

Granting the force of this objection, the error which is involved by leaving it out of consideration will be material only for very young or very old lives in possession, and even this error

may be corrected in a manner which will be subsequently explained.

Construction of the Copyhold Mortality Table.

The copyhold table can be easily constructed on any defined assumption of the effect of alienation.

Adopting the usual actuarial notation, the natural life table, or table of the number of persons out of the whole number observed entering on the $x+1$th year of age, is $l_x, l_{x+1}, l_{x+2} \ldots l_{x+n}$, &c., to the limit of life.

The natural mortality table, or table of the number dying in the $x+1$th and subsequent years, is $d_x, d_{x+1}, d_{x+2} \ldots d_{x+n}$, &c., to the limit of life. So that $d_x = l_x - l_{x+1}$, $d_{x+1} = l_{x+1} - l_{x+2}$, and so on.

When the contingency of alienation is added to that of death, these tables will be modified.

Let $1 - m_1, 1 - m_2 \ldots 1 - m_n$, &c., be the rate of alienation in the $x+1$th and subsequent years of life, and suppose, in the first place, that all alienations take place at the very beginning of each year before any deaths occur.

Then, in the $x+1$th year of age,

    Number at beginning of year $= l_x$
        ,,   alienating $= (1 - m_1) l_x$
        ,,   dying $= m_1 d_x$
        ,,   surviving $= m_1 l_{x+1}$

In the $x+2$th year,

    Number at beginning of year $= m_1 l_{x+1}$
        ,,   alienating $= (1 - m_2) m_1 l_{x+1}$
        ,,   dying $= m_1 m_2 d_{x+1}$
        ,,   surviving $= m_1 m_2 l_{x+2}$

and so on, for subsequent years.

In the $x+n$th year of age,

    Number at beginning of year $= m_1 m_2 \ldots m_{n-1} l_{x+n-1}$
        ,,   alienating $= (1 - m_n) m_1 m_2 \ldots m_{n-1} l_{x+n-1}$
        ,,   dying $= m_1 m_2 \ldots m_n d_{x+n-1}$
        ,,   surviving $= m_1 m_2 \ldots m_n l_{x+n}$

Again, suppose that all the alienations take place at the very end of each year after all the deaths have occurred.

Proceeding, as in the former case, it will be found that, in the $\overline{x+n}$th year of age,

$$\text{Number at beginning of year} = m_1 m_2 \ldots m_{n-1} l_{x+n-1}$$
$$\text{,,} \quad \text{dying} \qquad = m_1 m_2 \ldots m_{n-1} d_{x+n-1}$$
$$\text{,,} \quad \text{alienating} \qquad = (1-m_n) m_1 m_2 \ldots m_{n-1} l_{x+n}$$
$$\text{,,} \quad \text{surviving} \qquad = m_1 m_2 \ldots m_n l_{x+n}$$

The actual number of extinctions by death and alienation in any year will be the mean of the number on these two extreme suppositions; thus, in the $\overline{x+n}$th year,

Extinctions by death $\qquad = \dfrac{1+m_n}{2} m_1 m_2 \ldots m_{n-1} d_{x+n-1}$

Extinctions by alienation $\quad = \dfrac{1-m_n}{2} m_1 m_2 \ldots m_{n-1}(l_{x+n-1} + l_{x+n})$

Total number of extinctions $= m_1 m_2 \ldots m_{n-1}(l_{x+n-1} - m_n l_{x+n})$

On the adopted assumption that the probability of alienation is the same at every age, $m_1 = m_2 = \&c. = m_n = m$, which may be called the measure of alienation in the manor.

The copyhold mortality, or tenancy extinction, table, may thus be set out as follows:

Natural Lives. Number Living	(1) Yearly Extinctions by Death	(2) Yearly Extinctions by Alienation	(3) Total Yearly Extinctions	Copyhold Tenancies. Number Surviving
l_x	$\dfrac{1+m}{2}(l_x - l_{x+1})$	$\dfrac{1-m}{2}(l_x + l_{x+1})$	$l_x - m l_{x+1}$	l_x
l_{x+1}	$\dfrac{1+m}{2} m(l_{x+1} - l_{x+2})$	$\dfrac{1-m}{2} m(l_{x+1} + l_{x+2})$	$m(l_{x+1} - m l_{x+2})$	$m l_{x+1}$
l_{x+2}	$\dfrac{1+m}{2} m^2(l_{x+2} - l_{x+3})$	$\dfrac{1-m}{2} m^2(l_{x+2} + l_{x+3})$	$m^2(l_{x+2} - m l_{x+3})$	$m^2 l_{x+2}$
l_{x+3}	$\dfrac{1+m}{2} m^3(l_{x+3} - l_{x+4})$	$\dfrac{1-m}{2} m^3(l_{x+3} + l_{x+4})$	$m^3(l_{x+3} - m l_{x+4})$	$m^3 l_{x+3}$
&c.	&c.	&c.	&c.	&c.
l_{x+n-1}	$\dfrac{1+m}{2} m^{n-1}(l_{x+n-1} - l_{x+n})$	$\dfrac{1-m}{2} m^{n-1}(l_{x+n-1} + l_{x+n})$	$m^{n-1}(l_{x+n-1} - m l_{x+n})$	$m^{n-1} l_{x+n-1}$
&c.	&c.	&c.	&c.	&c.

The expressions for the total copyhold expectation, annuity, and assurance found from this table in the usual way are:

$$e_x = \frac{1}{l_x} \{ ml_{x+1} + m^2 l_{x+2} + m^3 l_{x+3} + \&c., \text{ to limit of life} \}$$

$$a_{(x)} = \frac{1}{l_x} \{ vml_{x+1} + v^2 m^2 l_{x+2} + v^3 m^3 l_{x+3} + \&c., \text{ to limit of life} \}$$

$$A_{(x)} = \frac{1}{l_x} \{ v(l_x - ml_{x+1}) + v^2 m(l_{x+1} - ml_{x+2}) + v^3 m^2 (l_{x+2} - ml_{x+3}) + \&c., \text{ to limit of life} \}$$

$$= v - (1-v) \frac{1}{l_x} \{ vml_{x+1} + v^2 m^2 l_{x+2} + v^3 m^3 l_{x+3} + \&c., \text{ to limit of life} \}$$

$$= v - (1-v) a_{(x)}.$$

The ages of admission have been already determined, and it has been shown that the manorial interval in any manor is a function of the expectations of these ages. If, therefore, any particular value be given to m, the corresponding manorial interval can be found from the expectations.

Conversely, if any particular manorial interval be given, that value of m can be found which will determine the table of copyhold mortality for that interval.

Again, having found the value of m for any given manorial interval, the total copyhold annuity or assurance table, at any rate of interest, can be constructed from column (3) of the mortality table, and the separate parts of it expectant on death and alienation from columns (1) and (2) respectively.

But to find the value of m for various manorial intervals, and to construct the tables at 3 and 4 per-cent (the rates of interest generally required) would involve an amount of labour which the importance of the problem scarcely justifies.

Fortunately, the assumption of the effect of alienation renders the construction of separate copyhold tables comparatively unimportant.

The expression for the ordinary life annuity a_x is

$$\frac{1}{l_x} \{ vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + \&c., \text{ to limit of life} \}.$$

If this be compared with the expressions for the copyhold expectation $e_{(x)}$ and the copyhold annuity $a_{(x)}$, the following deductions are obvious:

First.—The table of expectations for any manor is the ordinary life annuity table at the rate of interest represented by

that value of v which is equal to m, the measure of alienation for that manor. It is the annuity table at the rate of interest $\frac{1}{m} - 1$, since $v = \frac{1}{1+i}$, or $i = \frac{1}{v} - 1$.

Conversely, the ordinary life annuity table at a rate of interest represented by v is the table of expectations for copyhold lives (or tenancies) in a manor in which v is the measure of alienation.

Secondly.—The copyhold annuity table, at any rate of interest represented by r, for a manor where m is the measure of alienation, is the ordinary life annuity table at a rate represented by vm.

Conversely, the ordinary life annuity table, at a rate of interest represented by r, is the copyhold annuity table at a rate represented by $\frac{r}{m}$.

For example, on the assumption of a constant age of admission of 44 years, the manorial interval of a manor is $1 + e_{(44)}$. Any ordinary life annuity table is then the table of expectations for a manor with a manorial interval of $1 + a_{44}$. By the 3 per-cent Carlisle life annuity table, $1 + a_{44} = 17 \cdot 13$. This table is therefore the table of expectations for a manor with a manorial interval of $17 \cdot 13$ years, and the 6 and 7 per-cent Carlisle annuity tables will be approximately the 3 and 4 per-cent copyhold annuity tables for the same manor in which the measure of alienation is the value of v at 3 per-cent.

On any other assumption as to the age or ages of admission, these tables would represent the copyhold expectation and annuity tables for a manor of a different interval.

It follows from this explanation that when the assumption as to the age or ages of admission is made, the copyhold expectation and annuity tables for any given manorial interval (within certain limits) can be found from the ordinary life annuity tables.

The values of enfranchisement can then be found by means of the proper enfranchisement formulæ, which must next be determined.

DETERMINATION OF THE ENFRANCHISEMENT FORMULÆ.

First.—On the assumption of the single set of ages of admission from which inheritors and purchasers are indifferently drawn.

Let the assurances of £1, expectant on the death or alienation of any age x, be denoted respectively by the symbols $A_{(dx)}$ $A_{(ax)}$, so that

$$A_{(dx)} + A_{(ax)} = A_{(x)}.$$

Turning to the copyhold mortality table, on page 14, the assurance of £1, expectant on death, may be calculated in the usual way from column (1), thus:

$$A_{dx} = \Sigma \frac{1+m}{2} v^n m^{n-1} \frac{l_{x-n-1} - l_{x-n}}{l_x}$$

$$= \frac{1+m}{2} \frac{1}{l_x} \{ v(l_x - l_{x+1}) + mv^2(l_{x+1} - l_{x+2}) + m^2 v^3(l_{x+2} - l_{x+3}) + \&c. \}$$

$$= \frac{1+m}{2m} \frac{1}{l_x} \{ mv(l_x - l_{x+1}) + m^2 v^2(l_{x+1} - l_{x+2}) + m^3 v^3(l_{x+2} - l_{x+3}) + \&c. \}$$

$$= \frac{1+m}{2m} \frac{1}{l_x} \{ mvl_x - (1 - mv)(mvl_{x+1} + m^2 v^2 l_{x+2} + m^3 v^3 l_{x+3} + \&c.) \}$$

$$= \frac{1+m}{2m} \{ mv - (1 - mv) a_x \} \quad \cdots \cdots \cdots \quad (1)$$

The value of the assurance of £1, expectant on alienation, may be similarly calculated from column (2), thus:

$$A_{ax} = \Sigma \frac{1-m}{2} v^n m^{n-1} \frac{l_{x+n-1} + l_{x+n}}{l_x}$$

$$= \frac{1-m}{2} \frac{1}{l_x} \{ v(l_x + l_{x+1}) + mv^2(l_{x+1} + l_{x+2}) + m^2 v^3(l_{x+2} + l_{x+3}) + \&c. \}$$

$$= \frac{1-m}{2m} \frac{1}{l_x} \{ mvl_x + (1 + mv)(mvl_{x+1} + m^2 v^2 l_{x+2} + m^3 v^3 l_{x+3} + \&c.) \}$$

$$= \frac{1-m}{2m} \{ mv + (1 + mv) a_x \} \quad \cdots \cdots \cdots \quad (2)$$

Adding (1) and (2) together the whole copyhold assurance A_x, or the value of £1, expectant on death or alienation, is

$$v - (1 - v) a_x ;$$

a result which has been before directly obtained from column (3) of the copyhold table.

Let y be the age in possession, and x the age of the mean annuity which will replace the various ages of admission.

Then, if the same fine of £1 be payable on death and alienation,

$$F_y = \frac{A_y}{1 - A_x} ;$$

But if these fines are different the value of enfranchisement must be found by calculating separately the values of the fines expectant on death and alienation, which may be represented by the symbols F_{dy}, F_{ay}, respectively.

Those expectant on death can be set out as follows:

1st Fine.—Assurance of £1 expectant on the death of y $\Big\} = A_{(dy)}$

2nd Fine.—Assurance of £1 on death of x, after y has either died or alienated $\Big\} = A_{(y)} \cdot A_{(dx)}$

3rd Fine.—Assurance of £1 on death of x, after x has died or alienated, after y has died or alienated $\Big\} = A_{(y)} \cdot A_{(x)} \cdot A_{(dx)}$

4th Fine.—Assurance of £1 on death of x, the third in succession $\Big\} = A_{(y)} \cdot A_{(x)}^2 \cdot A_{(dx)}$

&c.   &c.   &c.

nth Fine.—Assurance of £1 on death of x, the $\overline{n-1}$th in succession $\Big\} = A_{(y)} \cdot A_{(x)}^{n-2} \cdot A_{(dx)}$

&c.   &c.   &c.

Adding together all these successive fines to infinity,

$$F_{dy} = A_{(dy)} + \frac{A_{(y)} \cdot A_{(dx)}}{1 - A_{(x)}}$$

$$= \frac{A_{(dy)}(1 - A_{(dx)} - A_{(ax)}) + A_{(dx)}(A_{(dy)} + A_{(ay)})}{1 - A_{(x)}}$$

$$= \frac{A_{(dy)} + A_{(dx)} A_{(ay)} - A_{(ax)} A_{(dy)}}{1 - A_{(x)}} \quad \cdots \quad (3)$$

It may be similarly shown that

$$F_{ay} = \frac{A_{(ay)} - (A_{(dx)} A_{(ay)} - A_{(ax)} A_{(dy)})}{1 - A_{(x)}} \quad \cdots \quad (4)$$

If the fines on death and alienation are f_1 and f_2 respectively, the total value of enfranchisement $= f_1 F_{dy} + f_2 F_{ay}$

$$= \frac{f_1 A_{(dy)} + f_2 A_{(ay)} + (f_1 - f_2)(A_{(dx)} A_{(ay)} - A_{(ax)} A_{(dy)})}{1 - A_{(x)}}$$

As the values have to be computed from annuity, and not assurance, tables, the formulæ must be expressed in terms of the copyhold annuities.

Replacing $A_{(dx)}$ by $\dfrac{1+m}{2m}\{mv - (1 - mv)a_{(x)}\}$

              $A_{(ax)}$ by $\dfrac{1-m}{2m}\{mv + (1 + mr)a_{(x)}\}$

and A_{dy}, A_{ay} by similar expressions, and $1-A_x$ by $\overline{1-v}(1+a_x)$, the values of F_{dy}, F_{ay}, in terms of the copyhold annuities, will be found to be as follows:

$$F_{dy} = \frac{\dfrac{1+m}{2m}\left\{ v(m-\overline{1-m}.a_x)-\overline{1-v}.a_{(y)} \right\}}{1-v(1+a_x)} \quad . \quad . \quad . \quad (5)$$

$$F_{ay} = \frac{\dfrac{1-m}{2m}\left\{ v(m+\overline{1+m}.a_x)+\overline{1-v}.a_y \right\}}{1-v(1+a_x)} \quad . \quad . \quad . \quad (6)$$

The enfranchisement values, when no life is in possession and a fine is immediately due, may be derived from the above expressions by making $a_y=0$, and dividing by v. If these values be denoted by the symbols F_d, F_a, then

$$F_d = \frac{\dfrac{1+m}{2m}(m-\overline{1-m}.a_x)}{1-v(1+a_x)} \quad . \quad . \quad . \quad . \quad (7)$$

$$F_a = \frac{\dfrac{1-m}{2m}(m+\overline{1+m}.a_x)}{1-v(1+a_x)} \quad . \quad . \quad . \quad . \quad (8)$$

Combining (5) with (7), and (6) with (8), it will be found that

$$F_{dy} = F_d(v-\overline{1-v}.a_{(y)}) - \frac{1-m^2}{2m}a_{(y)}$$

$$= F_d.A_{y} - \frac{1-m^2}{2m}a_y . \quad . \quad . \quad . \quad . \quad (9)$$

$$F_{ay} = F_a.A_y + \frac{1-m^2}{2m}a_y . \quad . \quad . \quad . \quad (10)$$

When the same fine of £1 is payable on death and alienation,

$$F = F_d + F_a \quad \text{and} \quad F_y = F_{dy} + F_{ay}.$$

In this case, adding together (7) and (8),

$$F = \frac{1}{1-v(1+a_{x})} = \frac{1}{1-A_x}.$$

Adding together (5) and (6), or (9) and (10),

$$F_y = \frac{v-\overline{1-v}.a_{y}}{1-v(1+a_x)} = \frac{A_y}{1-A_x} = F.A_y.$$

This result has been previously and independently obtained.

By putting $r=1$ in (5) and (6) the values of the fines expectant on death and alienation become respectively the numbers of the fines expected; and their proportion is therefore the proportion of the numbers of inheritors and purchasers in the manor, or $\dfrac{n_1}{n_2}$.

Hence $\qquad \dfrac{n_1}{n_2} = \dfrac{\overline{1+m}\{m-(1-m)e_{(x)}\}}{\overline{1-m}\{m+(1+m)e_{(x)}\}}$ . . . . . (11)

If a_x is the value of the mean annuity in the ordinary life annuity table which is the copyhold table of expectations for the manor, $a_x = e_{(x)}$ and

$$\dfrac{n_1}{n_2} = \dfrac{\overline{1+m}\{m-(1-m)a_x\}}{\overline{1-m}\{m+(1+m)a_x\}}$$ . . . . . (11A)

When the number of inheritors is equal to the number of purchasers in the manor, $n_1 = n_2$, and

$$\overline{1+m}\{m-(1-m)a_x\} = \overline{1-m}\{m+(1+m)a_x\},$$

or $\qquad\qquad\qquad a_x = \dfrac{m^2}{1-m^2}.$

Hence the manorial interval, $1+a_x$, of the manor in which the numbers of inheritors and purchasers are equal, is

$$1 + \dfrac{m^2}{1-m^2} = \dfrac{1}{1-m^2}.$$

Now, if m be the value of r at 3 per-cent, $\dfrac{1}{1-m^2} = 17\cdot42$,

and if m be the value of v at $3\frac{1}{2}$ per-cent, $\dfrac{1}{1-m^2} = 15\cdot04$.

It will be subsequently shown that, on the present assumption as to the ages of admission and on the Carlisle mortality table:

The mean fine interval of a manor in which m is⎫
   the value of r at 3 per-cent, is⎭   16·57

The mean fine interval, where m is the value of r⎫
   at $3\frac{1}{2}$ per-cent, is⎭   15·60

Hence, the mean fine interval of the manor in which inheritors and purchasers are equal, lies between 16·57 and 15·60 years. It may be taken to be about 16 years.

Secondly.—The enfranchisement formulæ can be determined on the assumption of the separate sets of ages of admission for inheritors and purchasers respectively.

Let x_1 be the age of the mean annuity for inheritors, x_2 for purchasers, and y the age in possession.

The successive fines, expectant on death, may be set out as follows:

1st Fine.—The assurance of £1 on the death of $y = \mathrm{A}_{dy}$.

2nd Fine.—This consists of two terms, namely, the value of the expectation of y dying and x_1 succeeding and dying, and that of y alienating and x_2 succeeding and dying,

$$= \mathrm{A}_{dy} \mathrm{A}_{dx_1} + \mathrm{A}_{ay} \mathrm{A}_{dx_2}.$$

3rd Fine.—This must consist of four terms, since each term of the 2nd fine must give rise to two terms in the 3rd fine.

Thus x_1, having succeeded y, may die or alienate. In the former case the value of the expectation in the 3rd fine is $\mathrm{A}_{dy} \mathrm{A}_{dx_1} \mathrm{A}_{dx_1}$. In the latter case it is $\mathrm{A}_{dy} \mathrm{A}_{ax_1} \mathrm{A}_{dx_2}$.

Again, x_2, having succeeded, may either die or alienate. In the former case the value of the expectation in the 3rd fine is $\mathrm{A}_{ay} \mathrm{A}_{dx_2} \mathrm{A}_{dx_1}$. In the latter case it is $\mathrm{A}_{ay} \mathrm{A}_{ax_2} \mathrm{A}_{dx_2}$.

It thus appears that the value of the 3rd fine may be derived from that of the 2nd by substituting $\mathrm{A}^2_{dx_1} + \mathrm{A}_{ax_1} \mathrm{A}_{dx_2}$ for A_{dx_1}, and $\mathrm{A}_{dx_2} \mathrm{A}_{dx_1} + \mathrm{A}_{ax_2} \mathrm{A}_{dx_2}$ for A_{dx_2}.

The total value of the 3rd fine or of the 3rd set of death expectations

$$= \mathrm{A}_{dy} \{ \mathrm{A}^2_{dx_1} + \mathrm{A}_{ax_1} \mathrm{A}_{dx_2} \} + \mathrm{A}_{ay} \{ \mathrm{A}_{dx_2} \mathrm{A}_{dx_1} + \mathrm{A}_{ax_2} \mathrm{A}_{dx_2} \}.$$

4th Fine.—It will be readily seen from similar reasoning that the 4th fine may be derived from the 3rd in exactly the same way as the 3rd from the 2nd, by substituting in the last factor of each term $\mathrm{A}^2_{dx_1} + \mathrm{A}_{ax_1} \mathrm{A}_{dx_2}$ for A_{dx_1}, and $\mathrm{A}_{dx_2} \mathrm{A}_{dx_1} + \mathrm{A}_{ax_2} \mathrm{A}_{dx_2}$ for A_{dx_2}.

The total value of the 4th fine, or set of death expectations, will thus be

$$= \mathrm{A}_{dy} \{ \mathrm{A}_{dx_1} (\mathrm{A}^2_{dx_1} + \mathrm{A}_{ax_1} \mathrm{A}_{dx_2}) + \mathrm{A}_{ax_1} (\mathrm{A}_{dx_2} \mathrm{A}_{dx_1} + \mathrm{A}_{ax_2} \mathrm{A}_{dx_2}) \}$$
$$+ \mathrm{A}_{ay} \{ \mathrm{A}_{dx_2} (\mathrm{A}^2_{dx_1} + \mathrm{A}_{ax_1} \mathrm{A}_{dx_2}) + \mathrm{A}_{ax_2} (\mathrm{A}_{dx_2} \mathrm{A}_{dx_1} + \mathrm{A}_{ax_2} \mathrm{A}_{dx_2}) \}$$

Any number of succeeding fines can be similarly expressed, each one from that preceding, and the enfranchisement value then found by summing the whole series to infinity.

The fines expectant on alienation can also be similarly expressed and summed.

In order to avoid confusion here, the summation of these series is transferred to the Appendix (p. 43), where it is shown that:

$$F_{dy} = \frac{A_{(dy)}(1-A_{(ax_2)}) + A_{(ay)}A_{(dx_2)}}{(1-A_{(dx_1)})(1-A_{(ax_2)}) - A_{(dx_2)}A_{(ax_1)}} \quad \ldots \quad (12)$$

$$F_{ay} = \frac{A_{(ay)}(1-A_{(dx_1)}) + A_{(dy)}A_{(ax_1)}}{(1-A_{(dx_1)})(1-A_{(ax_2)}) - A_{(dx_2)}A_{(ax_1)}} \quad \ldots \quad (13)$$

Replacing $\qquad A_{(dy)}$ by $\dfrac{1+m}{2m}\{mv - (1-mv)a_{(y)}\}$

and $\qquad\qquad A_{(ay)}$ by $\dfrac{1-m}{2m}\{mv + (1+mv)a_{(y)}\}$

and $A_{(dx_1)}A_{(ax_1)}A_{(dx_2)}A_{(ax_2)}$ by similar expressions, it will be found that the values in terms of the copyhold annuities are as follows:

$$F_{dy} = \frac{\dfrac{1+m}{2m}\left\{v(m - \overline{1-m}\,.\,a_{(x_2)}) - \overline{1-r}\,.\,a_{(y)}\right\}}{1 - v\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}} \quad \ldots \quad (14)$$

$$F_{ay} = \frac{\dfrac{1-m}{2m}\left\{v(m + \overline{1+m}\,.\,a_{(x_1)}) + \overline{1-r}\,.\,a_{(y)}\right\}}{1 - v\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}} \quad \ldots \quad (15)$$

Making $a_{(y)} = 0$ in these expressions, and dividing by v,

$$F_d = \frac{\dfrac{1+m}{2m}(m - \overline{1-m}\,.\,a_{(x_2)})}{1 - v\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}} \quad \ldots \quad (16)$$

$$F_a = \frac{\dfrac{1-m}{2m}(m + \overline{1+m}\,.\,a_{(x_1)})}{1 - v\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}} \quad \ldots \quad (17)$$

Combining (14) with (16), and (15) with (17), it will be found that

$$F_{dy} = F_d\,.\,A_{(y)} - \frac{\dfrac{1-m^2}{2m}(1 + a_{(x_2)})}{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}}\,a_{(y)} \quad \ldots \quad (18)$$

$$F_{ay} = F_a\,.\,A_{(y)} + \frac{\dfrac{1-m^2}{2m}(1 + a_{(x_1)})}{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}}\,a_{(y)} \quad \ldots \quad (19)$$

When the same fine of £1 is payable on death and alienation,

$$F = F_d + F_a, \quad \text{and} \quad F_y = F_{dy} + F_{ay}.$$

In this case, adding together (16) and (17),

$$F = \frac{1 + \dfrac{1-m^2}{2m}(a_{(x_1)} - a_{(x_2)})}{\overline{1-v}\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}} \quad . \quad . \quad . \quad (20)$$

Adding together (14) and (15),

$$F_y = \frac{v - \overline{1-v}.a_{(y)} + \dfrac{1-m^2}{2m}(a_{(x_1)} - a_{(x_2)})v}{\overline{1-v}\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}}$$

$$= \frac{A_{(y)} + \dfrac{1-m^2}{2m}(a_{(x_1)} - a_{(x_2)})v}{\overline{1-v}\left\{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}\right\}} \quad . \quad . \quad . \quad (21)$$

Or, adding together (18) and (19),

$$F_y = F.A_{(y)} + \frac{\dfrac{1-m^2}{2m}(a_{(x_1)} - a_{(x_2)})}{1 + \dfrac{1+m}{2m}a_{(x_1)} - \dfrac{1-m}{2m}a_{(x_2)}}a_{(y)} \quad . \quad . \quad (21\text{A})$$

If $x_1 = x_2 = x$, the above expressions all become the same as those previously found in the case of a single set of ages of admission.

By putting $v = 1$ in (14) and (15), the values of the fines expectant on death and alienation become respectively the numbers of the fines expected, and their proportion is, therefore, the proportion of the numbers of inheritors and purchasers in the manor, or $\dfrac{n_1}{n_2}$.

Hence
$$\frac{n_1}{n_2} = \frac{\overline{1+m}\{m - (1-m)e_{(x_2)}\}}{1-m\{m + (1+m)e_{(x_1)}\}} \quad . \quad . \quad . \quad . \quad (22)$$

If a_{x_1}, a_{x_2}, are the values of the mean annuities for inheritors and purchasers in the ordinary life annuity table which is the copyhold table of expectations, $e_{(x_1)} = a_{x_1}$, and $e_{(x_2)} = a_{x_2}$, and

$$\frac{n_1}{n_2} = \frac{\overline{1+m}\{m - (1-m)a_{x_2}\}}{1-m\{m + (1+m)a_{x_1}\}} \quad . \quad . \quad . \quad (22\text{A})$$

Again, if x be the age of the mean annuity for inheritors and purchasers taken together,

$$e_{x}, \text{ or } a_x = \frac{n_1 a_{x_1} + n_2 a_{x_2}}{n_1 + n_2}$$

$$= \frac{\overline{1+m}\{m-(1-m)a_{x_2}\}a_{x_1} + \overline{1-m}\{m+(1+m)a_{x_1}\}a_{x_2}}{2m + (1-m^2)(a_{x_1} - a_{x_2})}$$

$$= \frac{\dfrac{1+m}{2} a_{x_1} + \dfrac{1-m}{2} a_{x_2}}{1 + \dfrac{1-m^2}{2m}(a_{x_1} - a_{x_2})} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

This determines the manorial interval of the manor in which m is known, for it is $1 + e_{x_1}$.

When the number of inheritors and purchasers is equal, $n_1 = n_2$,

or, $\overline{1+m}\{m-(1-m)a_{x_2}\} = \overline{1-m}\{m+(1+m)a_{x_1}\}$

or, $\dfrac{a_{x_1} + a_{x_2}}{2} = \dfrac{m^2}{1-m^2}.$

In this case the mean expectation of all the ages of admission is

$$\frac{e_{x_1} + e_{x_2}}{2} = \frac{a_{x_1} + a_{x_2}}{2} = \frac{m^2}{1-m^2}.$$

The manorial interval is therefore $1 + \dfrac{m^2}{1-m^2} = \dfrac{1}{1-m^2}.$

Now, if m be the value of v at 3 per-cent, $\dfrac{1}{1-m^2} = 17 \cdot 42$,

and if m be the value of v at $3\frac{1}{2}$ per-cent, $\dfrac{1}{1-m^2} = 15 \cdot 04.$

It will be subsequently shown that on the present assumption as to the ages of admission and on the Carlisle mortality table :

The manorial interval of a manor, in which m is⎫
    the value of v at 3 per-cent, is   ⎬ 16·65
                                   ⎭

The manorial interval, where m is the value of v⎫
    at $3\frac{1}{2}$ per-cent, is   ⎬ 15·47
                                   ⎭

Hence, the manorial interval, for which inheritors and purchasers are equal, lies between 16·65 and 15·47 years.   It

may be taken to be about 16 years, the same as that before arrived
at on the assumption of a single set of ages of admission.

Application of the Carlisle Tables.

The formulæ now found have to be applied to some standard
set of life annuity tables. The Carlisle tables appear to be most
convenient for this application, because they have been calculated
at a greater number of rates of interest than those founded on
any other mortality observations, and can therefore be utilised
as copyhold annuity tables for a greater number of manorial
intervals.

In David Jones' work on Annuities (vol. 1) these tables are
published at the following rates of interest:

$$0, \; 3, \; 3\tfrac{1}{2}, \; 4, \; 4\tfrac{1}{2}, \; 5, \; 6, \; 7, \; 8, \; 9, \; 10.$$

Each of these is the expectation table (or annuity table at
interest 0) for some particular manorial interval depending on
the assumption made as to the age or ages of admission.

Three separate assumptions have been already referred to,
namely:

(A) A constant age of admission, the assumed mean age of
    44 years.

(B) A single set of ages (with a mean of 44 years), from which
    inheritors and purchasers are indifferently drawn.

(C) Two sets of ages, one for inheritors (with a mean of 34
    years), and one for purchasers (with a mean of 54
    years), the mean of the two sets being 44 years.

On either assumption the manorial interval for which any
life annuity table is the copyhold expectation table can be
determined on the principles already explained.

For example, the manorial interval for which the 5 per-cent
life annuity table is the copyhold table of expectations is:

On assumption A, $1 + e_{44} = 1 + a_{44} = 13 \cdot 81$ years.

On assumption B it is $1 + e_x = 1 + a_x$

$$= 1 + \frac{1}{100} \{5a_{14} + 10a_{24} + 20a_{31} + 30a_{44} + 20a_{54} + 10a_{64} + 5a_{74}\}$$

$$= 1 + 12 \cdot 24 = 13 \cdot 24 \text{ years.}$$

On assumption C it is $1 + e_{(x)} = 1 + \dfrac{n_1 e_{(x_1)} + n_2 e_{(x_2)}}{n_1 + n_2}$

$$= 1 + \frac{\dfrac{1+m}{2}a_{x_1} + \dfrac{1-m}{2}a_{x_2}}{1 + \dfrac{1-m^2}{2m}(a_{x_1} - a_{x_2})} \qquad \bullet$$

In this expression

$m =$ value of v at 5 per-cent $= \cdot 95238$

$$a_{x_1} = \frac{1}{100}\{10a_{14} + 25a_{24} + 30a_{34} + 25a_{44} + 10a_{54}\}$$

$$= 14 \cdot 028$$

$$a_{x_2} = \frac{1}{100}\{10a_{34} + 25a_{44} + 30a_{54} + 25a_{64} + 10a_{74}\}$$

$$= 10 \cdot 337.$$

Substituting these values of m, a_{x_1}, a_{x_2}, in the above expression, the manorial interval will be found to be

$$= 1 + 11 \cdot 81 = 12 \cdot 81 \text{ years.}$$

The first available copyhold expectation table is the Carlisle annuity table at 3 per-cent. This is the table of expectations for a manorial interval between 16 and 17 years, whichever assumption, either A, B, or C, is made as to the ages of admission.

The Carlisle 6 per-cent table, which is the copyhold annuity table at 3 per-cent, approximately, for the same manorial interval is therefore the first published Carlisle table directly applicable to the calculation of enfranchisement values.

The tables from 3 to 5 per-cent, inclusive, are copyhold annuity tables at from 3 to 4 per-cent for manorial intervals which can only be determined from Carlisle tables at rates of interest between 0 and 3 per-cent, and such tables have not been published.

For the purpose of utilizing the Carlisle tables as copyhold annuity tables for manorial intervals greater than 17 years, annuity tables have now been computed on the Carlisle life data at $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ per-cent. With this addition, the published tables can be utilized as copyhold annuity tables at from 3 to 4 per-cent, for manorial intervals ranging from 20 to 12 years, the greatest and least hitherto observed.

Now, suppose that the assumption as to the ages of admission is made, either A, B, or C, and that the manorial interval in any manor has been ascertained by careful and comprehensive observation of the separate fine intervals.

The Carlisle life annuity table, which is the copyhold table of expectations for about such an interval, can then be found.

Let this be the Carlisle table at the rate of interest i or $\frac{1}{v} - 1$.

Then, it has been before shown that any other Carlisle table at a greater rate of interest, i', or $\frac{1}{v'} - 1$, is the copyhold annuity table for the same manorial interval for a value of $v = \frac{v'}{v}$ or at the rate of interest $\frac{v}{v'} - 1 = v\left(\frac{1}{v'} - \frac{1}{v}\right) = v(i' - i)$. When the rates of interest are small, this is equal to $i' - i$, approximately.

For example, on assumption A, the Carlisle 4 per-cent table, for which $v = \cdot9615$ is the copyhold expectation table for a manorial interval of $1 + a_{44}$, or $15\cdot31$ years. The 7 per-cent table is the copyhold annuity table for the same manorial interval at the rate of interest $\cdot9615(\cdot07 - \cdot04) = \cdot9615 \times \cdot03 = \cdot0288$, or 3 per-cent, approximately.

Table I in the Appendix has been constructed to show what copyhold annuity tables at what precise rates of interest the several Carlisle life annuity tables represent for the manorial intervals resulting from the three separate assumptions, A, B, and C, as to the ages of admission.

Reading the figures in this table horizontally, it will appear, for example, on assumption B, that the 6 per-cent Carlisle table represents

(1) The annuity table for natural lives } at 6°/₀ for a manorial interval of $25\cdot76$ years.

(2) The copyhold annuity table } at $5\cdot47$,, ,, not calculated.

(3) ,, at $4\cdot95$,, ,, ,,

(4) ,, at $4\cdot43$,, ,, of $20\cdot28$ years.

(5) ,, at $3\cdot92$,, ,, of $18\cdot89$,,

(6) ,, at $3\cdot41$,, ,, of $17\cdot66$,,

&c.   &c.   &c.

On assumptions A or C the manorial intervals will be different, as may be seen from the figures at the head of each vertical column.

Again, reading the figures in the table vertically, it will appear, for example, on assumption C, that for a manorial interval of 16·65 years

(1) The Carlisle 3 $\%$ table is the copyhold table of expectations.

(2)	,,	$3\frac{1}{2}$	,,	copyhold annuity table at rate $\%$		}	·48
(3)	,,	4	,,	,,	,,		·97
(4)	,,	$4\frac{1}{2}$	,,	,,	,,		1·46
(5)	,,	5	,,	,,	,,		1·94
(6)	,,	6	,,	,,	,,		2·91
	&c.			&c.		&c.	

These same tables are the copyhold annuity tables at the same rates of interest for a manorial interval of 17·13 years, on assumption A, and of 16·57 years, on assumption B.

REDUCTION OF ENFRANCHISEMENT FORMULÆ.

The general enfranchisement formulæ, and the copyhold annuity tables for various manorial intervals, having now been found, it remains to reduce the formulæ to the simplest possible form for convenience of actual calculation of enfranchisement values, at rates of interest from 3 to 4 per-cent.

The formulæ have accordingly been so reduced for various manorial intervals between 20 and 12 years, and are set out in Tables II and III in the Appendix.

> Table II is founded on the assumption, B, of a single set of ages of admission.

> Table III is founded on the assumption, C, of two sets of ages of admission for inheritors and purchasers respectively.

The formulæ on assumption, A, of a constant age of admission have not been calculated. This is only a simple form of B, and, as it is manifestly not so correct in fact, it seemed unnecessary to continue it to a practical result.

Table III is recommended for the purpose of enfranchisement

valuations because the assumption of two sets of ages of admission appears to be most in accordance with the opinion of experts.

It is, however, a matter of little practical consequence which set of formulæ is used, for a comparison of the two will show that the two assumptions as to the ages of admission give nearly the same result for the same manorial interval and rate of interest.

The method of reducing the formulæ may be illustrated by the following examples:

On assumption B the formula for enfranchisement from fines of £1 expectant on death, is

$$\frac{\dfrac{1+m}{2m}\{r(m-1-m\,.\,a_x)-1-r\,.\,a_y\}}{1-r(1+a_x)}$$

Turning to Table I, it will be seen that the 3 per-cent life annuity table is the copyhold expectation table for a manorial interval of 16·57 years on this assumption, and that the life annuity table at 7 per-cent is the copyhold annuity table at 3·88, or, approximately, 4 per-cent for this interval.

For this interval, therefore, the value of enfranchisement at 3·88 per-cent from fines of £1 expectant on death, for any age y in possession, will be found, by substituting in the above formula the proper values of m, v, and a_x.

In this case,

$$m = \text{value of } v \text{ at 3 per-cent} = \cdot 97087$$
$$mv = \qquad ,, \qquad \text{at 7 per-cent} = \cdot 93458$$
$$\therefore \quad r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad = \cdot 96262$$

$a_x = $ value found from the 7 per-cent Carlisle table of

$$\frac{1}{100}\{5a_{14}+10a_{24}+20a_{34}+30a_{44}+20a_{54}+10a_{64}+5a_{74}\}$$
$$= 9\cdot9951.$$

Substituting the values of m, r, a_x, thus found, and reducing to the simplest form, the formula becomes

$$1\cdot6159-\cdot0923a_y.$$

The formula for the fines expectant on alienation may be similarly reduced, and will be found to be

$$\cdot7263+\cdot0014a_y.$$

When a fine of £1 is paid on death and alienation, the total value of enfranchisement is the sum of the two preceding, namely:

$$2\cdot3422 - \cdot0909a_y,$$

and the value of F will be found to be $2\cdot4331$.

Again, on assumption C, the formula for enfranchisement from fines of £1 expectant on death, is

$$\frac{\dfrac{1+m}{2m}\left\{\overline{v(m-\overline{1-m}.a_{x_2})}-\overline{1-v}.a_y\right\}}{\overline{1-v}\left\{1+\dfrac{1+m}{2m}a_{x_1}-\dfrac{1-m}{2m}a_{x_2}\right\}}$$

Turning to Table I, it will be seen that the 4 per-cent life annuity table is the copyhold expectation table for a manorial interval of $14\cdot46$ years on this assumption, and that the life annuity table at 7 per-cent is the copyhold annuity table at $2\cdot88$, or 3 per-cent approximately, for this interval.

For this interval, therefore, the value of enfranchisement from fines of £1 expectant on death, for any age y in possession, will be found by substituting in the above formula the proper values of m, v, a_{x_1}, a_{x_2}.

In this case,

$$m = \text{value of } v \text{ at 4 per-cent} = \cdot96154$$

$$mv = \quad\text{,,}\quad \text{at 7 per-cent} = \cdot93458$$

$$\therefore \quad v \quad . \quad . \quad . \quad . \quad . \quad . \quad = \cdot97196$$

$a_{x_1} =$ value found from the 7 per-cent Carlisle table of

$$\frac{1}{100}\{10a_{14}+25a_{24}+30a_{34}+25a_{44}+10a_{54}\}$$
$$= 11\cdot1982.$$

$a_{x_2} =$ value found from the same table of

$$\frac{1}{100}\{10a_{34}+25a_{44}+30a_{54}+25a_{64}+10a_{74}\}$$
$$= 8\cdot6966.$$

Substituting the values of m, v, a_{x_1}, a_{x_2}, thus found, and reducing to the simplest form, the formula becomes

$$1\cdot8102 - \cdot0833a_y.$$

The formula for the fines expectant on alienation may be similarly reduced, and will be found to be

$$1\cdot2977 + \cdot0016a_y.$$

When a fine of £1 is paid on death and alienation, the total value of enfranchisement is the sum of the two preceding, namely :

$$3{\cdot}1079 - {\cdot}0817a_y.$$

and the value of F will be found to be $3{\cdot}1975$.

The Carlisle tables are not computed for any fractional rates of interest except $3\frac{1}{2}$ and $4\frac{1}{2}$ per-cent. It follows, therefore, that the copyhold annuity tables at 3 and 4 per-cent are wanting for several fine intervals. In these cases, the formulæ are calculated from that Carlisle table which is approximately the $3\frac{1}{2}$ per-cent copyhold annuity table.

For example, the Carlisle $3\frac{1}{2}$ per-cent table is the copyhold expectation table for the manorial interval of $15{\cdot}47$ years on assumption C.

For this interval, the copyhold annuity tables at 3 and 4 per-cent, approximately, would be the $6\frac{1}{2}$ and $7\frac{1}{2}$ per-cent Carlisle tables. These are wanting, and the formulæ are therefore calculated on the 7 per-cent Carlisle table, which is the copyhold annuity table at $3{\cdot}38$ per-cent, or $3\frac{1}{2}$ per-cent, approximately.

Examples.

Two points must be borne in mind in the practical application of the enfranchisement formulæ.

In the first place, the manorial interval of the manor in which any particular property proposed to be enfranchised is situated must be known or determined.

The interval may be known in some few manors where it has been subject to continuous and comprehensive observation, but in the great majority of cases it is unknown, and must be determined by the Valuer from his own personal knowledge and general experience of the district.

He may be assisted in this determination by the following general considerations.

The least manorial interval of actual observation is that of a town manor, where commercial activity is greatest, and where, therefore, the proportion of inheritors to purchasers is least. It is about 12 years.

The greatest manorial interval of actual observation is that of a country district, where commercial stagnation is greatest, and where, therefore, the proportion of inheritors to purchasers is greatest. This is about 20 years.

The interval of the average manor, or the manor in which the

number of inheritors is equal to the number of purchasers, has been shown to be about 16 years.

In the second place, it has been hitherto assumed that all properties in a manor are alike in kind, and that the general manorial interval is applicable alike to all.

This, however, is not quite the case. For the purpose of enfranchisement, properties may be roughly divided into two classes—land and building. It is generally admitted that the latter change hands more frequently than the former, and the fine interval for land will therefore be somewhat greater than for building. This difference of interval is estimated by Mr. Rouse and Mr. Ed. Smyth at about two years.

Hence, when any general manorial interval has been observed or determined as above explained, the interval for land may, in deference to these opinions, be fixed at one year more, and for building at one year less than the general interval of the manor.

The formulæ used in the following Examples are those in Table III.

Example 1.—The custom of a manor, the general manorial interval of which is $13\frac{1}{2}$ years, is the payment of an arbitrary fine of two years' net annual value on death and alienation.

It is also subject to the payment of a death heriot of the estimated value of £5.

The age of the copyhold tenant of an agricultural property, the net annual value of which is £80 is 70 years.

Required the value of enfranchisement at 3 per-cent.

The appropriate fine interval for land in this manor is one year in excess of the general manorial interval, or $14\frac{1}{2}$ years.

Turning to Table III, the enfranchisement formula for fines of £1, payable on death and alienation, for the interval of 14·46 years, at 2·88 or 3 per-cent, approximately, will be found to be

$$3\cdot108 - \cdot0817 a_y;$$

and the table for the determination of a_y to be the 7 per-cent Carlisle annuity table, in which $a_{70} = 5\cdot69$.

The formula thus becomes $3\cdot108 - \cdot0817 \times 5\cdot69$

$$= 3\cdot108 - \cdot465 = 2\cdot643.$$

The value of enfranchisement from fines is, therefore,

$$= £2 \times 2\cdot643 \times 80 = £422\cdot88.$$

Again, a death heriot is a fine expectant on death.

The formula for such a fine of £1, for the same interval and the same rate per-cent, will be found from Table III to be

$$1\cdot810 - \cdot0833a_y = 1\cdot810 - \cdot0833 \times 5\cdot69$$
$$= 1\cdot810 - \cdot473 = 1\cdot337.$$

The value of enfranchisement from the death heriot of £5 is, therefore,

$$£5 \times 1\cdot337 = £6\cdot685.$$

Example 2.—The custom of a manor, the general manorial interval of which is $15\frac{1}{2}$ years, is the payment of a fine of two years' value on death, and $1\frac{1}{2}$ on alienation.

The age of a copyhold tenant of an agricultural property, the annual value of which is £50, is 45 years.

Required the value of enfranchisement at 3 per-cent.

The appropriate fine interval for land in this manor is $16\frac{1}{2}$ years.

Turning to Table III, the formulæ for a fine interval of 16·65 at 2·91 or 3 per-cent, approximately, will be found to be

For fines of £1 expectant on death $\quad= 1\cdot794 - \cdot0751a_y$

„ „ alienation $= \cdot974 + \cdot0011a_y$

and the annuity table for the determination of a_y to be the Carlisle 6 per-cent table, in which $a_{45} = 11\cdot428.$ Thus

Fines of £1 expectant on death $\quad= 1\cdot794 - \cdot0751 \times 11\cdot428$
$$= 1\cdot794 - \cdot858$$
$$= \cdot936$$

Fines of £1 expectant on alienation $= \cdot974 + \cdot0011 \times 11\cdot428$
$$= \cdot974 + \cdot013$$
$$= \cdot987$$

Hence, the value of enfranchisement

$$= £50(2 \times \cdot936 + 1\cdot5 \times \cdot987)$$
$$= £50(1\cdot872 + 1\cdot480)$$
$$= £50 \times 3\cdot352$$
$$= £167\cdot6$$

Example 3.—The custom of a manor, the general manorial interval of which is $17\frac{1}{2}$ years, is the payment of a fine of one year's value on death, and a nominal fine on alienation.

The age of the copyhold tenant of a building, the net annual value of which is £25, is 20 years.

Required the enfranchisement value at 4 per-cent.

The appropriate fine interval for a building is one year less than the general mean interval of the manor. In this case, therefore, it is $16\frac{1}{2}$ years.

Turning to Table III, the enfranchisement formula for fines of £1 expectant on death for the interval 16·65 years at 3·88 or 4 per-cent, approximately, will be found to be

$$1\cdot533 - \cdot0829 a_y ;$$

and the table for the determination of a_y to be the Carlisle 7 per-cent table, in which $a_{20} = 12\cdot259$.

The formula thus becomes $1\cdot533 - \cdot0829 \times 12\cdot259$

$$= 1\cdot533 - 1\cdot016 = \cdot517.$$

The value of enfranchisement $= £25 \times \cdot517 = £12\cdot925.$

Example 4.—The custom of a manor, the general manorial interval of which is $13\frac{1}{2}$ years, is the payment of an arbitrary fine of two years' net annual value on death and alienation, except on admission of the customary heir, when the fine is nominal.

The age of the copyhold tenant of a building, the net annual value of which is £35, is 30 years.

Required the enfranchisement value at 4 per-cent.

The appropriate fine interval for a building in this manor is $14\frac{1}{2}$ years.

Turning to Table III, the enfranchisement formulas for a fine interval of 14·46 at 3·85 or 4 per-cent, approximately, will be found to be

For fines of £1 expectant on death $= 1\cdot545 - \cdot0911 a_y$

    ,,   ,,   alienation $= \cdot970 + \cdot0018 a_y$

and the table for the determination of a_y to be the 8 per-cent Carlisle annuity table, in which $a_{30} = 10\cdot498.$

Fines of £1 expectant on death $= 1\cdot545 - \cdot0911 \times 10\cdot498$

$$= 1\cdot545 - \cdot956$$

$$= \cdot589$$

Fines of £1 expectant on alienation $= \cdot970 + \cdot0018 \times 10\cdot498$

$$= \cdot970 + \cdot019$$

$$= \cdot989$$

It is usually assumed that the value of the fines payable by the customary heir is one-half of the value of all fines payable by inheritors.

Hence, the value of enfranchisement

$$= £35(·589 + 2 × ·989)$$
$$= £35(·589 + 1·978)$$
$$= £35 × 2·567$$
$$= £89·845$$

Example 5.—The custom of a manor, the general manorial interval of which is 19 years, is the payment of two years' net annual value on death, and one year's value on alienation.

Two persons, aged 15 and 60 years, are tenants in common, or coparceners, of two-thirds and one-third undivided parts respectively of an agricultural property, the net annual value of which is £15.

Required the value of enfranchisement at 3 per-cent.

In this case the interests of the two persons are separate, and must be separately calculated according to their respective shares of the property.

The appropriate fine interval for land in this manor is 20 years.

Turning to Table III the enfranchisement formulæ for an interval of 19·74 years at 2·94 or 3 per-cent, approximately, will be found to be

For fines of £1 expectant on death $= 1·773 - ·0671a_y$

"   "   alienation $= ·649 + ·0007a_y,$

and the table for the determination of a_y to be the 5 per-cent Carlisle annuity table.

For the life aged 15, the owner of two-thirds of the property, of the annual value of £10, $a_{15} = 16·227$

Fines of £1 expectant on death $= 1·773 - ·0671 × 16·227$
                                    $= 1·773 - 1·089$
                                    $= ·684.$

Fines of £1 expectant on alienation $= ·649 + ·0007 × 16·227$
                                     $= ·649 + ·011$
                                     $= ·660$

Hence the value of enfranchisement in respect of the life aged 15

$$= \pounds 10(2 \times \cdot 684 + \cdot 660)$$
$$= \pounds 10 \times 2\cdot 028 = \pounds 20\cdot 28$$

The value of enfranchisement in respect of the life aged 60, the owner of one-third of the property, of the annual value of £5, for which $a_{60} = 8\cdot 757$, may be similarly determined.

Example 6.—The custom of a manor, the general manorial interval of which is $15\frac{1}{2}$ years, is the payment of one year's net annual value on death and alienation.

An agricultural property of the net annual value of £50 is held by two persons of the ages 35 and 45 as joint tenants, so that no death fine is payable until the extinction of the longest life.

Required the value of enfranchisement at 3 per-cent.

In cases of this kind, the various joint lives must be replaced by a single equivalent life. Such a single life cannot be accurately found for want of copyhold annuity tables for joint lives, the construction of which would be complicated by the consideration that, while the probability of a fine payable on extinction of the tenancy by death is altered by the addition of one or more lives, the probability of a fine payable on extinction by alienation remains the same.

For the purpose of this and similar valuations it will, however, be sufficient to find the single equivalent life from the ordinary annuity tables at the rate of interest required, and then apply the proper enfranchisement formula to the single life so found.

The annuity on the longest of the two lives 35 and 45

$$= a_{35} + a_{45} - a_{35 . 45},$$

which, taking the values of single and joint annuities from the Carlisle 3 per-cent tables,

$$= 18\cdot 434 + 15\cdot 863 - 13\cdot 331$$
$$= 20\cdot 966 = a_{24}.$$

Hence, for the purpose of this valuation, the joint lives 35 and 45 may be replaced by the single life 24.

Turning now to Table III, the enfranchisement formula for fines of £1 payable on death and alienation, for the interval of 16·65 years at 3 per-cent, approximately, will be found to be

$$2\cdot 768 - \cdot 0740 a_y,$$

and the table for the determination of a_y to be the 6 per-cent
Carlisle table, in which $a_{24} = 13\cdot541$.

The formula thus becomes $2\cdot768 - \cdot0740 \times 13\cdot541$

$$= 2\cdot768 - 1\cdot002 = 1\cdot766.$$

The value of enfranchisement $= \text{£}50 \times 1\cdot766 = \text{£}88\cdot3$.

The foregoing examples are sufficient to show that enfranchise-
ment valuations in all ordinary cases can be simply made by the
application of the formulæ to the Carlisle life annuity tables which
every Surveyor possesses.

Enfranchisement tables are superfluous, but some few such
tables may be useful for the purpose of exhibiting the general
results of the method employed, and for comparison with those of
previous inquirers.

Three manorial intervals have therefore been selected from
Table III, namely, $16\cdot65$, $14\cdot46$, $12\cdot81$, and the values of
enfranchisement have been calculated, for every fifth year of age
from 5 to 100, from the appropriate formulæ for the two rates of
interest, which are approximately 3 and 4 per-cent.

The values are set out in the six tables in the Appendix,
numbered IV, IVA, V, VA, VI, VIA.

Again, it may be useful to compare the results of the two
assumptions, B and C, as to the ages of admission, for the same
manorial interval and rate of interest.

The tables numbered VII and VIII in the Appendix have been
constructed for this purpose from the formulæ for the intervals
$15\cdot60$ and $15\cdot47$ taken from Tables II and III, respectively, of
enfranchisement formulæ.

These two intervals are practically the same, and have been
selected because they are the nearest in the tables to 16 years,
which is the interval of the average manor.

The object of the remaining tables in the Appendix, IX, X,
XI, is explained below.

In comparing the enfranchisement tables in the Appendix with
other published tables, it must be borne in mind that the fine unit
of the latter is invariably the usual arbitrary fine of £2.   The
latter must therefore be reduced by one-half for comparison with
those in the Appendix, in which the unit is a fine of £1.

Modification of Assumption as to the Effect of Alienation.

An objection to the adopted assumption as to the effect of alienation has been referred to, namely, that as the very young have usually no power and the very old no disposition to sell, the probability of alienation must be less for these than for other periods of life.

It will be useful to ascertain the probable limit of the error involved if this objection is admitted.

This can be done by comparing a table constructed on the adopted assumption with one specially constructed on an extreme modification of it, for the same manorial interval and rate of interest.

It will also be convenient to fix the interval and interest of these two tables as nearly as possible at 14 years and $3\frac{1}{2}$ per-cent, in order that a further comparison may be made with the new official enfranchisement table issued by the Land Commissioners.

The special table is founded on the extreme assumption that no alienations are possible between the ages of 0 and 20 years and between 81 and the limit of life, and is constructed in the following manner:

The copyhold life and mortality table is first formed (see page 14) by making $m_1 = m_2 = \&c. = m_{20} = 1$; $m_{81} = m_{82} = \&c. = 1$ and $m_{21} = m_{22} = \&c. = m_{80} = m = $ value of v at 4 per-cent.

The copyhold expectation table is then formed, and this value of m is found to give a manorial interval, calculated from the copyhold expectations, on assumption, C, of the two sets of ages of admission, of 13·89 or 14 years, approximately.

Copyhold assurance tables at $3\frac{1}{2}$ per-cent are then constructed from the three columns of the mortality table, and the enfranchisement formulæ, expressed in terms of the copyhold assurances, are then applied to determine the values of enfranchisement.

The table, so constructed, is that numbered IX in the Appendix.

Again referring to Table III, it will be seen that the nearest interval and interest for the required comparisons is 13·58 years and 3·35 per-cent.

The enfranchisement table for this interval and interest is that numbered X in the Appendix.

The values in this latter table are somewhat too large for the interval of 14 years and interest $3\frac{1}{2}$ per-cent on account of a deficiency both of interval and interest, but the excess will not be great, and a comparison of Tables IX and X will fairly indicate

the difference resulting from the two assumptions of the effect of alienation.

As the modified assumption is extreme, the most correct enfranchisement values will probably be the mean of the two tables compared.

The correction in values for other intervals and rates of interest can only be precisely found by means of other similarly constructed special tables. As this would involve considerable labour, the correction may be roughly but perhaps sufficiently made by the following general rule:

Find the value of enfranchisement, for the interval and rate of interest required, from the formulæ or tables already given, and reduce it in the same proportion as it would be reduced if the value were found from Table X and corrected by Table IX, as above explained.

For example:

(1) Manorial interval about 14 years. Interest about $3\frac{1}{2}$ per-cent.
  Fine nominal on death and arbitrary on alienation.
  Age in possession 15 years.
  Assumption C as to ages of admission.

 By Table X in Appendix the value of enfran-$\Big\}= £1·271$
  chisements from fines of £1 on alienation

 By Table IX this value is   ·968
                $\overline{£2·239}$

 Mean value or corrected value of enfranchise-$\Big\}= £1·119$
  ment from fines of £1 on alienation

 Corrected value of enfranchisement from fines $\Big\}= £2·239$
  arbitrary

(2) Manorial Interval about $16\frac{1}{2}$ years. Interest about 3 per-cent.
  Fines arbitrary on death and alienation.
  Age in possession 15 years.
  Assumption C as to ages of admission.

 The enfranchisement value by Table X for fine $\Big\}= £1·917$
  of £1

 By Table IX for fine of £1 $= \ \ 1·521$
                $£3·438$

 Mean or corrected value $£1·719$

 By Table IV the value for fines of £1 $= £1·723$

 Corrected value $\dfrac{1·719}{1·917} \times 1·723 - = ·897 \times 1·723$ $= £1·545$

 Value for fines arbitrary $= £3·090$

THE NEW OFFICIAL ENFRANCHISEMENT TABLE.*

The old enfranchisement table of the (late) Copyhold Commissioners has now been replaced by a new one issued by the Land Commissioners in February, 1888, for the guidance of Surveyors.

The former is founded on an average fine interval of 13 years, and rate of interest 4 per-cent; the latter on an average fine interval of 14 years, and $3\frac{1}{2}$ per-cent interest.

The values in the new table for every fifth year of age from 5 to 100, reduced by one-half for a fine of £1 instead of the arbitrary fine of £2, which is the unit of the table, are set out in Table XI in the Appendix.

The new table is free from the obvious errors of the old, and may be the best single table possible with the uncertain conditions of the problem.

The following criticisms may, however, be reasonably made upon it.

Any single table, however perfect in itself, is insufficient for the purposes required, for two reasons.

In the first place, a single rate of interest is insufficient. Properties of different kinds—for example, land and buildings— are capitalized at different rates of interest. If, then, a property belongs in part to the copyholder and in part to the lord, *i.e.*, if the lord's share may be taken to be some proportion of the capital value, this share, which is the value of enfranchisement, should be calculated at the particular rate of interest appropriate to the entire property.

In the second place, no single table can be properly applied in any manor in which the manorial interval is known, or can reasonably be assumed, to be greater or less than that on which the table is founded.

Again, the average fine interval is assumed to be 14 years, The expression average fine interval is indefinite, and may be misleading. The Commissioners probably mean the average interval between changes of tenure in all manors taken together which have come under their consideration.

The interval, however, on which an average table should be calculated, is that of the average manor or the mean interval of the manor in which the changes of tenure by death and alienation are equal.

* The official table is reprinted on page 71.

These two intervals are not necessarily the same, and reasons have been given for estimating the latter at 16 years.

Again, the new table, like the old, is unexplained, but some general principle, which must at least be implied in its construction, may be inferred from the previous investigation of the problem.

Let the table be compared, first with Table X, which is founded on the assumption of a constant alienation rate, and then with Table IX, which is founded on the modified assumption as above explained. With this comparison, it is difficult to avoid the conclusion that, however the Commissioners' table is actually constructed, it must involve the assumption that the probabilities of alienation diminish in some way at the two extremes of life.

But it is essential to know what the alienation rate is, for upon it depend the relative values of the fines expectant on death and alienation (which together make up the total enfranchisement value), and therefore the value of enfranchisement in those numerous cases in which the fines on death and alienation are unequal.

It is true that the Commissioners, in a series of instructions issued with the table, explain how the value is to be arrived at in such cases.

They say, in effect: " The table being calculated on the assump-
" tion that the same fine is payable on death and alienation, *therefore*,
" when the fine on death or alienation is diminished, a proportionate
" reduction should be made in the enfranchisement value, and when
" the fine either on death or alienation is nothing, the value given
" in the table should be reduced one-half."

But this involves the assumption that the effect of death and alienation is the same. Hence, the death rate must be different for different manorial intervals, and for the same interval the alienation, like the death, rate must increase with the age, and become a certainty at the limit of life.

This is not only obviously unreasonable in itself, but is inconsistent with the assumption of alienation on which the table, by implication at least, appears to be constructed.

Moreover, the error resulting from this method of determining the enfranchisement value, when the fines on death and alienation are unequal, may be material.

For example, in the common case of a nominal fine on death,

and an arbitrary fine on alienation, the value of enfranchisement
for an age 40 in possession is:

> By Table IX, which most nearly coincides with that of the
> Commissioners,
> $$2 \times 1{\cdot}181 = 2{\cdot}362.$$
> By the Commissioners' table it is $= 1{\cdot}680.$

The Commissioners' table here shows a difference in defect of
·682 year's purchase.

Again, for an age 80 in possession the value is:

> By Table IX $2 \times {\cdot}998 = 1{\cdot}996.$
>
> By Commissioners' table it is $= 2{\cdot}405.$

The Commissioners' table here shows a difference in excess of
·409 year's purchase.

If it is borne in mind:

> *First.*—That manorial customs of unequal fines are
> numerous;
>
> *Secondly.*—That the rule given by the Commissioners for
> determining the value of enfranchisements in such cases
> is obviously unsound, and may lead to material error;
>
> *Thirdly.*—That the division of the total enfranchisement
> value, for the purpose required, can only be properly
> made when the effect of alienation is known or assumed;

then it must necessarily follow that the best, in fact the only
satisfactory, way of constructing the enfranchisement table is that
here advocated, namely, to start with the most reasonable assump-
tion of the effect of alienation, and to construct the table directly
upon that assumption.

APPENDIX.

SUMMATION OF THE SERIES REFERRED TO ON PAGE 21.

The calculation will be made more clear by abbreviating the symbols employed. Thus, let $A_{(dy)}$, $A_{(ay)}$, $A_{(dx_1)}$, $A_{(ax_1)}$, $A_{(dx_2)}$, $A_{(ax_2)}$ be replaced by Y, y, X_1, x_1, X_2, x_2, respectively.

The 1st term of the series is Y

  ,,   2nd   ,,   ,, $YX_1 + yX_2$

  ,,   3rd   ,,   ,, $Y(X_1^2 + x_1X_2) + y(X_2X_1 + x_2X_2)$

  ,,   4th   ,,   ,,

$$Y\{X_1(X_1^2 + x_1X_2) + x_1(X_2X_1 + x_2X_2)\}$$
$$+ y\{X_2(X_1^2 + x_1X_2) + x_2(X_2X_1 + x_2X_2)\}$$

On comparing the 4th with the 3rd term the following simple relation will be apparent:

If C_3, C'_3 be the coefficients of Y, y, respectively, in the 3rd term, and C_4, C'_4 in the 4th term,

$$C_4 = C_3X_1 + C'_3x_1$$
$$C'_4 = C_3X_2 + C'_3x_2.$$

This will be found to be a constant relation between the coefficients of Y and y in the successive terms, so that if C_n and C'_n be these coefficients for the nth term, the $\overline{n+1}$th term will be

$$Y(C_nX_1 + C'_nx_1) + y(C_nX_2 + C'_nx_2).$$

The successive terms may therefore be written thus:

$$\text{1st term} = C_1Y + C'_1y$$
$$\text{2nd ,,} \ = C_2Y + C'_2y$$
$$\text{3rd ,,} \ = C_3Y + C'_3y$$
$$\&c. \ = \ \&c.$$
$$n\text{th ,,} \ = C_nY + C'_ny$$

and so on to infinity.

Where $\qquad C_1 = 1 \qquad\qquad C'_1 = 0$

$$C_{n+1} = C_n X_1 + C'_n x_1$$
$$C'_{n+1} = C_n X_2 + C'_n x_2.$$

Let $\qquad S = C_1 + C_2 + C_3 +$ &c., *ad inf.*

$$S' = C'_1 + C'_2 + C'_3 +$$ &c.,   ,,

Then the sum of the whole series $= SY + S'y$.

Now $\quad C_{n+1} = C_n X_1 + C'_n x_1$

$$= C_n X_1 + (C_{n-1} X_2 + C'_{n-1} x_2) x_1$$
$$= C_n X_1 + C_{n-1} X_2 x_1 + (C_{n-2} X_2 + C'_{n-2} x_2) x_2 x_1$$
$$= C_n X_1 + C_{n-1} X_2 x_1 + C_{n-2} X_2 x_2 x_1$$
$$\qquad + (C_{n-3} X_2 + C'_{n-3} x_2) x_2^2 x_1$$
$$= C_n X_1 + C_{n-1} X_2 x_1 + C_{n-2} X_2 x_2 x_1 + C_{n-3} X_2 x_2^2 x_1$$
$$\qquad + (C_{n-4} X_2 + C'_{n-4} x_2) x_2^3 x_1$$
$$= C_n X_1 + C_{n-1} X_2 x_1 + C_{n-2} X_2 x_2 x_1 + C_{n-3} X_2 x_2^2 x_1$$
$$\qquad + C_{n-4} X_2 x_2^3 x_1 +$$ &c. $+ (C_1 X_2 + C'_1 x_2) x_1 x_2^{n-2}$

Hence, since $C_1 = 1$ and $C'_1 = 0$,

$$C_1 = 1$$
$$C_2 = C_1 X_1$$
$$C_3 = C_2 X_1 + C_1 X_2 x_1$$
$$C_4 = C_3 X_1 + C_2 X_2 x_1 + C_1 X_2 x_1 x_2$$
$$C_5 = C_4 X_1 + C_3 X_2 x_1 + C_2 X_2 x_1 x_2 + C_1 X_2 x_1 x_2^2$$
$$C_6 = C_5 X_1 + C_4 X_2 x_1 + C_3 X_2 x_1 x_2 + C_2 X_2 x_1 x_2^2 + C_1 X_2 x_1 x_2^3$$

  &c. $=$ &c.   &c.

Summing to infinity,

$$S = 1 + SX_1 + SX_2 x_1 + SX_2 x_1 x_2 + SX_2 x_1 x_2^2$$
$$+ SX_2 x_1 x_2^3 + \&c.$$
$$= 1 + SX_1 + SX_2 x_1 (1 + x_2 + x_2^2 + \&c., \ ad \ inf.)$$
$$= 1 + SX_1 + S\frac{X_2 x_1}{1 - x_2}$$

$$\therefore \quad S = \frac{1 - x_2}{(1 - X_1)(1 - x_2) - X_2 x_1}.$$

Again, $C'_{n+1} = C'_n x_2 + C_n X_2$

$$= C'_n x_2 + (C'_{n-1} x_1 + C_{n-1} X_1)X_2$$
$$= C'_n x_2 + C'_{n-1} x_1 X_2 + (C'_{n-2} x_1 + C_{n-2} X_1)X_1 X_2$$
$$= C'_n x_2 + C'_{n-1} x_1 X_2 + C'_{n-2} x_1 X_1 X_2$$
$$+ (C'_{n-3} x_1 + C_{n-3} X_1)X_1^2 X_2$$
$$= C'_n x_2 + C'_{n-1} x_1 X_2 + C'_{n-2} x_1 X_1 X_2 + C'_{n-3} x_1 X_1^2 X_2$$
$$+ (C'_{n-4} x_1 + C_{n-4} X_1)X_1^3 X_2$$
$$= C'_n x_2 + C'_{n-1} x_1 X_2 + C'_{n-2} x_1 X_1 X_2 + C'_{n-3} x_1 X_1^2 X_2$$
$$+ C'_{n-4} x_1 X_1^3 X_2 + \&c. + (C'_1 x_1 + C_1 X_1)X_1^{n-2} X_2$$

Hence, since $C_1 = 1$ and $C'_1 = 0$,

$$C'_1 = 0$$
$$C'_2 = C'_1 x_2 + X_2$$
$$C'_3 = C'_2 x_2 + C'_1 x_1 X_2 + X_1 X_2$$
$$C'_4 = C'_3 x_2 + C'_2 x_1 X_2 + C'_1 x_1 X_1 X_2 + X_1^2 X_2$$
$$C'_5 = C'_4 x_2 + C'_3 x_1 X_2 + C'_2 x_1 X_1 X_2 + C_1 x_1 X_1^2 X_2 + X_1^3 X_2$$
$$\&c. = \qquad \&c. \qquad\qquad \&c.$$

Summing to infinity,

$$S' = S'x_2 + S'x_1X_2 + S'x_1X_1X_2 + S'x_1X_1^2X_2$$
$$+ \&c. + X_2 + X_1X_2 + X_1^2X_2 + \&c.$$

$$= S'x_2 + S'\frac{x_1X_2}{1-X_1} + \frac{X_2}{1-X_1}$$

$$\therefore \quad S' = \frac{X_2}{(1-x_2)(1-X_1)-x_1X_2}.$$

But the sum of the whole series

$$= SY + S'y$$
$$= \frac{(1-x_2)Y + X_2y}{(1-X_1)(1-x_2)-x_1X_2}.$$

Replacing the original symbols, the sum of the whole series,

$$F_{dy} = \frac{\Lambda_{(dy}(1-\Lambda_{(ax_2)}) + \Lambda_{(ay)}\Lambda_{(dx_2)}}{(1-\Lambda_{(dx_1)})(1-\Lambda_{(ax_2)}) - \Lambda_{(ax_1)}\Lambda_{(dx_2)}}.$$

The value of the fines of £1 expectant on alienation may be similarly found; but it may be simply deduced from the above expression by interchanging a and d, and x_1 and x_2. Thus,

$$F_{ay} = \frac{\Lambda_{(ay)}(1-\Lambda_{(dx_1)}) + \Lambda_{(dy)}\Lambda_{(ax_1)}}{(1-\Lambda_{(dx_1)})(1-\Lambda_{(ax_2)}) - \Lambda_{(ax_1)}\Lambda_{(dx_2)}}.$$

TABLE I.—*For transforming the Carlisle Life Annuity Tables into Copyhold Annuity Tables, for various Manorial Intervals, at various Rates of Interest.*

MANORIAL INTERVALS ON ASSUMPTIONS A, B, AND C

	On Assumption A	On Assumption B	On Assumption C
(leftmost column)	25·58	25·76	32·32
	20·66	20·28	21·80
	19·36	18·89	19·74
	18·19	17·66	18·05
	17·13	16·57	16·65
	16·17	15·60	15·47
	15·31	14·73	14·46
	14·52	13·91	13·58
	13·81	13·24	12·81
	12·55	12·01	11·53
	11·49	11·00	10·51
	10·60	10·14	9·67
	9·83	9·41	8·96
	9·16	8·78	8·36

RATES OF INTEREST OF THE NATURAL TABLES WHEN USED AS COPYHOLD TABLES FOR THE VARIOUS MANORIAL INTERVALS

Rates of Interest of the Natural Life Tables	25·58 / 25·76 / 32·32	—	20·66 / 20·28 / 21·80	19·36 / 18·89 / 19·74	18·19 / 17·66 / 18·05	17·13 / 16·57 / 16·65	16·17 / 15·60 / 15·47	15·31 / 14·73 / 14·46	14·52 / 13·91 / 13·58	13·81 / 13·24 / 12·81	12·55 / 12·01 / 11·53	11·49 / 11·00 / 10·51	10·60 / 10·14 / 9·67	9·83 / 9·41 / 8·96	9·16 / 8·78 / 8·36
·00															
·50	·00														
1·00	·49	·00													
1·50	·99	·49	·00												
2·00	1·49	·99	·49	·00											
2·50	1·99	1·49	·99	·49	·00										
3·00	2·49	1·98	1·48	·98	·49	·00									
3·50	2·98	2·48	1·97	1·47	·98	·48	·00								
4·00	3·48	2·97	2·46	1·96	1·46	·97	·48	·00							
4·50	3·98	3·47	2·96	2·45	1·95	1·46	·97	·48	·00						
5·00	4·48	3·96	3·45	2·94	2·44	1·94	1·45	·96	·48	·00					
6·00	5·47	4·95	4·43	3·92	3·41	2·91	2·42	1·92	1·44	·95	·00				
7·00	6·47	5·94	5·42	4·90	4·39	3·88	3·38	2·88	2·39	1·90	·94	·00			
8·00	7·46	6·93	6·40	5·88	5·37	4·85	4·34	3·85	3·35	2·86	1·89	·93	·00		
9·00	8·46	7·92	7·39	6·86	6·34	5·82	5·31	4·81	4·31	3·81	2·83	1·87	·93	·00	
10·00	9·45	8·91	8·38	7·84	7·32	6·80	6·28	5·77	5·26	4·76	3·77	2·80	1·85	·92	·00

TABLE II.—*Enfranchisement Formulæ, at Rates of Interest from 3 to 4 per-cent, for various Manorial Intervals, on the Assumption, B, of one set of Ages of Admission, from which Inheritors and Purchasers are indifferently drawn, the Average Age being 44 Years.*

Manorial Interval Years	Copyhold Table of Expectations = Carlisle Annuity Table at Rate per-cent	Copyhold Annuity Table for determining a_y at Rate per-cent	Carlisle Annuity Table at Rate per-cent	VALUE OF ENFRANCHISEMENT			
				From Fines of £1 expectant on			When no Life is on the Rolls, and a Fine of £1 is immediately due and payable on Death and Alienation
		$=$		Death	Alienation	Death and Alienation	
				£	£	£	£
20·28	1·5	2·96	4·5	$1·941 - ·0722a_y$	$·186 + ·0005a_y$	$2·127 - ·0717a_y$	2·498
18·89	2	$\{2·94$ / $3·92$	$\{5$ / 6	$1·921 - ·0763a_y$ / $1·639 - ·0811a_y$	$·648 + ·0008a_y$ / $·484 + ·0009a_y$	$2·569 - ·0755a_y$ / $2·123 - ·0832a_y$	2·644 / 2·206
17·66	2·5	3·41	6	$1·745 - ·0813a_y$	$·693 + ·0010a_y$	$2·438 - ·0833a_y$	2·521
16·57	3	$\{2·91$ / $3·88$	$\{6$ / 7	$1·886 - ·0815a_y$ / $1·616 - ·0923a_y$	$·973 + ·0013a_y$ / $·726 + ·0011a_y$	$2·859 - ·0832a_y$ / $2·342 - ·0909a_y$	2·912 / 2·433
15·60	3·5	3·38	7	$1·719 - ·0925a_y$	$·971 + ·0016a_y$	$2·690 - ·0909a_y$	2·780
14·73	4	$\{2·88$ / $3·85$	$\{7$ / 8	$1·856 - ·0928a_y$ / $1·595 - ·1006a_y$	$1·297 + ·0018a_y$ / $·969 + ·0020a_y$	$3·153 - ·0910a_y$ / $2·564 - ·0986a_y$	3·244 / 2·662
13·94	4·5	3·35	8	$1·695 - ·1008a_y$	$1·249 + ·0022a_y$	$2·944 - ·0986a_y$	3·042
13·24	5	$\{2·86$ / $3·81$	$\{8$ / 9	$1·829 - ·1011a_y$ / $1·579 - ·1089a_y$	$1·622 + ·0025a_y$ / $1·211 + ·0026a_y$	$3·451 - ·0986a_y$ / $2·790 - ·1063a_y$	3·550 / 2·896
12·01	6	$\{2·83$ / $3·77$	$\{9$ / 10	$1·808 - ·1095a_y$ / $1·564 - ·1173a_y$	$1·917 + ·0063a_y$ / $1·454 + ·0031a_y$	$3·755 - ·1063a_y$ / $3·018 - ·1139a_y$	3·861 / 3·132

TABLE III.—*Enfranchisement Formulæ, at Rates of Interest from 3 to 4 per-cent, for various Manorial Intervals, on the Assumption, C, of two sets of Ages of Admission, one for Inheritors with an Average Age of 34 Years, and one for Purchasers with an Average Age of 54 Years.*

Manorial Interval Years	Copyhold Table of Expectations = Carlisle Annuity Table at Rate per-cent	Copyhold Annuity Table for determining a_y at Rate per-cent	Carlisle Annuity Table at Rate per-cent	VALUE OF ENFRANCHISEMENT			
				From Fines of £1 expectant on			When no Life is on the Rolls, and a Fine of £1 is immediately due and payable on Death and Alienation
				Death	Alienation	Death and Alienation	
				£	£	£	£
19.74	2	$\begin{cases} 2.94 \\ 3.92 \end{cases}$	5 / 6	$1.773 - .0671a_y$ / $1.517 - .0718a_y$	$.649 + .0007a_y$ / $.485 + .0007a_y$	$2.422 - .0664a_y$ / $2.002 - .0741a_y$	2.493 / 2.080
18.05	2.5	3.41	6	$1.635 - .0752a_y$	$.694 + .0008a_y$	$2.329 - .0743a_y$	2.409
16.65	3	$\begin{cases} 2.91 \\ 3.88 \end{cases}$	6 / 7	$1.794 - .0751a_y$ / $1.533 - .0829a_y$	$.974 + .0011a_y$ / $.727 + .0012a_y$	$2.768 - .0740a_y$ / $2.260 - .0817a_y$	2.849 / 2.348
15.47	3.5	3.38	7	$1.652 - .0881a_y$	$.972 + .0014a_y$	$2.624 - .0817a_y$	2.712
14.16	4	$\begin{cases} 2.88 \\ 3.85 \end{cases}$	7 / 8	$1.810 - .0833a_y$ / $1.545 - .0911a_y$	$1.298 + .0016a_y$ / $.970 + .0018a_y$	$3.108 - .0817a_y$ / $2.515 - .0893a_y$	3.197 / 2.612
13.58	4.5	3.35	8	$1.664 - .0913a_y$	$1.249 + .0020a_y$	$2.913 - .0893a_y$	3.010
12.81	5	$\begin{cases} 2.86 \\ 3.81 \end{cases}$	8 / 9	$1.823 - .0915a_y$ / $1.557 - .0994a_y$	$1.623 + .0022a_y$ / $1.212 + .0024a_y$	$3.446 - .0893a_y$ / $2.769 - .0970a_y$	3.514 / 2.874
11.53	6	$\begin{cases} 2.83 \\ 3.77 \end{cases}$	9 / 10	$1.835 - .0999a_y$ / $1.566 - .1078a_y$	$1.946 + .0029a_y$ / $1.454 + .0032a_y$	$3.781 - .0970a_y$ / $3.020 - .1046a_y$	3.888 / 3.134

Table IV.

Enfranchisement Table.—Assumption (C) as to ages of admission—
Two sets of ages: one with a mean age of 34 years for inheritors,
and one with a mean age of 54 years for purchasers.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 16·65 years.   Interest 2·91 per-cent.

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·718	·990	1·708
10	·709	·990	1·699
15	·733	·990	1·723
20	·755	·989	1·744
25	·783	·989	1·772
30	·816	·988	1·804
35	·850	·988	1·838
40	·893	·987	1·880
45	·936	·987	1·923
50	·995	·986	1·981
55	1·079	·984	2·063
60	1·170	·983	2·153
65	1·247	·982	2·229
70	1·343	·981	2·324
75	1·437	·979	2·416
80	1·504	·978	2·482
85	1·576	·977	2·553
90	1·624	·976	2·600
95	1·604	·977	2·581
100	1·674	·976	2·650
Fine due	1·846	1·003	2·849

TABLE IVₐ.

Enfranchisement Table.—Assumption (C) as to ages of admission—
Two sets of ages: one with a mean age of 34 years for inheritors,
and one with a mean age of 54 years for purchasers.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 16·65 years.   Interest 3·88 per-cent.

Age in Possession.	VALUE OF ENFRANCHISEMENT FROM FINES OF £1 EXPECTANT ON		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·491	·742	1·233
10	·478	·743	1·221
15	·498	·743	1·241
20	·516	·743	1·259
25	·540	·742	1·282
30	·567	·742	1·309
35	·596	·741	1·337
40	·633	·741	1·374
45	·671	·740	1·411
50	·724	·740	1·464
55	·802	·738	1·540
60	·890	·737	1·627
65	·965	·736	1·701
70	1·061	·734	1·795
75	1·156	·733	1·889
80	1·225	·732	1·957
85	1·299	·731	2·030
90	1·351	·730	2·081
95	1·330	·730	2·060
100	1·403	·729	2·132
Fine due	1·593	·755	2·348

Table V.

Enfranchisement Table.—Assumption (C) as to ages of admission—
Two sets of ages: one with a mean age of 34 years for inheritors,
and one with a mean age of 54 years for purchasers.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 14·46 years.   Interest 2·88 per-cent.

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·763	1·318	2·081
10	·751	1·318	2·069
15	·771	1·318	2·089
20	·789	1·318	2·107
25	·813	1·317	2·130
30	·841	1·317	2·158
35	·870	1·316	2·186
40	·907	1·315	2·222
45	·944	1·315	2·259
50	·998	1·314	2·312
55	1·077	1·312	2·389
60	1·165	1·310	2·475
65	1·240	1·309	2·549
70	1·336	1·307	2·643
75	1·431	1·305	2·736
80	1·501	1·304	2·805
85	1·576	1·302	2·878
90	1·627	1·301	2·928
95	1·606	1·302	2·908
100	1·680	1·300	2·980
Fine due	1·862	1·335	3·197

Table Va.

Enfranchisement Table.—Assumption (C) as to ages of admission—
Two sets of ages: one with a mean age of 34 years for inheritors,
and one with a mean age of 54 years for purchasers.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 14·46 years. Interest 3·85 per-cent.

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·526	·990	1·516
10	·513	·990	1·503
15	·530	·990	1·520
20	·544	·990	1·534
25	·565	·989	1·554
30	·588	·989	1·577
35	·613	·988	1·601
40	·645	·988	1·633
45	·678	·987	1·665
50	·726	·986	1·712
55	·800	·984	1·784
60	·885	·983	1·868
65	·957	·981	1·938
70	1·052	·980	2·032
75	1·148	·978	2·126
80	1·219	·976	2·195
85	1·297	·975	2·272
90	1·351	·974	2·325
95	1·328	·974	2·302
100	1·404	·973	2·377
Fine due	1·605	1·007	2·612

Table VI.

Enfranchisement Table.—Assumption (C) as to ages of admission—
Two sets of ages: one with a mean age of 34 years for inheritors,
and one with a mean age of 54 years for purchasers.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 12·81 *years.   Interest* 2·86 *per-cent.*

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·799	1·648	2·447
10	·786	1·648	2·434
15	·803	1·648	2·451
20	·818	1·647	2·465
25	·838	1·647	2·485
30	·863	1·646	2·509
35	·887	1·645	2·532
40	·920	1·645	2·565
45	·952	1·644	2·596
50	1·001	1·643	2·644
55	1·075	1·641	2·716
60	1·160	1·639	2·799
65	1·232	1·637	2·869
70	1·328	1·635	2·963
75	1·425	1·632	3·057
80	1·496	1·631	3·127
85	1·573	1·629	3·202
90	1·628	1·627	3·255
95	1·605	1·628	3·233
100	1·682	1·626	3·308
Fine due	1·875	1·669	3·544

Table VIa.

Enfranchisement Table.—Assumption (C) as to ages of admission—
Two sets of ages: one with a mean age of 34 years for inheritors,
and one with a mean age of 54 years for purchasers.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 12·81 *years.   Interest* 3·81 *per-cent.*

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·557	1·236	1·793
10	·541	1·237	1·778
15	·556	1·236	1·792
20	·569	1·236	1·805
25	·586	1·236	1·822
30	·607	1·235	1·842
35	·628	1·235	1·863
40	·657	1·234	1·891
45	·685	1·233	1·918
50	·729	1·232	1·961
55	·798	1·231	2·029
60	·881	1·228	2·109
65	·950	1·227	2·177
70	1·045	1·224	2·269
75	1·142	1·222	2·364
80	1·214	1·220	2·434
85	1·294	1·218	2·512
90	1·351	1·217	2·568
95	1·326	1·218	2·544
100	1·406	1·216	2·622
Fine due	1·616	1·258	2·874

Table VII.

Enfranchisement Table, for comparing the result of the two assumptions (B) and (C) as to the ages of admission, for a manorial interval of about 15½ years, or nearly that of the average manor.

Assumption as to the effect of alienation—A constant alienation rate.

Assumption (B) as to ages of admission.

Manorial interval 15·60 years. Interest 3·38 per-cent.

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·556	·991	1·547
10	·543	·991	1·534
15	·565	·991	1·556
20	·585	·991	1·576
25	·612	·990	1·602
30	·643	·989	1·632
35	·674	·989	1·663
40	·716	·988	1·704
45	·757	·988	1·745
50	·817	·987	1·804
55	·904	·985	1·889
60	1·003	·983	1·986
65	1·086	·982	2·068
70	1·193	·980	2·173
75	1·298	·978	2·276
80	1·376	·977	2·353
85	1·459	·975	2·434
90	1·516	·974	2·490
95	1·492	·975	2·467
100	1·574	·973	2·547
Fine due	1·777	1·003	2·780

Table VIII.

Enfranchisement Table, for comparing the result of the two assumptions (B) and (C) as to the ages of admission, for a manorial interval of about 15½ years, or nearly that of the average manor.

Assumption *as to the effect of alienation—A constant alienation rate.*

Assumption (C) as to ages of admission.

Manorial interval 15·47 years.   Interest 3·38 per-cent.

Age in Possession.	Value of Enfranchisement from Fines of £1 expectant on		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·607	·990	1·597
10	·595	·990	1·585
15	·616	·989	1·605
20	·633	·989	1·622
25	·657	·989	1·646
30	·685	·988	1·673
35	·713	·988	1·701
40	·751	·987	1·738
45	·786	·987	1·773
50	·842	·986	1·828
55	·920	·984	1·904
60	1·009	·983	1·992
65	1·083	·982	2·065
70	1·179	·980	2·159
75	1·274	·978	2·252
80	1·344	·977	2·321
85	1·418	·976	2·394
90	1·469	·975	2·444
95	1·448	·976	2·421
100	1·522	·974	2·496
Fine due	1·707	1·005	2·712

TABLE IX.

Enfranchisement Table.—Assumption (C), as before, as to ages of admission.

Assumption as to the effect of alienation—An alienation rate of 0 between the ages 0 and 20 years (inclusive) and between 81 and the limit of life (inclusive), and a constant rate for all intermediate ages.

Manorial interval 13·89 *years.   Interest* 3·50 *per-cent.*

Age in Possession.	VALUE OF ENFRANCHISEMENT FROM FINES OF £1 EXPECTANT ON		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·471	·710	1·181
10	·492	·821	1·313
15	·553	·968	1·521
20	·647	1·219	1·866
25	·635	1·187	1·822
30	·671	1·185	1·856
35	·695	1·183	1·878
40	·727	1·181	1·908
45	·759	1·179	1·938
50	·807	1·176	1·983
55	·881	1·170	2·051
60	·966	1·163	2·129
65	1·041	1·152	2·193
70	1·142	1·135	2·277
75	1·254	1·098	2·352
80	1·373	·998	2·371
85	1·457	·994	2·451
90	1·497	1·022	2·519
95	1·517	1·035	2·552
100	1·527	1·042	2·569
Fine due	1·696	1·158	2·854

TABLE X.

Enfranchisement Table.—Assumption (C), as before, as to ages of admission.

Assumption as to the effect of alienation—A constant alienation rate.

Manorial interval 13·58 years.   Interest 3·35 per-cent.

Age in Possession.	VALUE OF ENFRANCHISEMENT FROM FINES OF £1 EXPECTANT ON		
Years	Death	Alienation	Death and Alienation
	£	£	£
5	·643	1·271	1·914
10	·629	1·272	1·901
15	·646	1·271	1·917
20	·661	1·271	1·932
25	·681	1·271	1·952
30	·705	1·270	1·975
35	·730	1·269	1·999
40	·762	1·269	2·031
45	·795	1·268	2·063
50	·843	1·267	2·110
55	·917	1·265	2·182
60	1·003	1·263	2·266
65	1·074	1·262	2·336
70	1·170	1·260	2·430
75	1·266	1·258	2·524
80	1·337	1·256	2·593
85	1·415	1·255	2·670
90	1·469	1·253	2·722
95	1·446	1·254	2·700
100	1·523	1·252	2·775
Fine due	1·719	1·291	3·010

TABLE XI.

Enfranchisement Table, based upon the Official Table issued by the Land Commissioners, February 1888.

Average fine interval 14 *years.　Interest* 3·50 *per-cent.*

Age in Possession	Value of Enfranchisement from Fines of £1 expectant on Death and Alienation	Age in Possession	Value of Enfranchisement from Fines of £1 expectant on Death and Alienation
	£		£
5	1·145	55	1·950
10	1·215	60	2·050
15	1·290	65	2·155
20	1·365	70	2·250
25	1·440	75	2·335
30	1·520	80	2·405
35	1·600	85	2·460
40	1·680	90	2·500
45	1·765	95	2·540
50	1·855	100	2·580

DISCUSSION.

The PRESIDENT (Mr. W. Sutton), in opening the discussion, said the subject of the paper was one which did not come before actuaries as a rule in practice, but it was one which exemplified the extended area over which actuarial principles and methods might hereafter be found to apply. It was a subject of considerable interest and importance to other professions, notably to surveyors. The main point, for actuarial purposes, developed by the paper was the recognition and actuarial treatment of the fact that tenures of copyholds might be terminated otherwise than by death. Put into ordinary insurance language, it would be to the effect that a policy did not always become a claim, but sometimes was surrendered and sometimes lapsed. He could not help thinking, after closely studying the paper, that for the purposes the author had in view, and considering that he did not profess to be an actuary in any proper sense of the word, he had treated the matter most ably. That which was rightly called by Mr. Mathews for his special purpose "alienation", was known in insurance matters under its other names of "secession" or "lapse"; and the growing importance of the question of rates of

secession was very noticeable. It almost appeared that rates of mortality would, for many purposes at all events, be hereafter considered as settled, and the vital matter to be considered would be the rate of lapse, and those who had to deal with friendly societies knew what an important element this had become. It was very striking to have this question, which had come so much to the front in the last 20 years, brought to bear upon such an out-of-the-way subject as copyhold fine enfranchisement.

Mr. E. SMYTH said that the author had stated that all previous solutions had been unsound. He (Mr. Smyth) must also apply that statement to the solution now before them, and to any other solution which might be offered, because they had not those data as to the ages of copyholders which would allow them to make satisfactory copyhold annuity tables. The question was simply which author's solution was the least unsound. His own papers, alluded to by Mr. Mathews, consisted of three. The first was read in 1870 to the Surveyors' Institution, and dealt with the nature and history of copyholds, with the legislation respecting their enfranchisement, and with his own opinion as to the mode of calculation (with a brief table) which should be adopted for that purpose. Subsequent consideration satisfied him that there was a very fair amount of success in that table, although amending the views with which he had broadly supported it in that tentative paper. His revised arguments were fully detailed in a second paper, read before the Institute of Actuaries (*J.I.A.*, xxi, 381) in 1879, which included an expansion of the table. A brief epitome of those arguments and a reprint of the table were accepted by the Surveyors' Institution in 1885; but the actual subject of that paper was the Copyhold Enfranchisement Bill of 1884–5, of which it was a hostile criticism, mainly on two grounds. One was the design of that bill to enforce enfranchisement in just those cases in which the lord and the copyholder both wished to be let alone, which tyrannical provision was afterwards swept out of the bill by a Select Committee of the House of Lords. The other was that a phrase in the bill would have led to the perpetuation of the official table of 1855 (since defunct). The table by which that was officially superseded last year was stated on its face to assume the fine on the admission of a tenant as being two years' value of the property; and that was the legal maximum. Mr. Mathews had proceeded to point out that in a majority of manors the fines were unequal; and he subsequently mentioned the case of the customary heir paying only a nominal fine, but there were very few manors in which that custom existed. The other instance mentioned in the paper was that the lords of most manors had been in the habit of requiring a rather smaller fine in the case of purchase than in that of inheritance, namely, one and a half instead of two years' value; but on adopting the Land Commissioners' estimate, that (in the aggregate) fines after inheritance were about equal in number to fines after purchase, and then adopting their further recommendation of varying their tabular years' purchases to fit the consequent conclusion that the average fine would be not two but one and three-quarters years' value, the amount of error still remaining after an adjustment of this nature would be very small as compared with the uncertainties

enveloping the whole subject. Mr. Mathews' calculations began with
extensions of the formula for the value of successive lifeholds; but the
ultimate regard for them must be governed by a consideration of the
assumptions which he afterwards interweaved with them as substitutes
for the statistics wanting as to copyholders' ages. In the "Assumption
of the Effect of Alienation", the *crux* of the case, instead of his
(Mr. Smyth's) having agreed with—as supposed in the paper—he
had differed from the view that the probability of alienation might
be taken as the same at every age. At his outset he had indeed
attached an exaggerated importance to the cases of some purchasers
deferring their admission, but in his second paper (that in the
Journal) he had clearly shown that he assented to the late Mr. E.
J. Smith's opinion that while it was ordinarily the older persons
who purchased estates it was the younger who disposed of them.
Mr. E. J. Smith had felt this so strongly as to support it by the
statement of his experience being that "the average age of persons
purchasing over those selling estates was fully half as much again";
and he (Mr. Smyth) had proceeded in that paper to show how his table
met the apparent fact that (notwithstanding occasional cases of the
procrastination just referred to) alienations occurred most frequently
among the young. Mr. Mathews had adopted, for the construction
of his copyhold tables, the different assumption that "the probability
of alienation was the same at every age", which assumption enabled
him to make that highly ingenious though loosely-fitting use of a
table of the value of a life-annuity at such rates of interest as
happened to have been computed. Mr. E. J. Smith was, however, a
surveyor of large and varied experience and a shrewd observer. His
novel statement as to the ages of vendors appeared to him
(Mr. Smyth) to be well corroborated by considering how often it
must happen that the heir was located differently from his prede-
cessor, and for that or other reasons felt less attraction towards the
property; and that the realization of its value would be further
prompted very often by a desire to invest money in business in
which he might be already engaged, or might wish to embark. He
must, therefore, contend that the computations and tables which
Mr. Mathews, with great skill, had created upon his stated assumption
of "a constant alienation rate" were deficient as regards practical value.
The next matter in the paper was the greater or less frequency of
fines, first drawn attention to by Mr. Rouse, who had recommended
an examination of the court rolls of the manor under which the
copyhold was held, to see what had been the fine interval in that
manor, and which he deemed would be found to range from 13 to 18
years. He (Mr. Smyth) had always argued that it was better to
regard the frequency of alienation as dependent on the nature of the
locality of the manor; and careful investigations had shown him five
typical cases of fine interval, ranging from 12 to 20 years. It was
satisfactory to him to observe that his conclusions had been adopted
by Mr. Mathews. This variety of fine interval, which, in the
magnitude of its effect, was really the most important matter of all,
was again disregarded by the Commissioners in their new table, the
last subject of the paper. The table was not obligatory, yet he would
suggest its adding to the interest of the discussion if the table and its

accompanying directions were published *in extenso*.* With regard to
its graduation for successive ages, the table showed a virtual agree-
ment, for some reason, with his (Mr. Smyth's) own conclusion, that
at the age of 35 the charge for enfranchisement should be a single
fine (two years' value), less than the value of the series of fines, if
one were immediately payable, for it made precisely this allowance
at the age 36.   He wished time permitted him to argue that this
allowance at the age 35 or 36 might be regarded as right, not
merely in the official table for its single fine interval, but right also
for all fine intervals whatever as in the successive columns of his own
table; and, at the same time, to show that it was just in this way
that his table allowed for the admission ages, varying according to the
variation of fine intervals which Mr. Mathews had in view.   It was
a circumstance that was regarded by both of them, although they had
approached the enfranchisement problem in different ways.   [Some
further observations by Mr. Smyth are given on page 67.—ED. *J.I.A.*]

Mr. T. B. SPRAGUE said they had heard that the subject of these
fines and enfranchisements did not often come before the actuary.
That, he supposed, agreed with the professional experience of most of
them.   Nevertheless, in the course of his life the matter had come
before him on various occasions, and in various capacities.   He had
for a good many years been resident in Scotland, and there the
subject had been brought to his notice, though under a different
name.   They did not hear anything in Scotland about copyhold
estates and fines, but they did hear of feus and casualties, and these
were substantially the same things.   In Scotland it was customary
for the owner of land, who was there called the superior, to let it in
perpetuity, subject to what in England would be called a ground
rent, but in Scotland a feu duty, and the person who paid the feu
duty was called the feuar, or the vassal.   Just as in England, when
a life in a copyhold dropped, a fine had to be paid and a new life put
in, so it was in Scotland, but the fine was there called a casualty.
When he bought his house in Edinburgh he had to pay not only
an annual feu duty, but he found upon the death of the vendor
that he had also to pay a casualty, or fine, equal to a year's feu
duty.   Just as in some manors in England, one fine was payable
upon the death of the tenant, and another upon alienation, so
in Scotland sometimes there was one casualty payable on the
succession of heirs, and another in the case of what are called
singular successors.   The author had asked whether it was right and
just to the lord that the life of the old tenant should be left in the
lease.   In doing so he had, perhaps, gone a little beyond the fair
scope of his paper: he had seemed, to a certain extent, to be pleading
in the interest of the lords to extend their feudal rights and ancient
customs as against the tenants, who appeared to him (Mr. Sprague)
to be more deserving of consideration.   These subjects of copyholds
were apparently remnants of the old feudal system, which had
remained in force in Scotland much longer than in almost any other
country.   But the incidents seemed to be very much the same in
Scotland in their substance as they were in England.   The subject

* These are given in full on pages 69-71.

had come under his notice in another way.   Some years ago he bought a reversion to a small estate in Berkshire, about 26 acres. When the life-tenant died, and he went to take possession, he found there were various fines to pay.   Included in those 26 acres there were, he was told, three copyhold estates of two or three acres each. There was no doubt about their existence, but nobody could tell their boundaries.   It appeared from the rolls of the manor that there were these copyhold estates with certain rights to the lord of the manor, and he had to pay the fines.   This was not the worst of it, for he was told that the property was subject to a heriot—a most objectionable custom.   He found that, in consequence of the existence of this custom, upon his death his best beast would be liable to seizure— not merely the best beast he might happen to have on that estate, but in London or elsewhere, the best cow on his little farm, or his best carriage horse; and if he did not happen to have any beast, then they might come and seize his best piece of furniture—perhaps worth more than the capital value of the land in respect of which the heriot was claimable.   On inquiry it turned out that the lord of the manor had been in the habit of receiving a composition of £20 instead of the best beast, this having been calculated with regard to the social circumstances of the people who formerly held the land, but there seemed to be nothing to prevent the lord insisting on his full pound of flesh.   These things were all relics of barbarism, and ought to be swept away.   It would be very interesting if some gentlemen could put together some statements upon the origin of these feudal customs, whether in England or in Scotland.   There must be a great deal of available information in the rolls of the different manors, and if surveyors or stewards of manors who had access to them would publish their information it would be a matter of extreme interest.

Mr. A. H. BAILEY thought there was very little doubt as to the origin of these customs, which, as Mr. Sprague had remarked, were remnants of feudal times.   Copyhold tenure was a survival of military service.   The landlord in feudal times had such service from those who regarded him as their chief; as time went on and military service was no longer required there was substituted a money payment in the shape of fine and heriot.   He had occasionally been asked to give opinions as to the value of copyhold fines, but in doing so he never thought of any contingency but that of death.   The various complications which the author had introduced had alarmed him, and he would hesitate before giving another opinion.   He thought that the bill to which Mr. Smyth had alluded had for its object to get rid of this feudal tenure by something like a compulsory process, and that therefore the value of the fines depended not merely on death or alienation, but on the actual age of the copyholder, because he had no intention either of selling or of dying.

Mr. SMYTH remarked that the lord could compel the tenant to enfranchise whenever the lord chose, and the tenant also could compel the lord to enfranchise whenever the tenant chose.   Therefore, what third operation was necessary he never could understand.

Mr. BAILEY said that in that case there would be a third element of consideration besides death or alienation, which would further increase the already sufficient complication which Mr. Mathews had brought before them.

Mr. R. P. HARDY said they learnt one thing from the paper, namely, that it was extremely dangerous for any London actuary to attempt to put a value upon these things without possessing much more legal knowledge than he was ever likely to have, because otherwise he might not only do considerable injustice to his client, but also get himself into considerable scrapes. Whenever such a case came before him again he should send it on to Mr. Mathews. He thought many of them would be astonished to find that these feudal services were still allowed to encumber the circulation of property. Here were rights which they were required to measure, but if two such eminent and experienced gentlemen as Mr. Mathews and Mr. Smyth could not agree how to do it, it was unlikely that the Institute of Actuaries could find a way out of the difficulty. The procedure was a very cumbersome one. The lord and the tenant each appointed valuers, the details for valuation had to be set out and submitted to the commissioners; and considering the varied nature of the elements which came into determination of this question, it was difficult, without a much larger experience than at present existed, to deduce a rate fairly applicable to the case of any particular manor. The author stated that the rate of succession was constant at all ages and at all periods after admission on the roll. The rate of succession was a function not merely of present age, but of the period of observation, and it was by no means constant, especially with regard to property which was changing in value every day. It was almost impossible to arrive at anything like a fair rate of succession. The author had given them what he said was a safe and reasonable approximation. He (Mr. Hardy) was not sure that any man could do better, and thought they were very much indebted to Mr. Mathews for the formula he had given.

Mr. T. Y. STRACHAN said the practical part of the paper did not come within their knowledge as actuaries, but that did not reduce their admiration for the way in which Mr. Mathews had elucidated the general principle and solved the problem. He felt that there was, perhaps, an attempt in the paper to find data which did not exist. From the little knowledge he had, he should have gone to the manor rolls and not distinguished death and alienation, unless death and alienation required different fines. He should want to know how often the interval occurred, what the lord obtained on an average, and have attempted to capitalize, in a rough and ready way. With the formulas before them they could see the many difficulties, and they would be ready to send copyhold cases to the surveyors. The difficulties as to the term of alienation, the manorial intervals, were, he presumed, affected not only by different manors, but by different customs in different manors. He knew of a custom in a manor where upon the death of an old tenant or upon sale the new tenant was not placed upon the roll. A solicitor often got himself put upon the roll as a purchaser, and thereafter, when a client came to take a conveyance of a copyhold, the solicitor with the conveyance to himself gave a declaration that he held the property in trust for the real purchaser; that purchaser might sell it again, and then there was another declaration of trust, and so there were very many transactions of sale in that particular manor not recorded on the manorial roll.

If a fine was payable, he could understand that the custom might spread. This was an illustration of the difficulty that arose in finding the data necessary to put the formula into practice.

Mr. MATHEWS (in reply) said that Mr. Smyth had referred to his statement that all previous solutions were unsound, and had added that his (Mr. Mathews') proposed solution was also unsound, and necessarily so in consequence of the insufficiency of the data. This criticism he felt bound to dispute. He called a solution sound if it was a correct result of the premises laid down. He had been trying to find out from the discussion whether this was the opinion of the meeting on his solution, and thought he was entitled to assume so, since no criticism had been made on the principle of his method. It was true that the necessary data were wanting, and therefore some assumptions were necessary. His point was that as these assumptions must be involved in any solution, they ought to be made definitely and explicitly, and the valuation founded upon them. Mr. Smyth's observations were rather directed in support of his (Mr. Smyth's) own previous method than a criticism of the principle suggested in the present paper. He (Mr. Mathews) could not agree with Mr. Smyth's method, but felt a difficulty in discussing it at that meeting as the discussion was intended to be upon the paper now before them. He did not quite gather how far Mr. Smyth disagreed with the assumptions made to give practical effect to the principle of valuation. These assumptions were, however, matters of detail. The principle might be applied to any other assumptions, and though he was prepared to defend those made, he was quite willing to admit that upon them there was room for considerable difference of opinion. They could, however, be scarcely discussed in this Institute, they must rather be left to experts on the subject of enfranchisement. Mr. Bailey complained that he had introduced various complications into the subject. The complications were, however, there to start with, and his object was to disentangle them and give to each its appropriate place and weight. He had, in fact, endeavoured to lay down a principle of valuation, sound in itself, and which could be applied alike to different manorial intervals and all ordinary manorial customs. This could only be done by dividing the enfranchisement table for different intervals into two parts, one for fines expectant on death, the other for fines expectant on alienation; and this could only be done by some precise explicit assumption of the effect of alienation. With reference to Mr. Hardy's remark as to the difficulty of arriving at a fair rate of secession or alienation in any particular manor, he appreciated this difficulty. It might be impossible to deduce this with accuracy, but some rate or other must be assumed, even if it had to be guessed at; and, in fact, in whatever way the enfranchisement value was obtained, the alienation rate must be involved in the result.

As the time for the discussion after the reading of the paper
did not permit Mr. Smyth's observations to be made *in extenso,*
it was arranged that the remainder of the remarks he desired
to make should be appended to the report of the discussion.
They are as follows:

In the section headed "Assumption as to the Ages of Admission",
there is a reference by Mr. Mathews to a passage in the *Journal*
(*J.I.A.*, xxi, 392), in which I had explained my acquiescence in
Mr. Rouse's estimate that the mean age of copyholders at their
admission was about 44. The explanation consisted of showing why
32 might be regarded, upon an average, as the age of an heir at the
death of his father; and if heirships were (as appeared to be the
Commissioners' experience) about equal in number to purchases, then
it would be inferred from Mr. Rouse's 44 that the average age of
purchasers would be 56, and this was possibly not too old. "Possibly
not too old", or, probably rather too much so. On the other hand,
32 may be slightly too young for the average age of heirs of all
kinds, because, for example, a childless copyholder may prefer to
devise his copyhold to a brother rather than to a nephew. Yet a
slight diminution of "56", and a slight increase of "32", would
still leave, as their mean, Mr. Rouse's 44, to which in 1879, in the
passage above referred to, I stated my assent as being the probable
average admission age. Now, in that paper (pp. 390–5) I have
argued in detail why, instead of treating the average admission age as
indicating the tenancy of mean duration—and therefore calling for an
enfranchisement charge of one fine, or two years' purchase, less than
the value of the immediate series of fines, or "series-due"—the
assignment of this deduction (or discount of the series) to some
younger age, certainly not so young as Mr. Rouse's best possible age,
but (say) to 35, happens to constitute an allowance very apt, in both
nature and quantity, for the circumstance that the probability of a
copyhold tenant's alienating his interest does not increase with the
probability of his dying (as would have been implied in that rejected
treatment), nor yet is constant at all ages (as assumed in the paper
of Mr. Mathews), but, as pointed out by Mr. E. J. Smith, is greatest
among the young. However, I can imagine someone saying that my
said argument for treating the age 35 as corresponding with the
tenancy of mean duration, is based upon regarding 44 as the average
admission age when deaths and alienations are equally frequent; and,
as this equality is seen to be the Commissioners' opinion as regards
manors in the aggregate, it may well be conceived to happen in the
case of such fine-interval column in my table as is about central,
seeing that this is the mean of a number of intervals indicated by
observations of fines arising in the various classes of locality. But,
no matter what may be the precise fine-interval in which such equality
exists, it is incumbent upon me to show that 35 has been equally
appropriately treated in my table as corresponding with the tenancy
of mean duration in all the other fine-intervals; and this, notwith-
standing the fact that the average admission age, instead of being
always such 44, must be different in every one of these intervals,

according to the relative numbers, respectively, of elderly purchasers
and younger heirs. Now, in illustration of there being no *primâ
facie* objection to such uniform selection of the age 35 as that for a
deduction of the single fine, I had pointed out (p. 395) that
" although a uniform deduction, it is not a uniform allowance; but
" the discount is a smaller or larger percentage of the immediate
" series, consistently with the circumstance of the immediate series
" being nearer when the alienation fines are frequent, or further off
" when the alienation fines are rare. Thus, to a tenant, aged 35,
" the table will be seen to charge, for the enfranchisement of house
" property, about three-fifths of the immediate series, if the fines are
" deemed to happen so frequently as 13 years; but only about one-
" half of the immediate series if the fine-interval is 18 years."
However, I now meet the theoretical requirement which I have just
supposed to be started. It is the case that we may expect that in a
locality where the fine-interval is shortened through purchases being
more frequent than inheritance, the *greater than* the above-mentioned
44 will be the average admission age; but then my adoption of—not
that greater age, but—35 as corresponding with the tenancy of mean
duration, makes in this case the greater allowance for comparative
youthfulness in vendors which is suitable to their being here pro-
portionately more numerous through alienations outnumbering heir-
ships. On the other hand, in a locality where the fine-interval is
lengthened through purchases being less frequent than inheritance,
the *less than* the above-mentioned 44 will be the average admission
age; but then my adoption of—not that less age, but—35 as corre-
sponding with the tenancy of mean duration, makes in this case the
smaller allowance for comparative youthfulness in vendors which is
suitable to their being here proportionately less numerous through
heirships outnumbering alienations. Test this by supposing the
addition of about as many as 10 columns in continuation of my
enfranchisement table in order to carry to the utmost limit the latter
class of cases, and so reach the longest conceivable fine-interval,
namely, one arising from there being no purchases at all, but each
tenant holding on till his death. Vendors, therefore, having at this
point completely disappeared, their ages would cease to need any con-
sideration whatever, and the tenancy of mean duration would be
indicated simply by the average admission age of heirs (whose mean
after-lifetime, too, would be the fine-interval); it is therefore at this
average admission age that the deduction of two years' value should
in this case be allowed. Now, as in my printed table, so also in its
supposed extension, the deduction of such two years' value would, in
all fine intervals, be made at the age 35; and it has been shown, in
the opening portion of this minute, that the average admission age of
heirs—at which that deduction should, as just seen, be made in
the extreme case under consideration—is also very nearly, if not
exactly, 35.

COPYHOLD ENFRANCHISEMENT.

SCALE OF COMPENSATION *in ordinary cases of Enfran-*
    chisement of Copyholds of Inheritance, framed by the
    Land Commissioners for England, pursuant to section
    30 *of the Copyhold Act,* 1887.

In fine arbitrary cases when a fine is payable on Fine arbitrary cases.
alienation by, as well as on the death of, a tenant, the
compensation for fines should not exceed the number of
years' annual value of the property according to the age
of the tenant as set forth in the table hereto annexed.

The table is calculated on the principle that a fine of
two years' annual value is payable on each change of
tenancy; therefore, in those manors in which the
customary fine on alienation by, or on the death of, a
tenant is less than two years' annual value, a propor-
tionate reduction should be made in the amount of the
compensation.

In estimating the annual value of the property, no
deduction should be made for land tax, but the quit
rent should be deducted, and, where there are buildings,
allowance should be made for keeping the buildings in
repair. The gross annual value of the land for the poor
rate assessment may be used, when applicable, as the basis
for ascertaining the annual value.

When there are facilities for improvement or the land
has present or prospective building value, one twenty-fifth
part of the fee simple value may be taken as the annual
value.

In fine certain cases when a fine is payable on alienation Fine certain cases.
by, as well as on the death of, a tenant, the compensation
for fines may be calculated by multiplying the amount of
the fine by one-half of the number of years' purchase
given in the table according to the age of the tenant.

The amount of compensation for a relief, if payable, Reliefs.
to be calculated in like manner as a fine certain.

The compensation for a heriot payable on alienation Heriots.
by, as well as on the death of, a tenant, may be calculated
by multiplying the value of the heriot by one-half of the
number of years' purchase given in the table according to
the age of the tenant.

The value of a heriot may generally be ascertained
from the average value of the last three heriots taken or
paid in respect of the property to be enfranchised. If
that information cannot be obtained, or will not apply,
the following circumstances should be taken into con-
sideration in fixing the value of a heriot: namely, the
nature of the heriot, the character and value of the
property, the condition in life of the tenant, and also
whether the heriot can be seized as well without as within
the manor.

When fine payable only on one of the events of alienation or death.

The table being calculated on the assumption that fines and heriots are payable both on alienation by, and on the death of, a tenant, when a fine, whether arbitrary or certain, or a heriot, is payable only on one of those events, then only one-half of the compensation calculated as previously directed should be given.

When fine payable on death of lord.

In manors in which fines or heriots are payable on the death of the lord, as well as on alienation by, or on the death of, a tenant, the compensation on enfranchisement should be increased according to the nature and amount of the customary fine or heriot payable in the manor on the death of the lord.

Quit rents and other annual payments.

The compensation for quit rents, free rents, and other annual rents, services, or payments, should be calculated at 25 years' purchase.

Timber.

Compensation for timber should be ascertained as follows: When, by the custom of the manor, the lord can enter upon the land, and cut and carry away the timber without the consent of the tenant, its whole value, after making a sufficient allowance for repairs, should be given to the lord. But if the lord cannot enter and cut without the consent of the tenant, one-half only of its value, after making a sufficient allowance for repairs, should be given. If, however, there be any special custom in the manor relating to timber, such custom should be regarded.

Forfeitures, &c.

The compensation for forfeitures and all other incidents of copyhold tenure not hereinbefore provided for should not exceed 20 per-cent of the annual value of the property. The gross annual value for the poor rate assessment may be used, when practicable, as the basis for ascertaining the annual value. If the property has facilities for improvement or building, one-twenty-fifth part of the fee simple value may be taken as the annual value.

Escheat.

The right of escheat being reserved to the lord under the Copyhold Act, 1887, its value is not to be taken into consideration.

Special customs or circumstances.

If there be any special customs or circumstances connected with any manor which would affect the compensation payable for enfranchisement, they should be taken into consideration, and due allowance should be made in respect of them.

This scale is for guidance only, and is not binding as a matter of law in any particular case; but the party requiring enfranchisement should, in accordance with the Act, state to the other party whether or no he is willing to adopt the scale.

Table referred to in the foregoing Scale of Compensation for Enfranchisement.

Age of Tenant	Number of Years' Purchase	Age of Tenant	Number of Years' Purchase	Age of Tenant	Number of Years' Purchase
5 or under	2·29	37	3·26	70	4·50
		38	3·29	71	4·54
6	2·32	39	3·33	72	4·57
7	2·34	40	3·36	73	4·60
8	2·37	41	3·40	74	4·63
9	2·40	42	3·43	75	4·67
10	2·43	43	3·46	76	4·70
11	2·46	44	3·50	77	4·73
12	2·49	45	3·53	78	4·76
13	2·52	46	3·57	79	4·78
14	2·55	47	3·60	80	4·81
15	2·58	48	3·64	81	4·83
16	2·61	49	3·67	82	4·85
17	2·63	50	3·71	83	4·88
18	2·66	51	3·75	84	4·90
19	2·69	52	3·78	85	4·92
20	2·73	53	3·82	86	4·94
21	2·76	54	3·86	87	4·95
22	2·79	55	3·90	88	4·97
23	2·82	56	3·93	89	4·99
24	2·85	57	3·97	90	5·00
25	2·88	58	4·01	91	5·02
26	2·91	59	4·06	92	5·03
27	2·94	60	4·10	93	5·05
28	2·97	61	4·14	94	5·06
29	3·00	62	4·18	95	5·08
30	3·04	63	4·23	96	5·10
31	3·07	64	4·27	97	5·12
32	3·10	65	4·31	98	5·13
33	3·13	66	4·35	99	5·15
34	3·16	67	4·39	100 or upwds.	5·16
35	3·20	68	4·43		
36	3·23	69	4·47		

In constructing this table a fine arbitrary on admission has been taken as equivalent to two years' annual value, and whilst the average fine interval has been assumed to be 14 years, regard has been had to the age of the tenant on the rolls.

The Land Office,
   3 St. James's Square, London, S.W.
      February 1888.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, April 1889.

Examination for Admission to the Class of Associate (Part 1).

Examiners—H. W. Andras, Esq.; J. McGowan, Esq., B.A.; T. E. Young, Esq., B.A.

First Paper.

ARITHMETIC AND ALGEBRA.

1. Simplify

$$2 + \frac{25}{\cdot 625} + \frac{\cdot 05}{\frac{1}{20} \text{ of } \cdot 05} - \frac{\cdot 025}{\frac{1}{2} \text{ of } \frac{9}{10}}.$$

2. Multiply by the contracted method $348 \cdot 8414$ by $51 \cdot 30742$, reserving only four decimal places in the product, and explain the rationale of the method.

3. State the rules for the position of the decimal point in the product and quotient of multiplication and division of decimals respectively. By the use of algebraical symbols show the rationale of the rules.

4. If £7,927. 10s. be laid out in purchasing 3 per-cent stock at $98\frac{3}{8}$, what income will be derived from this investment after deducting income tax at $7d.$ per pound?

5. State the rule for the extraction of the square root, and find the square root of the sum of the squares of $\cdot 2$, $\cdot 4$, $\cdot 6$, and $\cdot 86$.

6. A number consists of 6 digits, of which the last to the left hand is 1. If the number is altered by removing the 1 and putting it in the unit's place, the new number is three times as great as the original one. Find the number.

7. Solve the equation

$$\left.\begin{array}{l} x + y + z = 6 \\ x^2 + y^2 - z^2 + 2xy = 24 \\ x - y = 1 \end{array}\right\}$$

8. Show by general reasoning that the whole number of ways in which a selection can be made out of n different things is $2^n - 1$.

9. Expand $(1-x)^{-4}$ to 5 terms, and write down the coefficient of x^m.

10. If a, β, be the roots of the equation $3x^2 + 6x + 2 = 0$, show that the equation whose roots are $-\dfrac{a^2}{\beta}$ and $-\dfrac{\beta^2}{a}$ will be $3x^2 - 18x + 2 = 0$.

BOOK-KEEPING.

11. Explain briefly the principle of book-keeping by double entry.
12. What is the use of a Journal in double entry?

Second Paper.

EUCLID (Books I to IV, and VI, and Definitions of Book V).

1. If two angles of a triangle be equal to one another, the sides also which subtend or are opposite to the equal angles shall be equal to one another.

2. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

3. If two isosceles triangles are on the same base, the straight line joining their vertices or that straight line produced will bisect the base at right angles.

ARITHMETIC AND ALGEBRA.

4. The co-efficient of the third term in the expansion of $(1-x)^{-n}$ is $\frac{2}{9}$; find n and the co-efficient of the fifth term.

5. Define a logarithm.

Given $\log 3 = \cdot 4771213$, find the log of

$$\frac{\left(1+\dfrac{143}{100}\right)^{\frac{3}{4}} \times (\cdot 81)^{\frac{7}{8}}}{\left(2+\dfrac{529}{100}\right)^{\frac{4}{5}} \times (3\frac{1}{3})^{\frac{1}{4}}}$$

6. Prove that the arithmetic and geometric mean between a and b, and the reciprocal of the arithmetic mean between $\dfrac{1}{a}$ and $\dfrac{1}{b}$, are in geometric progression.

7. From a bag containing 2 guineas, 3 sovereigns, and 5 shillings, a person is allowed to draw 3 coins indiscriminately. Find the value of his expectation.

8. What is the probability that the first card dealt out from a full pack will be a spade; and that being so, that the second will be a heart?   And a spade and a heart being already out, what probability remains that the third will be a spade?

FINITE DIFFERENCES.

9. Show how to obtain u_{x+n} in terms of u_x and its successive finite differences.

10. Express the nth leading difference of a series u_0, u_1, u_2, &c., in terms of the successive values of the function u.

11. (a) Given $u_0 = 2,844$,
$u_1 = 2,705$,
$u_2 = 2,501$,
$u_3 = 2,236$,
find $u_{\frac{1}{5}}$.

(b) Find by Finite Differences the first 10 terms of the series 1, 3, 5, 13, 33, &c.

It is optional on the part of the Candidate to answer one or more of the following questions; but due weight will be given to answers sent in.

PLANE CO-ORDINATE GEOMETRY.

12. Find the equation to a straight line
> (1) In terms of the intercepts on the axes.
> (2) In terms of the perpendicular from the origin and the inclination of this perpendicular to one of the axes.

13. Find the equation to the straight line which passes through two given points, whose co-ordinates are x_1, y_1 and x_2, y_2 respectively.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE, OR TO THE CLASS OF FELLOW (PART II).

Examiners—F. E. COLENSO, Esq., M.A.; G. H. RYAN, Esq.; W. J. H. WHITTALL, Esq.

First Paper.

1. A Company holds 10,000 Colonial Government 5 per-cent debentures redeemable at par in seven years, and standing in its books at 10,097. It is offered conversion into 4 per-cent inscribed stock at the rate of 111 for every 100 of debentures. Assuming that it may count upon realizing the stock at par in seven years' time, and in the meanwhile reinvesting at 4 per-cent, what difference in the rate of interest yielded by the investment during the term would the conversion make?

2. Obtain an expression for the amount of an annuity-certain for a term of n years, the annuity being payable k times a year, and interest being convertible m times a year. To what limit does the expression tend when k and m are indefinitely increased?

3. A loan of 1 is repayable in n years by an annuity of k which provides a (varying) sinking fund improving at rate j per annum, and yields the lender interest on his capital at rate i per annum for the first $\frac{n}{2}$ years, and i' per annum during the remainder of the term. Find k.

4. Find the probability that x will die in the nth year, y having predeceased him by at least t years, and z surviving him by at least t years.

5. What is the distinction between the column l_x of the life table and a column showing the corresponding population as enumerated at a census according to ages attained?

How can the former column be obtained from the latter?

6. The several columns of Table III in Hardy's *Valuation Tables* are headed:—

> Probable lifetime.
> Average duration of life.
> Average duration of life of those living at age x and upwards.
> Average age of those living at age x and upwards.
> Average age at death of those now living at age x.
> Average age at death of those now living at age x and upwards.

Explain how the series of values for these columns are obtained.

7. Deduce the approximate relation $a_x^{(m)} = a_x + \dfrac{m-1}{2m}$, and give the formula for the premium for a deferred annuity to x commencing after t years, both premium and annuity being payable m times a year.

8. Give the formulas for the value of a policy in terms involving

(1) P_x, P_{x+n}, and a_x;

(2) P_x, A_{x+n}, and i.

9. Given $A_x = \frac{1}{2}$, establish the following identity:

$$P_x = ({}_nV_x^{-1} - 1)P_{x+n} - {}_nV_x^{-1}d.$$

10. Investigate an expression for $A^2_{x:y}$, the value of an assurance payable at the end of the year in which x shall die provided he die after y.

11. A company is willing to forego payment of a constant extra annual premium of 1 per-cent on the sum assured in consideration of a reduction in the sum assured in case of death occurring within t years (where t is equal to the expectation of life), the reduction commencing at k and running off by $\dfrac{k}{t}$ each year. How would you proceed to find k?

12. Show how to construct a table of values of endowment assurance policies.

13. A borrower is given the choice of two methods of repayment of a loan granted for n years:—

(a) Besides the annual interest, he may pay the premium for an endowment assurance for n years to refund the principal.

(b) The amount advanced being repayable by an annuity-certain (including interest and sinking fund) for $n-1$ years, he may, in addition to this annuity, pay while he lives an annual premium to provide the annuity for the remainder of the term after his death.

Show algebraically that the total annual expenditure on the part of the borrower will be the same in each case, assuming net premiums only, and employing one rate of interest throughout.

The following question is optional on the part of the Candidate.

a. Express $\dfrac{d^n u}{dx^n}$ in terms of the successive differences of u.

Second Paper.

14. Give the formula for the value of an annuity to continue as long as *at least* r lives out of m lives survive.

Hence, find the value of an annuity payable while *exactly* r lives out of m lives are in existence.

15. The owner of a lease for the longest of three lives, x, y and z, has the right to renew upon the first death by substituting a new life aged w. Show how to obtain the general term of the series expressing the value of the lease.

16. Explain how you would proceed to prepare a mortality table from the records of a Life Assurance Company.

17. Define the "Force of Mortality," and show how its value may be approximately calculated.

If two mortality tables are formed by means of the formula $\mu_x = A + Bc^x$, and the constant B has the same value in each case, while the constant A is very much larger in one case than in the other, what conclusions would you draw as to the relative magnitude of the respective policy-values?

18. Show how to construct a premium conversion table in which $\bar{a}_x$ is the argument and $\bar{A}_x$ the result.

What would be the result of entering the table inversely with $e^{-n\delta}$?

19. A policy, subject to an annual premium of P, is issued on the life of a child aged x, for a sum payable on the attainment of age $x+n$, the total amount of premiums received, without interest, being returnable if death occur earlier. Assuming P to bear no loading, give an expression for the value of the policy after $n-t$ years.

20. Prove that

$$_nV_x + P_x = v - vp_{x+n}(1 - {}_{n+1}V_x),$$

and give an explanation of the term "Death Strain."

21. An annuity society is formed in which members may secure an annuity of m at age $x+n$ by payment of a single sum at age x. If k members aged x start the society and l new members of the same age join each subsequent year, find the total amount that will be payable in annuities at the end of $n+t$ years.

22. Give formulas for obtaining a table of net half-yearly premiums from the annual rates:—

>   *First*, if any unpaid instalment of a year's payments is to
>   be deducted from the policy-moneys at death;
>   *Secondly*, if no such deduction is to be made.

23. What do you understand by Select Mortality Tables? State what you know of Mr. A. J. Finlaison's Government Annuitants' Tables (published in 1884).

24. Prove that $_nV_x$ decreases as the rate of interest increases.

25. At the commencement of a given year, a company has on its books l_x persons who have been assured for 1 each for n years, and of whom a certain section will be subject throughout the year to a special rate of mortality, $q_x + w_x$. In respect of all claims occurring within this section during the given year, the company undertakes to pay only the reserves held against the policies. What profit may be expected to accrue from this arrangement?

The following questions are optional on the part of the Candidate.

b. Define the differential coefficient of a function, and apply your definition to obtain the differential coefficients of $\bar{a}_x$ and A_x.

c. State the method of integration by parts, and employ it to prove that $\bar{M}_x = D_x - \delta N_x$.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

Examiners—M. N. ADLER, Esq., M.A.; J. CHISHOLM, Esq.; A. J. FINLAISON, Esq., C.B.

First Paper.

1. Galloway's system of collecting the mortality experience of a life assurance society has been described as proceeding on the basis of the experience of "policy years", while the system adopted for the Institute Tables is described as proceeding on the basis of "calendar years."

What do you understand by the above-mentioned descriptions, and what is your opinion of the relative merits of the methods in question?

2. What orders of differences are retained in Woolhouse's formulas for the adjustment of the Institute Tables—

      (*a*) For the ages 17 and upwards?
      (*b*) For the ages 10–16?

Is there any, and if so what, distinction between the above-mentioned two systems of adjustment?

3. Sketch a form of schedule showing the information which would be required for the tabulation of the mortality, sickness, and secession, experience of a friendly society.

4. How would you construct a table, showing the annual death-rate from the year of birth to the age of nine years, from the materials afforded by two censuses, taken at an interval of ten years, and the records of births and deaths for the intervening period?

5. In what manner should additions to ordinary rates of premium for supposed impaired health be treated in a valuation?

What would be the effect of the system you propose on valuation reserves?

6. It has been stated that certain life assurance companies in the distribution of their realized profits deal differently with the profit arising from surplus interest and the profits arising from other sources.

Do you consider that any advantage arises from such an analysis, and what, in your opinion, would be the desirable method of dealing with the two amounts of profit respectively?

7. Explain the reinsurance method of valuation, and give your opinion as to its merits.

8. In what circumstances will the specific performance of a contract or agreement be decreed by the Court?

9. Into what classes is personal property divided?

Explain the nature of a "chose in action."

10. What is copyhold tenure? How did it originate? Is there any limit, and, if so, what limit to the amount of an arbitrary fine?

11. In what way do you consider that a loan by a life assurance society on one of its own policies should be secured? What covenants should be required for the protection of the society?

12. A is tenant for life, and B is tenant in tail in remainder. By what means can B convert his estate tail into an estate in fee simple?

It is optional on the part of the Candidate to answer one or more of the following questions, but due weight will be given to answers sent in.

13. Prove—

$$a_{60} = \frac{7}{D_{60}} (D_{67} + D_{74} + D_{81} + D_{88} + D_{96}) + 3 - 4(\mu_{60} + \delta).$$

14. Find by the Differential Calculus under what conditions the value of $_nV_x$ increases as the rate of interest decreases.

Second Paper.

15. On what occasions have the provisions of the Bank Act of 1844, with regard to the issue of notes, been suspended, and what were the results of the suspension in each case?

16. Describe—

1 Debentures,
2 Debenture Stock,
3 Preference Stock,
4 Ordinary Stock,
5 Preferred Stock,
6 Deferred Stock,

of railway or other companies.

17. What do you understand by the expressions " par of exchange " and " rate of exchange "?

Do you consider they are relatively dependent upon each other in any way?

18. Give your reasons for the method which in your opinion should be adopted for the valuation of such of the assets of a life assurance society that form part of the securities dealt in on the Stock Exchange.

19. Directions have been given in a trust deed for the investment of a uniform amount quarterly in Consols, in order to replace the loss expected, at the end of 30 years from the present time, from the expiration of a lease. What rate of interest would you adopt in order to calculate the amount to be set aside and what price would you estimate for the realization of the fund?

20. Freehold property stated to yield £1000 a year net income has been compulsorily purchased for £35,000 free of all charges.

The property is settled to the use of A, a male aged 70 years, for life, with remainder absolutely to B, a male aged 40 years. It is subject to a charge of £5,000 at 4 per-cent interest, and to a life annuity of £100 to a woman aged 70 years.

It is wished to divide the proceeds of the sale.

Give your opinion as to the principles upon which the apportionment should be made.

21. A sum of £135,000 is raised by the issue of debentures at par, bearing 6 per-cent interest and repayable by annual drawings

over a term of 15 years, with a bonus of 5 per-cent on each debenture.   What is the annuity required to provide for interest and the repayment of principal as proposed?

22. What is your opinion as to the relative amount of profit to be derived by an assurance company from whole-life and endowment assurances?   On what basis and with what loadings would you frame with and without-profit office premiums for endowment assurances, and what regulations would you make as to the declaration of reversionary bonuses on such assurances?

23. Sketch a brief report explaining the sources of profit of a life assurance business for the whole term of life, and indicate the proper limits of expenditure in the transaction of the business.

It is optional on the part of the Candidate to answer the following questions, but due weight will be given to the answers sent in.

24. Let $a, b, c, \ldots \kappa$ be n values of x not equidistant, for which the values of u_x are known.   Determine u_x generally.

25. What is meant by the law of facility of error?   Describe the method of Least Squares.

PROCEEDINGS OF THE INSTITUTE.—Session 1888–9.

First Ordinary Meeting, 26 November 1888.

The President (Mr. W. Sutton) in the Chair.

The President delivered an inaugural address.

Second Ordinary Meeting, 17 December 1888.

The President (Mr. W. Sutton) in the Chair.

The following gentleman was elected a Fellow of the Institute :
Stewart, John.

The following gentleman was elected an Associate of the Institute :
Gray, John.

Mr. Manly read a paper entitled " Formulas and Tables for applying Profits to making Policies payable during Life, or at an earlier Age."
The following gentlemen took part in the discussion :—Messrs. Terry, C. D. Higham, J. Chisholm, G. King, Bailey, Saunders, Todd, F. Bell, and Colenso.

Third Ordinary Meeting, 28 January 1889.

The President (Mr. W. Sutton) in the Chair.

Mr. Woolhouse read a paper entitled " An Easy Method of getting out a Rough Estimate Valuation of a Whole-Life Assurance Business."
Mr. Chisholm further read a paper on " The Approximate Calculation of Valuation Reserves."
The following gentlemen took part in the discussion :—Messrs. Wyatt, G. King, Ryan, Manly, R. P. Hardy, Strachan, C. D. Higham, and Rothery.

Fourth Ordinary Meeting, 25 *February* 1889.

The President (Mr. W. Sutton) in the Chair.

The following gentleman was elected a Corresponding Member of the Institute :

Léon Joseph Charles Mahillon.

Mr. George S. Mathews read a paper entitled " The Problem of Copyhold Fine Enfranchisement."

The following gentlemen took part in the discussion:—Messrs. Smyth, Sprague, Bailey, R. P. Hardy, and Strachan.

Fifth Ordinary Meeting, 25 *March* 1889.

The President (Mr. W. Sutton) in the Chair.

The following gentlemen were elected Associates of the Institute :

Bremner, Thomas William, \
Jacobs, Frederick Job.   /

Mr. H. R. Harding read a paper entitled " Life Offices: in reference to Expenses, New Business, Profit divided, and Competition."

Mr. T. Y. Strachan also contributed a series of Tables relating to the subject of the paper.

The following gentlemen took part in the discussion:—Messrs. Strachan, Macfadyen, Aug. Hendriks, Bailey, Browne, Saunders, G. King, Newbatt, and Crisford.

Sixth Ordinary Meeting, 29 *April* 1889.

The President (Mr. W. Sutton) in the Chair.

The following gentleman was elected a Fellow of the Institute :

Wallace, Thomas.

On the retirement of the President, the chair was occupied by Mr. Adler, Vice-President.

Mr. Searle read a paper entitled " A Table of Coefficients arising out of a given Mortality Table, for finding Annuity-Values at any Rate of Interest."

The following gentlemen took part in the discussion:—Messrs. Adler, G. King, R. P. Hardy, and Ackland.

The Forty-second Annual General Meeting, 1 *June* 1889.

The President (Mr. W. Sutton) in the Chair.

The Proceedings at the Annual General Meeting will be found on page 85.

REPORT, 1888–9.

" The Council have pleasure in reporting to the members upon the progress of the Institute during the session 1888-9, the forty-first year of its existence.

" The appended Schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending the 31st March

last. It is satisfactory to see an increase in the number of members of 42, following upon an increase last year of 37.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1888 .	2	160	229	121	9	521
ii. Withdrawals by						
(1) Death . . .	...	4	2	1	...	
(2) Resignation . .	...	...	2	6	...	23
(3) Default in payment of Subscriptions . .	...	1	2	5	...	
	2	155	223	109	9	498
iii. Additions to Membership . . .	...	1	1	62	1	65
	2	156	224	171	10	563
iv. Transfers						
(1) After Examination:						
from Associates	...	...	8	...	...	...
to Fellows .	...	8	...	...	...	...
	2	164	216	171	10	563
(2) By Examination:						
from Students	...	...	...	3	...	...
to Associates .	...	...	3	...	...	...
	2	164	219	168	10	563
(3) By Election:						
from Associates	...	...	1	...	...	...
to Fellows .	...	1	...	...	...	...
v. Number of Members in each class on 31 March 1889 .	2	165	218	168	10	563

"The total expenditure in respect of the *Text-Book*, Part II, has now been ascertained and paid; it has amounted in all to £1,159. 16s. 4d., and the amount realized by sales up to the close of the year is £450. 10s. 3d., showing a net cost of £709. 6s. 1d.; and against this we have the stock in hand of 994 copies, together with the stereotyped plates of the whole work.

"Of the total cost, £322. 1s. 4d. has appeared in the accounts for the past year, met to the extent of £48 by sales; this sum may therefore be considered as representing extraordinary outlay and not as part of the current expenditure.

" The stock in hand of the Institute publications is as follows :

No. of Copies					Description of Work
710	.	.	.	.	*Text-Book*, Part I.
994	.	.	.	.	,, ,, II.
219	.	.	.	.	Mortality Experience, 21*s.*
57	.	.	.	.	,, ,, 30*s.*
11	.	.	.	.	Logarithm Cards.

" The Library Committee have paid much attention during the past year to the revision and rearrangement of the books, and the Council are pleased to report to the members that they have secured the valuable assistance of two gentlemen to act as Honorary Librarians,—Mr. Henry Cockburn and Mr. W. J. H. Whittall,—by which means the arrangement and use of the Library will have continuous supervision.

" The proposal to establish a Library Fund has been acted upon, and the sum now standing to the credit of that Fund amounts to £286. 14*s.*

" The Council invite the members of the Institute to take an interest in the subject, and refer them to the notes upon the Library of the Institute which appear on the cover of the *Journal.*

" The Council have great pleasure in informing the members that Mr. Sprague has generously renewed his offer of a sum of £70, to be accorded, if thought worthy, to the writers of the two best essays on a subject which he has himself chosen, and which is one of which the Council entirely approve, namely :

" ' An examination of the published experiences of one or more life offices or groups of life offices, for the purpose of determining, with the greatest accuracy obtainable—(1) The rate of mortality among recently-selected lives ; (2) The rate at which that mortality increases during the 10 years after insurance ; and (3) Whether such rate of increase depends upon the rate of discontinuance or not.'

" At the first meeting of the session, held on 26 November 1888, the President (Mr. William Sutton) delivered an address.

" The following papers also have been submitted :

" 17 *December* 1888 —' Formulas and Tables for applying Profits to making Policies payable during Life, or at an earlier Age'—by Mr. H. W. Manly.

" 28 *January* 1889—' An Easy Method of getting out a Rough Estimate Valuation of a Whole-Life Assurance Business'—by Mr. W. S. B. Woolhouse.

' On the Approximate Calculation of Valuation Reserves'—by Mr. James Chisholm.

" 25 *February* 1889—' The Problem of Copyhold Fine Enfranchisement'—by Mr. George S. Mathews.

" 25 *March* 1889—' Life Offices : in reference to Expenses, New Business, Profit divided, and Competition'—by Mr. H. R. Harding.

" 29 *April* 1889—' A Table of Coefficients arising out of a given Mortality Table for finding Annuity-Values at any Rate of Interest that may be required'—by Mr. T. J. Searle.

" The first Intermediate Examination in Part I was held on 27 October 1888, when twenty candidates were examined, of whom nine passed, namely :

Aldcroft, W. H.	Hudson, A. J.
Cross, H. J.	Laing, W. C.
Dawson, C. P.	Melhuish, W. J.
Gayford, H.	Taylor, A.

Wingfield, H. W.

" For the examinations held on 12 and 13 April last, seventy-five candidates, the largest number that has yet been recorded, entered their names.

" For the examination in the First Part, thirty-seven appeared, of whom nineteen passed, namely :

Allin, S. J. H. W.	Ladelle, O. G.
Bull, E. J.	Le Maitre, F. W.
Cook, W. P.	Lighton, H. J.
Day, W. R.	Lisle, G.
Fulford, F. W.	McConway, J. R.
Harris, A. S.	McDonald, J.
Hart, J. R.	Quick, J. R.
Hurnall, C. W. L.	Stanham, W. C.
Hutcheson, W. A.	Thomas, E. C.

Williams, D.

" Twenty-four candidates were examined for the Second Part, of whom there passed nine, namely :

Fellows, R. H.	Laughton, A. M.
Fraser, D. C.	Makeham, W. R.
Gamble, A. F. M.	Slimon, W. J.
Kingdon, S. M.	Stirling, Jas. (Jr.).

Tilt, R. R.

" For the Third Part fourteen candidates were examined, and the following nine passed, namely :

Allen, A. G.	Cooper, W. G.
Anderson, J.	Cross, R.
Bell, F.	Faulks, J. E.
Chatham, J.	Tarn, A. W.

Thiselton, H. C.

" The names in each case are alphabetically arranged.

" The classes for instruction in Parts I and II of the examination have been satisfactorily attended.

" The audited accounts and balance sheet for the year are appended.

" The annual subscriptions (including admission fees) amount to £1,059, or an excess of those over 1887 of £29.

" The expenditure for the year is £1,170, which includes the final net payment on account of the *Text-Book*, and is less by £448 than the corresponding expenditure for the previous year.

" A profit of £120 arose from the sale of £500 Metropolitan Railway 4 per-cent Debenture Stock.

" The Council would specially invite the members of the Institute to contribute more largely to the interest of our sessional meetings by preparing papers for reading or insertion in the *Journal*. Much more might be done if a larger proportion, especially of the younger members, would devote the time and consideration necessary for investigating the varied subjects of general interest, which from time to time arise in the course of their official duties."

" WILLIAM SUTTON,
" *President.*"

" STAPLE INN HALL:
" 16 *May* 1889."

Dr.   *Receipts and Expenditure for Year ending 31st March 1889.*

1888-9.	£ s. d.	£ s. d.	£ s. d.
Amount of Funds, 31st March 1888—			
Messenger Legacy Fund		318 4 3	
Brown Prize Fund		256 13 2	
General Fund		2,367 2 2	2,942 0 2
Annual Subscriptions—			
Town Fellows	£264 12 0		
Country	81 18 0		
Fellows admitted since Issue of Bye-Laws:			
(1) By Examination	58 16 0		
(2) By Ballot	25 4 0	430 10 0	
Town Associates	256 4 0		
Country	87 3 0		
Associates admitted since Issue of Bye-Laws:			
(1) By Examination	6 6 0		
(2) By Ballot	21 0 0	370 13 0	
Students		182 14 0	953 17 0
Entrance Fees—			
Fellow, on Election	£5 5 0		
Associate, on Election	21 5 0		
Students	48	75 12 0	
Examination Fees—			
April 1888	61 19 0		
October 1888	22 1 0	84 0 0	
Dividends		159 12 0	
Profit on Sale of Metropolitan Railway Stock		73 19 1	
Donations to the Library Fund		119 16 0	286 14 0
			£4,565 19 1

Cr.

1888-9.	£ s. d.	£ s. d.	£ s. d.
Journal Expenses		471 5 6	
Less Sales		108 17 0	362 8 3
Text-Book, Part I, Expenses		20 16 8	
,, II, ,,		322 1 4	
		342 18 0	
Less Sales—Part I	£50 12 0		
,, II	48 1 1		
Logarithm Cards	1 1 3		
Experience Tables	4 19 0	84 13 10	258 4 2
Messenger Prize			50
Library—Purchase of Books	8 14		
Binding, &c.	12 7		
Clerical Assistance	8 7 0		28 9 10
General Expenditure—			
Rent	250 0 0		
Salaries	194 10 0		
Lecturers	52 10 0		
Examination Charges	33 4 8		
Law Charges	59 5 6		
General Meetings	38 13 0		
House Expenses (Gas, Coal, &c.)	25 15 1		
Corporation Duty	23 15 0		
Fire Insurance	12 14 0		
Postage and Stationery	85 16 0		
Sundries	16 11 2		770 15 0
Amount of Funds, 31st March 1889—			
Messenger Legacy Fund	274 6 3		
Brown Prize Fund	264 3 6		
Library Fund	286 14 1		
General Fund	2,270 12 0		3,096 1 10
			£4,565 19 1

Examined and found correct, 25th April 1889.

Balance Sheet, 31st March 1889.

Dr.   LIABILITIES.

	£ s. d.	£ s. d.
Messenger Legacy Fund (£211. 1s. 10d. Consols), cost	203 17 8	
Accumulated Dividends	70 9 1	274 6 9
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock), cost	200 0 0	
Accumulated Dividends	64 8 3	264 8 3
Library Fund		286 14 1
General Fund		2,270 12 10
Total Funds, as per Revenue Account		3,096 1 10
Examination Fees received, applying to Account 1889-90		66 3 0

Cr.   ASSETS.

	£ s. d.	£ s. d.
2¾ per-cent Consols, cost		1,246 4 6
Metropolitan Railway 4 per-cent Debenture Stock, cost		715 1 0
On Deposit at the London and Westminster Bank		700 0 0
On Current Account at ,, ,,		462 0 1
Cash in hand, including Subscriptions		12
Arrears of Subscriptions—		
Country Fellows	48 8 0	
Town Associates	8 8 0	
Country ,,	8 11	
Students	8 11	26 5 0

Examined and found correct, 25th April 1889.

(Signed)   W. G. GLENSIE, } Auditors.
          ROBT. CROSS,

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the Members was held at Staple Inn Hall, on Saturday, 1 June, the President, Mr. W. Sutton, M.A., in the chair.

The Report of the Council (given on p. 80) having been read,

The PRESIDENT, in moving the adoption of the Report, said that there were several matters to which he felt it his duty to call attention. In the first place, he referred with regret to the deaths of four Fellows of the Institute, namely, Mr. Thomas Carr; Mr. Samot, of Holland; Mr. Stott; and Mr. White. The number of members was rapidly increasing. In 1884 there were 374 members, and the number had increased by nearly 200 since that time. A great part of the increase arose, of course, in the class of Students. It was mentioned in the Report of the Council last year that attention had been called to the use of the Institute initials by gentlemen who were not members of the Institute; but on this occasion he had to mention a more serious fact, that certain Students were openly advertising themselves as "members of the Institute of Actuaries", in such a way as was calculated to mislead. He hoped they would at once discontinue that objectionable practice, otherwise the Council might have to interfere. He referred to the expenditure in respect of the *Text-Book*, and pointed out that this was to a great extent an investment of the Institute funds, and there was every reason to believe the money expended would be returned within a few years. He was glad to see in the Report a record given of the amount of stock in hand of the Institute publications. He was sure everybody would agree that the appointment of Mr. Cockburn and Mr. Whittall, to undertake the duties of honorary librarians, was evidence that the library would receive satisfactory attention; and the Institute was indebted to Mr. Newbatt for the trouble he had taken with regard to this matter. As regards the library fund, he would ask Mr. Newbatt to second the adoption of the Report, and would leave that subject in his hands. He took the opportunity of mentioning that he would like to see, in the rooms of the Institute, portraits of those who had in the past served the Institute as its chiefs for the time being, and that at the present time there was a serious gap. The portraits of Mr. Bailey, Mr. Sprague, and Mr. Day, were not there, and they ought to be. Mr. Sprague had again offered a sum of money for two prizes, and they had reason to be extremely obliged to him, because his first offer met with such poor success. The subject which Mr. Sprague had chosen was one which no member was more competent to deal with than himself, but he had considered it a proper subject for the younger Fellows to take up, and the Council were quite in accordance with that view; and he (the President) would look forward with some confidence to the offer of these prizes bringing forward at least one or two very valuable contributions. He might be allowed to refer to another matter closely connected with that just mentioned. In the introduction to the *Twenty Offices' Experience*, written by Mr. Samuel Brown, the first sentence was as follows: "Nearly a quarter of a century having elapsed

" since the period to which the combined experience of seventeen life assur-
" ance offices was collected, it began to be felt amongst actuaries and the
" managers of companies that a large mass of valuable materials had
" accumulated which, if combined, would tend to throw further light on the
" law of mortality amongst assured lives, and on other points affecting the
" interest and prosperity of assurance companies." That sentence was
written by Mr. Brown in connection with the Twenty Offices' Experience,
which extended to the end of 1863, and as nearly 26 years had passed since
that time, in his opinion—no doubt, in the opinion also of many members of
the Institute—the words were equally applicable to the present day. The
Institute, he believed, loved work. The collection of the Twenty Offices'
Experience had occupied a great deal of its time and attention. Since then, it
had taken up the question of a Text-Book on Interest and Annuities-Certain
and Life Contingencies, which had been successfully completed. He did not
think that a more important subject could be undertaken by the Institute in
the immediate future than that of the further investigation of the mortality
of assured lives, and he felt, too, that if the Council should resolve to take it
up, every member of this Institute would do everything in his power to help.
There were other people who had to be consulted, if the investigation were
to be a complete success. There were what are known as "the Scotch
offices", and these offices transact an important part of the life assurance
business of this country. It would accordingly be necessary, to ensure the
complete success of such an undertaking, that the Scotch offices should
co-operate with the English offices. He had recently, with other office-bearers
of the Institute, attended a dinner of the Faculty of Actuaries in Scotland,
and from the most hospitable and cordial manner in which they were received,
he had not the least doubt that the Scotch offices would do their share in
co-operating in this matter with the Institute. At the ordinary meetings of
the past year, three of the papers submitted were by Mr. Woolhouse, Mr. Manly,
and Mr. Chisholm, and the remaining three papers were the productions of
gentlemen who had not hitherto favoured the Institute with any contribu-
tions. During the last year no less than 95 candidates had been examined—
75 at the annual examinations and 20 at the intermediate. The large
increase in the number of candidates would be looked upon with satisfaction
by those who believed that the examination work which the Institute had
done was one of the best and most important parts of its duties. It was
curious that for the last three years nearly the same proportion of the
candidates had been successful, amounting to 50 per-cent, a decided increase as
compared with the few years preceding. In congratulating the nine gentle-
men who had obtained their Fellowship by passing the final examination, he
stated that he had great pleasure in presenting each of them with a copy of
the Scottish Life Offices' mortality experience, as taken out some years ago,
which Mr. Meikle had generously given him to dispose of at his discretion.
He concluded by reminding the members that the five years during which,
under the Charter, the qualifications for election as Fellows had been left, to
some extent, in the hands of the Council to determine, had nearly come to
an end, and he expressed the hope that the younger members would never

forget that the object of the Charter could only be completely fulfilled by the Institute ultimately having as its members every man in the profession qualified to act.

Mr. NEWBATT referred to the satisfactory increase in membership recorded in the Report. He quoted, with approval, from a speech made by Mr. Jellicoe in July 1851 (*J.I.A.*, i, 387), in which Mr. Jellicoe said, in effect, that the object of the Institute was to train men fit to conduct the affairs with which we had to do, where the exclusively scientific man was most likely to err and the merely practical man was almost sure to mislead,—to, in fact, train the actuary, who, coming between them and combining the knowledge of both, was by that means enabled to be of real service to his fellow men. He believed that the Institute had been fulfilling the work which Mr. Jellicoe laid down as its chief function. He had been specially invited to refer to the work in which he had taken special interest during the last three years as Vice-President, namely, the work of the library; and he expressed his satisfaction that two Fellows of the Institute had taken upon themselves the duty of giving that personal attention to the library, without which its efficiency could not be established and continued. But, concurrently with the desire to make the library efficient and useful, there had been in the minds of a good many of the members a desire to extend the usefulness of the Institute into channels which had not hitherto been recognized. With that view, and prompted by Mr. Young, he had started a subscription list for the purpose of obtaining what he hoped would prove to be a sufficient endowment fund—a fund sufficient in itself to practically relieve the general funds of the Institute from the cost of maintaining the room for the purposes in view. Those purposes had not been put as plainly as seemed necessary. He wished to see the Hall of the Institute made a common ground of meeting for members of the Institute. He also hoped that there would be established junior classes and associations for the better training of the junior members. They had obtained over £300, but required £500. He did not ask a large subscription of any individual member, but he thought every member should, according to his ability, contribute to the fund. He had great pleasure in seconding the adoption of the Report.

The motion was agreed to unanimously.

ELECTION OF OFFICERS.

A ballot having been taken, the Scrutineers, Mr. H. C. L. SAUNDERS and Mr. F. E. COLENSO, reported that the following list had been unanimously adopted :

President.
WILLIAM SUTTON, M.A.

Vice-Presidents.

MARCUS NATHAN ADLER, M.A.	AUGUSTUS HENDRIKS.
ALEXANDER J. FINLAISON, C.B.	THOMAS EMLEY YOUNG, B.A.

Council.

Thomas Gans Ackland.
Marcus Nathan Adler, M.A.
Arthur Hutcheson Bailey.
Thomas G. C. Browne.
*Arthur Francis Burridge.
Henry Cockburn.
George Stephen Crisford.
Griffith Davies.
Archibald Day.
Edward Docker, M.A.
Alexander J. Finlaison, C.B.
Henry Harben.
Ralph P. Hardy.
*Augustus Hendriks.
George Humphreys, M.A.

George King.
Henry William Manly.
Thomas Marr.
Frank McGedy.
Benjamin Newbatt.
*Arthur Pearson.
*John George Priestley.
Gerald Hemmington Ryan.
Thomas Bond Sprague, M.A.
William Sutton, M.A.
George Todd, M.A.
*Andrew Hugh Turnbull.
William Wallis.
Frank Bertrand Wyatt.
Thomas Emley Young, B.A.

* New Members of Council.

Treasurer.

George Humphreys, M.A.

Honorary Secretaries.

Thomas G. C. Browne. | George Stephen Crisford.

On the motion of Mr. T. J. Searle, seconded by Mr. W. R. Makeham, Messrs. A. B. Woods, W. G. Turpin, and J. P. Longstaff, were elected Auditors for the ensuing year.

Mr. A. Hendriks proposed, and Mr. T. C. Dewey seconded, a vote of thanks to the President, Vice-Presidents, Council, and Officers, of the Institute for their services during the past year.

The resolution was cordially adopted, and Mr. A. J. Finlaison responded.

A vote of thanks was accorded to the Auditors, on the motion of Mr. A. H. Turnbull, seconded by Mr. R. Watson.

The proceedings then terminated.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

" I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

Vol. XXVIII.—Part II.

OCTOBER 1889.

CONTENTS OF NO. CLIV.

NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

INSTITUTE OF ACTUARIES.

The Mortality Experience of the Norwegian General Widows'
Fund. By OSCAR SCHJÖLL, Manager (Kontorchef) of the
"Idun"* Life Assurance Company, Christiania.

THE *Norwegian General Widows' Fund (Den norske almindelige*
enkekasse) is a public institution, guaranteed by the Norwegian
Government, and consisting of two sections. The older of these,
which ceased to receive new members in 1848, contains (1) the
members of the *Norwegian Widows' Fund,* established on
20 March 1814, and (2) the Norwegian members of the old
Danish-Norwegian Widows' Fund, who were transferred to it in
1821 in accordance with the Treaty of Kiel.

The Fund has thus the same historical origin as the *Danish*
Widows' Fund, which may be traced back to 3 January 1707,
when a *Widows' Fund for the Army* was established by a royal
edict. With this fund was united, in 1739, another fund, called
the *Annuity Fund for the Widows and Children of Military*
Officers. The contributions were single premiums of from 100 to
1,000 Rigsdaler, ($£11\frac{1}{9}$ to $£111\frac{1}{9}$); and the annuity paid to the

---

* Iduna (Idhun), a North-Teutonic goddess, the custodian of the apples, the
possession of which secured eternal youth to the gods. She was the daughter of
the dwarf Ivald, and the wife of Bragi, the god of poetry; and there are various
myths about her.

widow of a contributor was 40 per-cent of the single premium,
irrespective of the ages.   If a widow remarried, she lost her
annuity; but if she became a widow again, she regained it.   A
contributor who became a widower, could secure a reversionary
annuity for a second wife, by paying half the ordinary contribution.
The fund also granted provisions for children, which took the
same form of annuities of 40 per-cent of the single premiums,
payable from the death of the father until the child attained the
age of 24.   In the following year (1740) the fund was opened to
all classes of men, but civilian members had to pay an extra
contribution of 10 per-cent.

It is nowadays obvious to actuaries that an institution on such
a basis, could not possibly fulfil its engagements permanently;
but more than a quarter of a century elapsed before its unsound-
ness became generally known.   In 1773 a commission was
appointed to examine the state of the fund; and its investigations
led to the closing of the old fund, and the establishment of a new
one, the constitution of which was dated 30 August 1775.   In
1782 the assets of the old fund were exhausted, and since that
time the Government has paid more than £330,000 to the
annuitants.

The object of the new fund of 1775 was " to give support and
" relief to the widows of all the King's subjects, and particularly
" to those of the King's officers—civil and military"; and it was
therefore styled: *Den almindelige enkekasse for de kongelige riger,
hertugdómmer og lande* (The General Widows' Fund of the Royal
Kingdoms,* Duchies, and Countries), or, briefly, *Den almindelige
enkekasse*.   According to the above-mentioned constitution, every
public officer, civil or military, was obliged, when he married, to
purchase for his widow, a reversionary annuity of from 40 to
400 Rigsdaler (£4. 8s. 11d. to £44. 8s. 11d.).   The granting of
annuities for children was discontinued.   Contributors who were not
public officers were charged 10 per-cent extra.   The rates of the new
fund were calculated on Süssmilch's Mortality Table, but after the
lapse of only 10 years were changed for others about 20 per-cent
higher, which were based on the Süssmilch-Baumann Mortality
Tables, both for males and females,† with interest at 4 per-cent.

---

* *i.e.,* Denmark and Norway.

† The calculator, Mr. Lous, in his paper, "Zuverlässiger Bericht von der bey
" Anlegung der neuen allgemeinen Wittwen-Kasse angenommenen Theorie und
" gebrauchten Rechnungs-Art; Copenhagen, 1778", nevertheless mentions
Mr. Wargentin's Swedish tables, which give the mortality for males and females
separately, and would have been a far better basis.

All members were required to supply evidence of age, and proof of the husband's good health was also required. Only single premiums were accepted; but a public officer, if unable to pay the whole premium at once, could pay by instalments, giving as security one-fourth of his yearly pay in addition to his property. As in the older fund, the annuity ceased when the widow remarried, and commenced again if she became again a widow.

For many years the *General Widows' Fund* appeared to flourish. In 1797* the assets amounted to about £330,000, with an estimated surplus of about £44,000; in 1813 the assets had increased to about £550,000, and the estimated surplus was £17,000, although the fund had had a loss of about £78,000, owing to the deranged monetary relations of Denmark and Norway at that time. Of course all parties interested were satisfied that the fund was in a good position, and the opinion even came to be held that the premiums were too high. Unfortunately, however, the calculations had been made on the basis of the above mentioned Süssmilch-Baumann Tables, which (as is now very well known) much under-estimated the vitality of the annuitants. When fresh calculations were made, based upon the mortality experience of the fund itself, it was found that the fund was just solvent in 1797, while in 1813 there was a deficiency of about £89,000.

The separate Norwegian Fund, established, as already mentioned, in 1814, had the same regulations as the fund of 1775; and when the Norwegian members of the old fund were transferred to it, it received about £139,000 as its share of the assets. This daughter-institute, of course, inherited the constitutional weakness of its parent. Accordingly, it was found in 1835 to have a deficiency of £94,000 on a fund of only £250,000; in 1845 the deficiency had increased to about £236,000, although the Government had, during the intervening years, contributed more than £83,000 to its funds; and, since 1845, the Government has been obliged to pay yearly considerable sums, in order to fulfil the obligations of the fund.

At last, by an Act of 29 June 1848, this older section of the fund was closed, and a new section founded, of which the most important provisions are as follows: The new section has its funds kept distinct, the Norwegian Government still guaranteeing the

---

* T. N. Tetens's paper, "Nachricht vom Zustande der allgemeinen Wittwen-Kasse zu Copenhagen am Schluss des Jahres 1797."

contracts; the new rates are based on the mortality experience of
the older section up to 1845, with 4 per-cent interest, no loading
being charged. As in the older section, only reversionary annuities
are granted, and only single premiums are even now accepted from
the members. As formerly, every public official is obliged to
purchase a reversionary annuity for his wife proportional to his
yearly pay, usually equal to one-fifth of it. These officials are
admitted without any proof of health, and may pay by instalments
as above described. Other married men must, on their admission
as members, furnish proof of good health, and pay the whole
premium at once.

Every fifth year a valuation was to be made, but at present
one is made every third year. According to the valuation up to
31 December 1885, the following was the position:

OLD SECTION.

Fund	.	.	.	.	.	.	£51,444
Liability	.	.	.	.	.	.	86,333
	Deficiency		.	.	.		£34,889

At the end of 1886 there were in existence, 139 reversionary
annuities for £3,060, and 446 immediate annuities for £12,200.

The Government contributes about £5,000 a year to the
section.

NEW SECTION.

Fund	.	.	.	.	.	.	£588,222
Liability	.	.	.	.	.	.	355,333
	Surplus	.	.	.	.		£232,889

At the end of 1886 there were in existence, 1,657 reversionary
annuities for £44,025, and 546 immediate annuities for £12,634.

At present the immediate annuities are paid at the rate of
139 per-cent of the nominal amount.

The two appended tables, A and B, exhibit the mortality
experience of the fund. A relates to the male lives, and gives
the experience during 110 years (1775–1885); and B relates
to the female lives, and gives the experience only for 70 years
(1814–1885), because the necessary particulars concerning the
female lives cannot be obtained for the earlier years. With
regard to the construction of columns (2) and (3), it should be
mentioned that every life has been traced, not from the date of
entry, but from the first birthday after entry, either down to death,

or to the birthday in the year 1885; every person who died before his (or her) first birthday after entry being omitted; and every person who died in 1885 *after* the birthday in that year, being considered as alive at the close of the observations. Hence, as there are no discontinuances, the exposures are given by the formula

$$E_x = E_{x-1} - d_{x-1} + n_x - e_x,$$

where E represents the "exposed to risk"; d, the "deaths"; n, the "new entrants"; and e, the "existing."

Although the data are not extensive (containing only 3,812 and 3,525 deaths respectively), they are, I think, of considerable value, because the deaths are more uniformly distributed among the different ages than is commonly the case; so that, for instance, in Table B the number of deaths at ages 21–92 is never less than 10, nor more than 113. As regards the character of the data in other respects, I need only remark that the medical examination originally required by the constitution of the Danish Fund, was practically of no value; but, on the other hand, the fact that all members at the time of admission were married, and most of them newly married, operated, I believe, as a sort of selection. The Table B is certainly the more valuable of the two; and my object in compiling the data for it was to obtain for the Norwegian Life Office, *Iduna*, a safe basis for the calculation of life annuities. The figures in Table A are given rather for the sake of completeness than on account of their intrinsic value.

I have graduated the results by the graphic method, first grouping the numbers as shown in columns (5), (6), and (7). The groups in the first half of the Table B have been taken in accordance with a suggestion made by Mr. Sprague, and it is also at his suggestion that I now submit these results to the readers of the *Journal of the Institute*. Columns (8) and (9) show the corresponding rates of mortality, and column (10) the graduated results. Columns (11), (12), and (13) have been added to show how far the graduated mortality agrees with the actual. The values of l_x and d_x deduced from the female experience, and the corresponding 4 per-cent annuity-values, are given in Table C.

In the diagram are shown the probabilities of dying in a year, for both males and females, as first approximately obtained by the grouping, and next as finally adjusted. I have added, for the purpose of comparison, the probabilities among the English Government Female Annuitants, four years after entry, as given in Mr. Sprague's Table W, *J.I.A.*, xxv, 107.

Table A.—*Mortality Experience of the Norwegian General Widows' Fund (Males), 1775-1885.*

	OBSERVATIONS								RESULTS OF GRADUATION			
Age	Number at Risk	Actual Deaths	Rate of Mortality per-cent	Grouped Ages	Number at Risk in Groups	Actual Deaths in Groups	Rate of Mortality in Groups per-cent	Middle Ages of the Groups	Rate of Mortality per-cent	Expected Deaths	Expected Deaths in Groups	Differences (12)-(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
20	3	0	...	20–23	133	0	0·000					
21	12	0	...									
22	34	0	...									
23	84	0	...									
24	172	1	0·581	24–27	1,759	7	0·398	25½	0·50	0·9	8·7	+ 1·7
25	308	0	0·000						0·47	1·4		
26	527	1	0·190						0·49	2·6		
27	752	5	0·665						0·51	3·8		
28	1,050	3	0·286	28–29	2,427	14	0·577	28½	0·55	5·8	14·1	+ 0·1
29	1,377	11	0·799						0·60	8·3		
30	1,668	13	0·779	30–32	5,896	41	0·695	31	0·65	10·8	40·8	– 0·2
31	1,975	14	0·709						0·69	13·6		
32	2,253	14	0·621						0·73	16·4		
33	2,527	20	0·791	33–37	14,816	123	0·830	35	0·76	19·2	121·3	– 1·7
34	2,765	21	0·759						0·78	21·6		
35	2,982	23	0·771						0·81	24·2		
36	3,197	28	0·876						0·84	26·9		
37	3,345	31	0·927						0·88	29·4		
38	3,174	29	0·835	38–39	7,058	65	0·921	38½	0·92	32·0	66·4	+ 1·4
39	3,584	36	1·004						0·96	34·4		
40	3,655	35	0·958	40–43	14,972	165	1·102	41½	1·01	36·9	164·8	– 0·2
41	3,733	49	1·313						1·07	39·9		
42	3,780	30	0·794						1·13	42·7		
43	3,804	51	1·341						1·19	45·3		
44	3,813	54	1·405	44–49	23,012	334	1·451	46½	1·26	48·4	336·1	+ 2·1
45	3,858	45	1·166						1·33	52·4		
46	3,870	54	1·395						1·41	54·6		
47	3,842	67	1·744						1·49	57·2		
48	3,815	64	1·678						1·58	60·3		
49	3,784	50	1·321						1·67	63·2		
50	3,756	59	1·571	50–54	18,330	362	1·975	52	1·77	66·5	364·2	+ 2·2
51	3,725	60	1·611						1·87	69·7		
52	3,696	78	2·110						1·98	73·2		
53	3,615	89	2·462						2·10	75·9		
54	3,538	76	2·148						2·23	78·9		
55	3,459	73	2·110	55–57	10,125	257	2·538	56	2·37	82·0	256·9	– 0·1
56	3,377	86	2·547						2·53	85·4		
57	3,289	98	2·980						2·72	89·5		
58	3,173	87	2·742	58–61	12,077	405	3·353	59½	2·94	93·3	400·1	– 4·9
59	3,082	92	2·985						3·19	98·3		
60	2,978	103	3·459						3·46	103·0		
61	2,844	123	4·325						3·71	105·5		
62	2,684	94	3·502	62–63	5,240	207	3·950	62½	3·94	105·7	211·8	+ 4·8
63	2,556	113	4·421						4·15	106·1		
64	2,410	104	4·315	64–67	8,783	419	4·771	65½	4·37	105·3	419·6	+ 0·6
65	2,264	92	4·064						4·62	104·6		
66	2,136	118	5·524						4·92	105·1		
67	1,973	105	5·322						5·30	104·6		
68	1,816	101	5·562	68–69	3,489	214	6·134	68½	5·80	105·3	210·7	– 3·3
69	1,673	113	6·754						6·30	105·4		

Table A—(continued).

	OBSERVATIONS								RESULTS OF GRADUATION			
Age	Number at Risk	Actual Deaths	Rate of Mortality per-cent	Grouped Ages	Number at Risk in Groups	Actual Deaths in Groups	Rate of Mortality in Groups per-cent	Middle Ages of the Groups	Rate of Mortality per-cent	Expected Deaths	Expected Deaths in Groups	Differences (12)−(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
70	1,517	99	6·526	70–73	5,261	392	7·451	71½	6·80	103·2	394·9	+ 2·9
71	1,384	98	7·081						7·30	101·0		
72	1,254	100	7·974						7·80	97·8		
73	1,106	95	8·590						8·40	92·9		
74	974	91	9·343	74–78	3,818	395	10·316	76	9·00	87·7	392·3	− 2·7
75	858	70	8·159						9·70	83·2		
76	767	89	11·604						10·40	79·8		
77	657	80	12·177						11·20	73·6		
78	562	65	11·566						12·10	68·0		
79	479	55	11·482	79–83	1,721	254	14·759	81	13·10	62·7	252·5	− 1·5
80	411	64	15·572						14·10	58·0		
81	336	51	15·179						15·10	50·7		
82	277	51	18·412						16·00	44·3		
83	218	33	15·138						16·90	36·8		
84	183	32	17·486	84–87	546	99	18·132	85½	17·85	32·7	104·1	+ 5·1
85	148	19	12·838						18·80	27·8		
86	123	26	21·138						19·80	24·4		
87	92	22	23·913						20·90	19·2		
88	69	21	30·435	88–91	170	45	26·471	89½	22·10	15·2	40·3	− 4·7
89	47	11	23·404						23·50	11·0		
90	33	8	24·242						25·30	8·3		
91	21	5	23·810						27·70	5·8		
92	15	3	20·000	92–96	40	14	35·000	94	30·70	4·6	15·4	+ 1·4
93	12	3	25·000						34·70	4·2		
94	8	4	50·000						40·00	3·2		
95	4	3	75·000						60·00	2·4		
96	1	1	100·000						100·00	1·0		
...	139,673	3,812	...	...	...	...	...	...	...	...	3815·0	+ 22·3 −19·3

TABLE B.—*Mortality Experience of the Norwegian General Widows'*
Fund (Females), 1844–1885.

	OBSERVATIONS								RESULTS OF GRADUATION			
Age	Number at Risk	Actual Deaths	Rate of Mortality per-cent	Grouped Ages	Number at Risk in Groups	Actual Deaths in Groups	Rate of Mortality in Groups per-cent	Middle Ages of the Groups	Rate of Mortality per-cent	Expected Deaths	Expected Deaths in Groups	Differences (12)—(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
16	4	0	...						0·78	0·0		
17	15	0	...						0·86	0·1		
18	61	0	...	16–20	568	6	1·057	18	0·94	0·6	5·7	− 0·3
19	166	2	1·205						1·00	1·7		
20	322	4	1·242						1·04	3·3		
21	550	10	1·818						1·07	5·9		
22	837	11	1·314	21–23	2,533	31	1·224	22	1·09	9·1	27·5	− 3·5
23	1,146	10	0·873						1·09	12·5		
24	1,479	13	0·879						1·085	16·0		
25	1,752	20	1·112	24–27	7,159	78	1·046	25½	1·08	18·9	79·7	+ 1·7
26	1,992	17	0·853						1·07	21·3		
27	2,236	28	1·252						1·05	23·5		
28	2,448	30	1·225						1·04	25·5		
29	2,656	25	0·941	28–31	10,844	110	1·015	29½	1·03	27·4	111·4	+ 1·4
30	2,800	24	0·857						1·02	28·6		
31	2,940	31	1·054						1·016	29·9		
32	3,045	29	0·952						1·012	30·8		
33	3,137	27	0·861						1·01	31·7		
34	3,257	30	0·921	32–36	16,101	163	1·012	34	1·008	32·8	162·2	− 0·8
35	3,328	40	1·202						1·006	33·5		
36	3,334	37	1·110						1·003	33·4		
37	3,390	40	1·180	37–38	6,826	67	0·982	37½	1·00	33·9	67·6	+ 0·6
38	3,436	27	0·786						0·98	33·7		
39	3,509	37	1·054						0·95	33·3		
40	3,504	34	0·970	39–42	14,059	125	0·889	40½	0·90	31·5	124·0	− 1·0
41	3,523	27	0·766						0·86	30·3		
42	3,523	27	0·766						0·82	28·9		
43	3,540	24	0·678	43–44	7,089	54	0·762	43½	0·78	27·6	55·6	+ 1·6
44	3,549	30	0·845						0·79	28·0		
45	3,554	27	0·760	45–46	7,113	60	0·844	45½	0·81	28·8	58·3	− 1·7
46	3,559	33	0·927						0·83	29·5		
47	3,549	33	0·930	47–48	7,086	61	0·861	47½	0·85	30·2	61·7	+ 0·7
48	3,537	28	0·792						0·89	31·5		
49	3,514	30	0·854	49–50	7,005	65	0·928	49½	0·94	33·0	67·9	+ 2·9
50	3,491	35	1·003						1·00	34·9		
51	3,478	52	1·495						1·14	39·6		
52	3,420	46	1·345	51–53	10,262	134	1·306	52	1·27	43·4	129·8	− 4·2
53	3,364	36	1·070						1·39	46·8		
54	3,316	46	1·387						1·50	49·7		
55	3,245	46	1·418						1·61	52·2		
56	3,207	56	1·746	54–58	15,976	271	1·696	56	1·71	54·8	271·2	+ 0·2
57	3,138	61	1·944						1·80	56·5		
58	3,070	62	2·020						1·89	58·0		
59	3,004	57	1·897						1·98	59·5		
60	2,921	61	2·088	59–61	8,765	183	2·088	60	2·08	60·8	182·2	− 0·8
61	2,840	65	2·289						2·18	61·9		
62	2,756	63	2·286						2·29	63·1		
63	2,670	81	3·034						2·43	64·9		
64	2,555	52	2·035	62–66	12,828	335	2·611	64	2·60	66·4	336·6	+ 1·6
65	2,472	65	2·629						2·81	69·5		
66	2,375	74	3·116						3·06	72·7		

TABLE B—(*continued*).

Age	OBSERVATIONS								RESULTS OF GRADUATION			
	Number at Risk	Actual Deaths	Rate of Mortality per-cent	Grouped Ages	Number at Risk in Groups	Actual Deaths in Groups	Rate of Mortality in Groups per-cent	Middle Ages of the Groups	Rate of Mortality per-cent	Expected Deaths	Expected Deaths in Groups	Differences (12)−(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
67	2,279	84	3·686						3·38	77·0		
68	2,168	80	3·690	67–70	8,426	330	3·916	68½	3·73	80·9	326·4	− 3·6
69	2,054	88	4·284						4·06	83·4		
70	1,925	78	4·052						4·42	85·1		
71	1,811	90	4·970						4·80	86·9		
72	1,700	106	6·235						5·16	87·7		
73	1,552	77	4·961	71–75	7,833	438	5·592	73	5·56	86·3	441·9	+ 3·9
74	1,439	79	5·490						6·20	89·2		
75	1,331	86	6·461						6·90	91·8		
76	1,223	113	9·240						7·70	94·2		
77	1,087	90	8·280						8·6	93·5		
78	973	96	9·866	76–80	4,880	460	9·426	78	9·6	93·4	457·0	− 3·0
79	855	89	10·409						10·6	90·6		
80	742	72	9·704						11·5	85·3		
81	641	70	10·920						12·3	78·8		
82	557	86	15·440						13·3	74·1		
83	458	56	12·227	81–85	2,375	332	13·979	83	14·4	66·0	335·6	+ 3·6
84	396	63	15·909						15·6	61·8		
85	323	57	17·647						17·0	54·9		
86	258	53	20·543						18·6	48·0		
87	194	39	20·103						20·4	39·6		
88	151	32	21·192	86–90	801	173	21·598	88	22·2	33·5	170·2	− 2·8
89	114	21	18·421						24·0	27·4		
90	84	28	33·333						25·8	21·7		
91	52	17	32·692						27·8	14·5		
92	35	13	37·143						29·8	10·4		
93	20	4	20·000	91–95	131	43	32·824	93	31·8	6·4	39·7	− 3·3
94	15	6	40·000						34·0	5·1		
95	9	3	33·333						36·2	3·3		
96	6	1	16·667						38·6	2·3		
97	5	1	20·000						41·0	2·1		
98	4	2	50·000						43·6	1·7		
99	2	0	0·						46·6	0·9		
100	2	0	0·	96–105	26	6	23·077	100½	49·8	1·0	12·8	+ 6·8
101	2	0	0·						54·0	1·1		
102	2	1	50·000						60·0	1·2		
103	1	0	0·						68·0	0·7		
104	1	0	0·						80·0	0·8		
105	1	1	100·000						100·0	1·0		
...	158,986	3,525	...	...	...	...	...	...	...	...	3,525·0	+ 25·0 −25·0

Table C.—*Norwegian General Widows' Fund (Females),
1814–1885.—Values of l_x, d_x, and a_x (4 per-cent).**

x	l_x	d_x	a_x (4 %)	x	l_x	d_x	a_x (4 %)
0	100,000	3,480	19·639	53	61,765	859	12·998
1	96,520	1,438	20·161	54	60,906	911	12·709
2	95,082	932	20·285	55	59,992	966	12·419
3	94,150	668	20·305	56	59,026	1,009	12·127
4	93,482	533	20·268	57	58,017	1,044	11·831
5	92,949	437	20·200	58	56,973	1,077	11·530
6	92,512	379	20·107	59	55,896	1,107	11·222
7	92,133	341	19·997	60	54,789	1,140	10·907
8	91,792	330	19·874	61	53,649	1,170	10·584
9	91,462	320	19·744	62	52,479	1,202	10·253
10	91,142	310	19·606	63	51,277	1,246	9·913
11	90,832	318	19·460	64	50,031	1,301	9·566
12	90,514	326	19·309	65	48,730	1,369	9·215
13	90,188	379	19·154	66	47,361	1,449	8·860
14	89,809	485	19·004	67	45,912	1,552	8·506
15	89,324	581	18·872	68	44,360	1,655	8·155
16	88,743	666	18·755	69	42,705	1,734	7·810
17	88,077	749	18·653	70	40,971	1,811	7·466
18	87,328	812	18·565	71	39,160	1,880	7·124
19	86,516	865	18·489	72	37,280	1,924	6·783
20	85,651	891	18·423	73	35,356	1,966	6·438
21	84,760	907	18·361	74	33,390	2,070	6·090
22	83,853	914	18·302	75	31,320	2,161	5·752
23	82,939	904	18·244	76	29,159	2,245	5·425
24	82,035	890	18·183	77	26,914	2,315	5·113
25	81,145	876	18·118	78	24,599	2,362	4·818
26	80,269	859	18·048	79	22,237	2,357	4·523
27	79,410	834	17·973	80	19,880	2,286	4·285
28	78,576	817	17·890	81	17,594	2,164	4·035
29	77,759	801	17·801	82	15,430	2,052	3·785
30	76,958	785	17·706	83	13,378	1,926	3·540
31	76,173	774	17·604	84	11,452	1,787	3·301
32	75,399	763	17·496	85	9,665	1,643	3·067
33	74,636	754	17·382	86	8,022	1,492	2·843
34	73,882	745	17·262	87	6,530	1,332	2·633
35	73,137	736	17·135	88	5,198	1,154	2·440
36	72,401	726	17·002	89	4,044	971	2·262
37	71,675	717	16·861	90	3,073	793	2·095
38	70,958	695	16·712	91	2,280	634	1·937
39	70,263	667	16·553	92	1,646	491	1·790
40	69,596	626	16·380	93	1,155	367	1·653
41	68,970	593	16·190	94	788	268	1·520
42	68,377	561	15·983	95	520	188	1·396
43	67,816	529	15·760	96	332	128	1·273
44	67,287	532	15·519	97	204	84	1·155
45	66,755	541	15·269	98	120	52	1·042
46	66,214	550	15·009	99	68	32	0·914
47	65,664	558	14·740	100	36	18	0·794
48	65,106	579	14·461	101	18	10	0·650
49	64,527	607	14·175	102	8	5	0·517
50	63,920	639	13·882	103	3	2	0·357
51	63,281	721	13·583	104	1	1	0·000
52	62,560	795	13·289				

* For ages under 20 the values are substantially those according to the Mortality Table for females (Oppermann's Tables), used in the Danish Insurance and Annuity Institution of 1871 (Dansk Forsikrings-og Forsörgelsesanstalt af 1871), a mutual institution guaranteed by the Danish Government, which grants insurances and annuities. It was established with new rates in 1871, when a similar, but older, institution was closed for new entrants.

PART II.
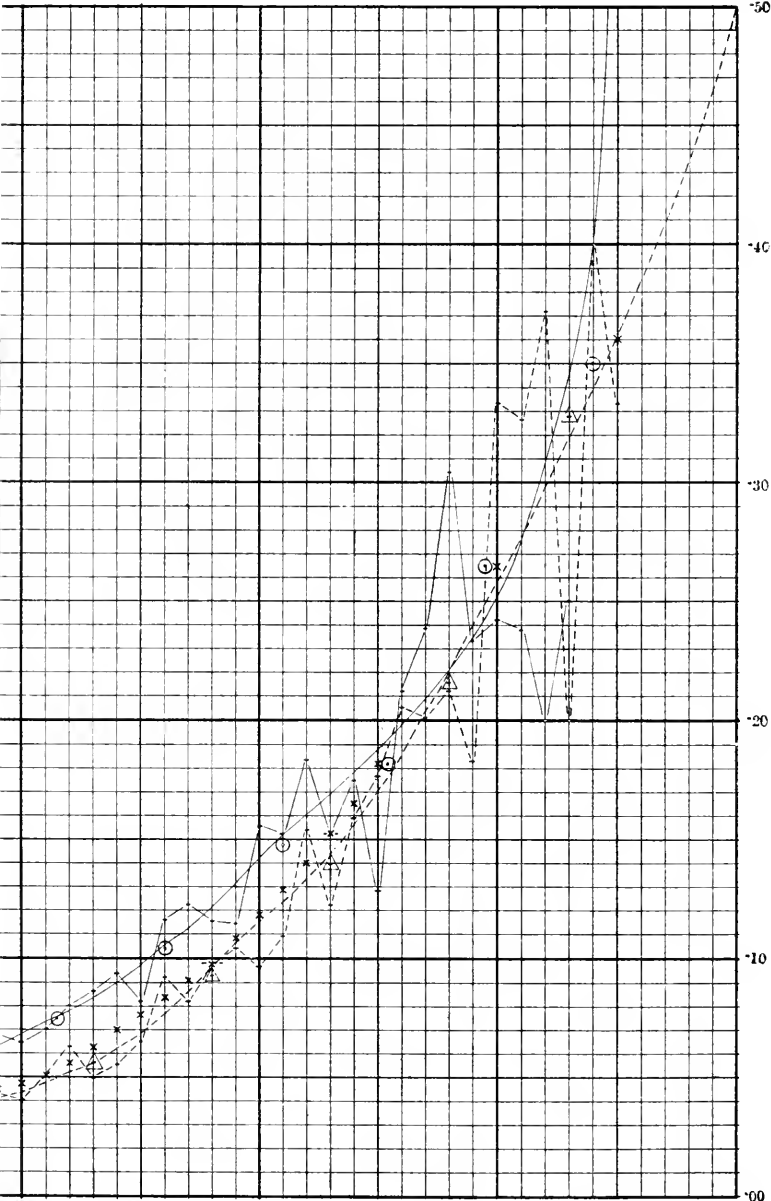
NORWEGIAN GENERAL WIDOWS' FUND.

DIAGRAM.

The abscissa represents the age (x), and the ordinate the rate of mortality (q_x).

———— Males of the Norwegian Widows' Fund, 1775–1885.
- - - - - - Females „ „ „ 1814–1885.
. . . . . . Female Government Annuitants, **four years and upwards after purchase** (Mr. Sprague's adjustment, *J.I.A.*, xxvi, 107)
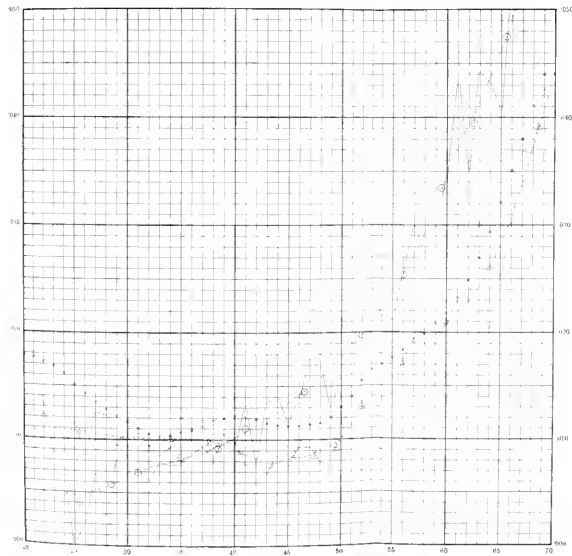
PART I.

PART II.
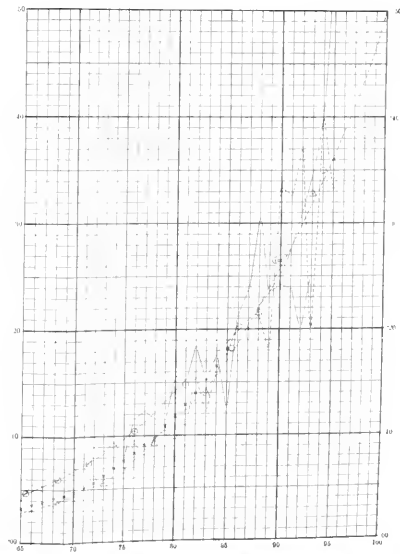
The Liability of Life Assurance Companies to Income Tax.

CLERICAL, MEDICAL AND GENERAL LIFE ASSURANCE SOCIETY
v. CARTER (SURVEYOR OF TAXES).

THIS case was adjudicated upon in the Court of Appeal—consisting of the Master of the Rolls (Lord Esher) and Lords Justices Bowen and Fry—on 16 February last. The judgment, like that of the Divisional Court below (consisting of Mr. Justice Manisty and Mr. Justice Charles), was unanimous in upholding the original decision of the Income Tax Commissioners; and, under these circumstances, the *Clerical* Office and the offices associated with it will probably be thought well-advised in having accepted as conclusive a verdict so weighty in itself and so strenuously adverse to their contention. Understanding, at all events, that it has been decided not to carry the matter to the House of Lords, the time has arrived for placing on record in the *Journal* the materials necessary for enabling our readers to understand what, for the time at least, must be taken to be the law on the question at issue. We accordingly append—(1.) the original case as stated by the Income Tax Commissioners, dated 2 November 1886; (2.) the judgment of the Divisional Court, dated 16 July 1888; and (3.) the judgment in the Court of Appeal above referred to.

The case itself may be said to be a corollary to that of *Last* v. *The London Assurance Corporation* (*J.I.A.*, xxv. 327 *et seq.*). In a paper in the same volume " On the Income Tax, especially in reference to its incidence on Assurance Companies", Mr. Bailey (p. 326) thus summarizes the result of the protracted proceedings in Last's case.

> " The Crown contended that the only profits of life assurance
> " companies arise from the interest on their funds. *The Court*
> " *decided that the income from this source is not profit at all,*
> " *and ought not to be taxed.* The assurance company said that
> " it made profit from its life business, such profit being not the
> " interest on the life fund, but the share of the surplus of that
> " fund appropriated to the company. The House of Lords
> " decided that the assured's share of the surplus must be
> " included in this profit."

The words put into italics state the position which formed the starting-point of the case we are now considering. We have to go to the judgment of Mr. Justice Day in Last's case to find the doctrine here so concisely stated. The learned judge's language is

well-known, and has formed the subject of much discussion. It will be found at p. 333 of vol. xxv:

> " The third question," he said, " is as to the right of the " Crown to levy the duty upon what has been termed the life " fund. This should, in my opinion, be answered in the " negative."

Recognizing the ambiguity of these words, their true meaning would seem, nevertheless, to admit of no real doubt. We think the intention was to remove the life fund in every aspect of it from the purview of the tax-gatherer. This, in fact, was the interpretation put upon the judgment, both by the London Assurance Corporation and by the Crown itself. Not only, as Mr. Bailey said (p. 323), had no attempt been made, at the time he wrote, to contest the decision that the Crown has not the right to levy tax upon the life funds, but in the assessment agreed to (after final judgment) between the Crown and the London Assurance Corporation, *the whole amount of tax paid by the Corporation on the interest of its life fund*—the only part of the fund, *i.e.*, capable, under any view, of being taxed—*was allowed in reduction of the duty to which the Corporation was liable on its total net profits.*

Here, then, there seemed to be a clearly defined position; and on the Income Tax authorities seeking to obtain from the *Clerical* Office by direct assessment income tax on a portion of their interest which had escaped taxation at its source, it was determined to resist the attempt, and a combination of offices was formed for the purpose. Though logically involving the much larger issue of the right to a return of overpaid tax, the case as stated and argued was short and simple, and strictly confined to the point here indicated. It was thus put by Mr. Finlay, Q.C., in the Appeal Court. " When the " profits of a company are derived from interest on investments, is " the interest liable to be taxed as such, and are they liable to pay " income tax upon interest received in full upon a portion of their " investments, although they have already paid income tax by " deduction at the source on an amount which exceeds their annual " profits?"

This question was answered uncompromisingly in the affirmative, and the grounds of the judgment (which carries with it the overthrow of *Last v. The London Assurance Corporation*), are so startling as to open up very serious prospects, not only for the assurance companies, but for dealers in money of every class.

The judgment turned, almost entirely, on the technical, grammatical construction of the last clause of Schedule D in Section 2 of the 16 & 17 Victoria, chap. 34, which runs as follows: " And for " and in respect of all interest of money, annuities, and other annual

" profits and gains not charged by virtue of any of the other schedules
" contained in this Act." It was held that the words " and for and
" in respect of all interest of money" must be read literally, and that
it was not permissible to interpolate words—such, for example, as the
words " profit arising from ", already to be found in Schedule C—
so as to bring the language of the clause into harmony with the
avowed object of the statute, which is entitled, " An Act for granting
" to Her Majesty, duties on profits arising from property, possessions,
" trades and offices."

Mr. Justice Charles, in the Divisional Court, held that there was
in the Income Tax Acts an apparent " intention to tax interest of
money in all cases as being in itself a profit or gain;" and this was
certainly the view taken in the Appeal Court, Lord Justice Fry, in
the course of the argument. going so far as to say " that it might be
" that the Legislature had intended to tax interest specifically as it
" had taxed tobacco," and concluding his judgment emphatically
thus: " The Act appears to me to create a plain charge in respect of
" all interest of money, and that this is interest of money, and it is
" impossible to escape from the charge." We cannot suppose that a
judgment which, so far as the financial world, at all events, is
concerned, is so contrary to the usage of everyday life, can long
be allowed to express the law, though we do think that whatever
change may be brought about in it must result from legislation, and
not from litigation.

LAW REPORTS.

IN THE HIGH COURT OF JUSTICE, QUEEN'S BENCH DIVISION.

(Queen's Remembrancer.)

Between The Clerical, Medical & General Life
      Assurance Society . . . . Appellants,

and

      Carter (Surveyor of Taxes) . . . Respondent.

CASE,

Stated for the Opinion of the Court, under Part III. of the
Act of 37 Vict., cap. 16, sec. 9.

At a meeting of the Commissioners for the General Purposes of
the Income Tax Acts and for executing the Acts relating to the
Inhabited House Duties for the Parish of St. James, in the City of
Westminster, held at No. 12, Savile Row, in the said parish, on

Tuesday, the 2nd day of November, 1886. Mr. Benjamin Newbatt, on behalf of the Clerical, Medical and General Life Assurance Society, appealed against an assessment made upon that Society under Schedule D of 16 & 17 Vict., cap. 34, for the year 1885, in the sum of £2,000, in respect of interest arising from securities belonging to the Society on which the tax had not been deducted at its source, and which had not been otherwise specifically charged.

The Clerical, Medical and General Life Assurance Society carries on the business of life insurance, and derives a large portion of its income from the interest on its investments. This interest (which includes the interest in question from which the tax had not been deducted at its source), together with its other sources of income, is brought into the accounts of the Society, and the printed accounts of the Society show that after debiting all expenses of management and other outgoings, there remained for 1885-6, an average net balance of profit not exceeding £74,250. 12s. 7d. The Appellant showed, by a printed statement (copy of which is attached hereto), that while the average net balance of the profits of the Society had thus amounted to no more than £74,250. 12s. 7d., the Society had paid, by way of deduction, Income Tax on £107,378. 13s. 8d. in respect of the interest derived from its investments.

The Appellant stated that the interest which had so escaped taxation at its source did not amount to the sum assessed, but he admitted that over and above the interest taxed by deduction the Society had received a small amount, namely, £165, in respect of interest from securities in full, without deduction for Income Tax. He, however, contended, on the authority of the case of Last *v.* The London Assurance Corporation, that as the interest from the investments which had been subjected to Income Tax by deduction exceeded the Society's net balance of profits, the interest from the securities in question which had not been taxed by deduction was not liable to Income Tax, and he claimed that the assessment of £2,000 in respect thereof should be wholly discharged.

If the contention of the Appellant be correct, that tax being paid upon an amount of interest from investments exceeding the amount of net profits should exempt the interest derivable from the securities in question from liability to tax, the assessment should have been discharged, but the Commissioners were of opinion that such interest became chargeable under the rules and regulations of Schedule D of the 16 & 17 Vict., cap. 34, and they thereupon reduced the assessment to the amount actually received by the Society, namely, £165, instead of discharging it altogether, whereupon the Appellant declared his dissatisfaction with the decision of the Commissioners, and duly required them to state and sign a case for the opinion of the High Court of Justice, which we have stated and signed accordingly, in pursuance of the 59th section of the 43 & 44 Vict., cap. 19.

Clerical, Medical & General Life Office.   INCOME-TAX ASSESSMENT.   October 1886.

(A)

RETURN of LONDON ASSURANCE CORPORATION, as adjudicated upon by the Income Tax Commissioners for the City of London, which formed the foundation of the case of LAST *v.* LONDON ASSURANCE CORPORATION.

GROSS PROFITS.

MARINE	£27,635 11 9	
FIRE	75,766 19 5	
LIFE	27,454 9 5	
		£130,857 0 7
Deduct Expenses	57,597 10 0	
		£73,259 10 7

Income Tax paid at source on Interest on Investments amounting to . £84,154 15 10

Income Tax overpaid on . £10,895 5 3

Shareholders' proportion only.

(B)

FORM OF RETURN after Final Judgment of House of Lords, as agreed upon between Mr. LAST (on behalf of the Board of Inland Revenue) and the LONDON ASSURANCE CORPORATION.   YEAR........

MARINE PROFITS, viz., Premiums, less Losses, Commission, and Bad Debts £

FIRE   Do.   do.   do.   £

LIFE † Do. viz.:—
  Surplus at Quinquennial Valuation on 31 December, 1880 . £
  Less Balance undivided at previous Quinquennium . . £
                        1/5th) £

Interest, all, except on Life Funds . £

Profit on Realization of Securities (except on Life Account included in Surplus) £

Deduct Expenses of the Year . . . £

Net Profit . . . . £

Taxed Interest . . . £

Amount remaining to be Assessed, or Amount on which the Tax has been overpaid, as the case may be . £

† All Life Profits.

(C)

CORRESPONDING RETURN OF CLERICAL, MEDICAL AND GENERAL LIFE ASSURANCE SOCIETY on basis of (B).

YEAR 1885-6.

MARINE PROFITS . . . . *Nil*

FIRE   Do.   Do.   viz.:— . *Nil*

LIFE † Do. viz.:—
  Surplus at Quinquennial Valuation as at 30 June 1881 . £366,693 19 5
  Less Balance undivided at previous Quinquennium . 7,940 16 4
                        £358,753 3 1
                1/5th) £71,750 12 7

Interest, all, except on Life Funds . £2,500 0 0

Profit on Realization of Securities (except on Life Account included in Surplus) . *Nil*
                                  £74,250 12 7

Deduct Expenses of the Year; as the above is *not* Profit, the deduction is . *Nil*

Net Profit £74,250 12 7

Taxed Interest . . . 107,378 13 8

Amount on which the Tax has been overpaid . £33,128 1 1

† All Life Profits.

THE FOLLOWING ARE THE POINTS intended to be insisted upon on the part of the Appellants on the argument of the special case herein, namely :—

1. That the amount of the annual profits or gains of the Appellants within the meaning of Schedule D of 16 & 17 Vict., cap. 34, did not exceed £74,250. 12s. 7d.

2. That the Appellants are liable, under Schedule D of 16 & 17 Vict., cap. 34, to pay Income Tax on the amount of their annual profits or gains, and on no further or other sum.

3. That the sum of £165, in the special case mentioned, being a part of the life fund of the Appellants, is not profits or gains within the meaning of the said schedule or subject to assessment thereunder.

4. That the Appellants have paid Income Tax on more than the full amount of their annual profits or gains.

5. That the Respondent is not entitled to disregard the fact that the Appellants have, in fact, paid Income Tax under Schedule D upon £107,378. 10s.

6. That the Respondent is not entitled to disregard the fact that the Appellants have, in fact, paid Income Tax under Schedule D upon £33,128. 1s. 1d. in excess of their annual profits or gains.

POINTS for Argument on behalf of Respondent the (the Crown).

On the argument of this case, the following (among other) points will be relied upon on behalf of the Respondent (the Crown), namely :—

1. That the Appellants are chargeable with Income Tax in respect of all interest paid to them without deduction of Income Tax under 5 & 6 Vict., cap. 35, sec. 102, and 16 & 17 Vict., cap. 34, sec. 2, and that the sum assessed is interest so paid to them.

2. That the said sum is chargeable with Income Tax as yearly interest of money or other annual payment under 5 & 6 Vict., cap. 35, sec. 102.

3. That the said sum is chargeable with Income Tax as interest of money or other annual profits or gains under 16 & 17 Vict., cap. 34, sec. 2, Schedule D.

4. That there is nothing in any of the Acts relating to Income Tax to exempt the said sum from the payment of Income Tax.

5. That the decision of the Commissioners is good in law, and ought to be affirmed.

IN THE HIGH COURT OF JUSTICE, QUEEN'S BENCH DIVISION.

Royal Courts of Justice,
Monday, 2 *July* 1888.

Before Mr. Justice Manisty and Mr. Justice Charles.

The Clerical, Medical and General Life
    Assurance Society   .   .   .   .   .   Appellants,
                  and
Carter (Surveyor of Taxes) .   .   .   .   .   Respondent.

Mr. *Finlay*, Q.C., and Mr. *A. T. Lawrence*, appeared as Counsel for the Appellants.

The *Attorney-General*, the *Solicitor-General*, and Mr. *Dicey*, appeared as Counsel for the Respondent.

Monday, 16 *July* 1888.

JUDGMENT.

Mr. *Justice Manisty*.—In this case my brother Charles has been kind enough to write a judgment, and he will read it.   I shall have to make one or two observations which I will make afterwards.

Mr. *Justice Charles*.—In this case the Clerical, Medical and General Life Assurance Society appealed to the Commissioners for General Purposes of the Income Tax Acts for Westminster against an assessment under Schedule D of 16 & 17 Vict., cap. 34, in the sum of £2,000, in respect of interest arising from securities belonging to the Society on which the tax had not been deducted at its source, and which had not been otherwise specifically charged.

The Society carries on the business of life insurance, and derives a large portion of its income from interest on its investments.   This interest, which includes the interest in question from which the tax had not been deducted at its source, together with its other sources of income, is brought into the accounts of the Society.   Those accounts show that after debiting all expenses of management and other out-goings there was, for 1885-86, an average net balance of profit not exceeding £74,250. 12s. 7d.   The Society had, however, paid by way of deductions, Income Tax on £107,378. 13s. 8d., in respect of interest derived from its investments.   They stated that the interest which had escaped taxation at its source did not amount to the sum assessed, but admitted that they had received £165 in respect of interest from securities in full without deduction for Income Tax. The Commissioners thereupon reduced the assessment to the amount actually received, namely, £165, but refused to discharge it altogether. The Society, who contended that the assessment should be discharged altogether on the ground that Income Tax was already overpaid, declared their dissatisfaction with the Commissioners' decision, and required them to state this case for the opinion of the Court.

It was contended for the Appellants that the case of Last *v.* The London Assurance Corporation (12 Queen's Bench Division, 389; 14 Queen's Bench Division, 239; 10 Appeal Cases, 438) has decided that where an insurance society has paid in taxed interest on investments a sum exceeding what they would have to pay on their

net balance of profits, they can be called upon to pay no more. And after an examination of that case it appears to us that this contention is well-founded to this extent, but to this extent only, that it was there assumed that the Corporation were entitled to credit for Income Tax paid at the source on interest on investments, and in the form of return after the final judgment in the House of Lords, agreed upon between the Board of Inland Revenue and the Corporation, the Corporation are credited with taxed interest accordingly. Although, however, the account was adjusted on this basis, the case did not expressly decide the point now raised, which is, whether, where interest on the investments of a life insurance society has not been taxed at its source but paid in full, the Crown is entitled to charge Income Tax upon it, although the taxed interest paid at the source exceeds the sum which would be payable on trade profits ? And we therefore proceed to consider the question apart from the authority of that case.

Now, the schedules of charge are at present to be found in 16 & 17 Vict., cap. 34, sec. 2, which imposes the duty upon the various properties therein mentioned in the schedules. Schedule C is as follows: " For and in respect of all profits arising from interest, " annuities, dividends, and shares of annuities payable to any person, " body politic or corporate, company or society, whether corporate or " not corporate, out of any public revenue." This schedule, it will be seen, is limited to profits arising from interest, &c., payable out of any public revenue. Schedule D runs thus: " For and in respect of " the annual profits or gains arising or accruing to any person " residing in the United Kingdom from any kind of property what- " ever, whether situate in the United Kingdom or elsewhere, and for " and in respect of the annual profits or gains arising or accruing to " any person residing in the United Kingdom, from any profession, " trade, employment, or vocation, whether the same shall be " respectively carried on in the United Kingdom or elsewhere. And " for and in respect of the annual profits or gains arising or accruing " to any person whatever, whether a subject of Her Majesty or not, " although not resident within the United Kingdom, from any " property whatever in the United Kingdom, or any profession, trade, " employment, or vocation exercised within the United Kingdom. " And for and in respect of all interest of money, annuities, and " other annual profits and gains not charged by virtue of any of the " other schedules contained in this Act." It will be seen that there are three distinct heads mentioned, the third of which is general in its terms and applies to all interest of money which, like an annuity, is in our opinion *primâ facie* a profit or gain, and is made specifically a subject-matter of charge. The rules, according to which the duties contained in Schedule D are to be charged, are contained in the earlier Income Tax Act, 5 & 6 Vict., cap. 35, sec. 102, and the last-mentioned section provides for the deduction of the tax on interest or other annual payment by the person liable to pay it, and for the allowance of such deduction by the person entitled to receive it; and further enacts as follows : " In every case where any annual payment " as aforesaid shall, by reason of the same being charged on any " property or security in the British plantations, or in any other of " Her Majesty's dominions, or any foreign property or foreign

" security or otherwise, be received or receivable without any such
" deduction as aforesaid, then and in every such case there shall be
" charged upon such interest, &c., the duty above mentioned."   We
agree in the argument addressed to us, an argument which was
supported by the authority of Foley *v.* Fletcher, 3 Hurlston and
Norman, 769, that this section is not itself a charging section.   It is,
however, among the rules which regulate the levy of duties under
sec. 2 of the later Act, 16 & 17 Vict., cap. 34, which undoubtedly
is a charging section, and specifically charges interest of money
invested.   If deducted at its source it cannot be reclaimed except
only in the case provided for by 5 & 6 Vict., cap. 35, sec. 163,
which exempts persons whose incomes are less than £150 a year from
liability, a circumstance which seems to us to indicate the intention
to tax interest on money in all cases as being in itself a profit or gain.
In the present instance it has not been deducted, and according to
the language of the latter part of sec. 102 is taxable in the hands
of the recipient.   The Society contend that where the interest arises
from an investment necessary to conduct their business, that it is
only taxable as a part of the trade profit, and that if there is no
profit, it should escape taxation altogether, or that if there be a profit
less than the amount invested, the Crown should return the difference
between the amount paid on interest and that which would have to be
paid on the trade profit.   The language of the Act, however, makes
no distinction between the case of investments necessary to carry on
the business and other investments.   The words of the charging
section and of sec. 102 appear to us expressly applicable to this
case, and we are, therefore, of opinion that the Commissioners were
right, and that our judgment must be for the Crown, with costs.

Mr. *Justice Manisty.*—I entirely concur in the conclusion at
which my learned brother has arrived, and which he has just stated
as the opinion of us both; but I want to make one or two general
observations.

It seems to me that a life insurance company stands by itself.
It is very peculiar.   Its main income is derived from investments,
and the interest received from those investments.   It seems to me
that there is a good deal of difficulty arising from the fact that that
seems in the minds of some to be confounded with the profits
other than the income derived from interest, and that that has
given rise to a great deal of argument, and no doubt some difficulty,
in the case which went to the House of Lords of Last *v.* The London
Assurance Company.   The two things, to my mind, are essentially
different.   By the Act, income consisting of interest, or annuity, or
dividends, or shares of annuities, are taxable both by Schedule C and
Schedule D of the Act of 1853.   Here is a life insurance company.
It has a large sum of money invested.   The interest of that money
amounts to a large sum, and upon the whole of that very large sum—
£107,000—of taxed interest, the tax has been paid, and it happens as
an incident, or an accident almost, that £165 escaped taxation at the
source.   Now, what an extraordinary thing it would be, and how
contrary to all one's notions of sense would it be that the whole of the
income, or the whole of the interest is liable to be taxed and has been
taxed, and yet a mere sum like this, where the interest was not taxed,
is to escape taxation.   There is no provision, as my learned brother

has pointed out, for getting back the taxed interest on the £107,000.
Then upon what principle is it that that which has not been taxed is
to escape ?　Suppose that none of this had been taxed, and that all
had been paid without deduction, could it be said that there was no
tax payable ?　It seems to me to follow, as a matter of course, that if
the whole of that interest, save £165, was subject to taxation, the
£165 also was subject to taxation.　The difficulty, and the arguments
founded upon it, arises very much from confounding the interest
with the profit ultimately taxable, if any.　It may be there is none.
It may be there is a good deal of profit, apart altogether from this
question; but the Act has made this taxable in the first instance, and
there is no means of getting it back.　The ordinary case is quite
different.　Say a man has £100,000 in consols.　He receives the
interest of that, but he also engages in a trade.　It may be he is a
coal owner, a lead-mine owner, or a salt-mine owner.　All these
things are the subject of taxation, if there be a profit made by it.
But because a man has a large income derivable, say, from land, or
has money in the funds, and also engages in a trade or business, the
income which he derives from that or from the funds is not to be
taxable, provided he uses it in a trade or business, where he, perhaps,
loses it all, is, to my mind, a fallacious argument.　The two things are
essentially different; and it has been decided by the House of Lords, in
the Mersey Dock *v.* Lucas, that you have nothing to do with how the
income is used.　The income is taxable.　It may be it is all lost; it
may be it is so used that a large profit is made in some other way.
But I wish it to be clearly understood that in this case we are not
dealing with the question of the profit which ultimately may be made,
and how it is to be divided, and how it is to be taxed.　We are simply
dealing with the one question.　Is that £165 taxable just the same as
the whole £107,000 was?　It seems to me it cannot admit of a doubt,
and that our Judgment must be for the Crown, with costs.

IN THE SUPREME COURT OF JUDICATURE,
COURT OF APPEAL.

Royal Courts of Justice,
Friday, 15 *February* 1889.

Before The Master of the Rolls, Lord Justice Bowen,
and Lord Justice Fry.

The Clerical, Medical and General Life Assurance Society,
v.
Carter (Surveyor of Taxes).

Mr. *Finlay*, Q.C., M.P., and Mr. *A. T. Lawrence*, appeared as
Counsel for the Appellants.

The *Attorney-General*, the *Solicitor-General*, and Mr. *Dicey*,
appeared as Counsel for the Crown.

JUDGMENT.

The *Master of the Rolls.*—In this case the question which is
raised seems to me to be this: The Crown has assessed the Defendants
in respect of certain interest on money which has been paid to the

Defendants, and it is admitted that it is such a payment of interest
as, if it is not met by the objection put forward on behalf of the
Defendants, is an interest which would be liable to assessment; but it
is said, on behalf of the Defendants, that this interest is not an
assessable subject-matter, because it is interest which comes to the
Defendants in the course of trade and business in this way: that that
interest is an item in their trade and business, which is an item
of their gross profits in their trade or business, and that when you
come to set off against their gross profits, including this item, it will
be found that they have made no profit at all.   That is the point
raised.   The question is whether this interest is the interest which is
spoken of in Schedule D of the Act of 1853, the 16 & 17 Vict., cap. 34.
If this interest is the interest which is mentioned in the last
paragraph of Schedule D of that Act, then there are words plainly
that the persons who are in receipt of that interest are assessable.   It
is suggested on behalf of the Defendants that that word "interest"
is not to include this interest, because this interest is, as I have said,
an item which is to be dealt with in order to arrive at the annual
profits and gains of the trade or business.   We have heard a long
argument, and a most able argument, and a puzzling argument, as all
arguments about this Income Tax Act seem to me to be.   It is
enough to puzzle one's head off, nearly; but we must, after all, deal
with this Income Tax Act according to the ordinary canons of
construction, and we must construe it according to those canons;
and, unless it produces a clear and manifest absurdity, we must
construe it according to the ordinary grammatical interpretation of
the English language as applied to such a subject-matter.

Now, I find that in the Act of 1842 there is no such section as
sec. 1 of the Act of 1853: the Act of 1853 has departed from
the forms of the Act of 1842, and departed from them at the very
beginning.   In the Act of 1842 there is no such section as sec. 1
of the Act of 1853; but when you come to the Act of 1853 there is
the first section of the Act, which, as I read it, does deal with the
interest on money as a distinct and separate subject-matter of
taxation.   When you find that that is so, and that that alteration is
made in the Act of 1853 from the Act of 1842, it seems to me that
the argument which was addressed to us, that Schedule D in the Act
of 1853 is not to add any subject-matter of taxation beyond those
which were taxable in the Act of 1842, falls to the ground.   They
have put in the Act of 1853, expressly at the beginning of it, that
interest on money shall be a separate and distinct subject of taxation.
If that is so, when you come to Schedule D you find these words:
"And for and in respect of all interest of money."   There is that
which in sec. 1 is made a separate subject-matter of taxation here
in the very same terms that people may be charged in respect of all
interest of money.   Looking at sec. 1, and looking at the ordinary
grammatical construction of this part of sec. 52, I cannot see my
way to add any words to it, or to add the words which have been
suggested, that "in respect of all interest of money" is to be read
"in respect of profits or gains, if any, in a trade or business arising
from every interest of money."   It seems to me to be too large a
paraphrase for us to make, and we must read the words in their

ordinary sense, and, if so, this interest, which is the subject-matter
of the contention in this case, is an interest which is taxable.   I
think that that view of the matter, arising from a perusal of the
whole Act, beginning at the beginning and going to this schedule,
is greatly fortified by the argument of Mr. Dicey.   I think that
when you come to look at the Act it is true to say that all interest,
where it can be, is taxable, and I agree with the argument that that
goes far to show that if all interest is deductable, all interest is
taxable; but I only use that argument myself as fortifying the
ordinary reading of the statute according to its ordinary grammatical
construction in the English language.   That is a cardinal rule, as it
seems to me; and that is the cardinal matter with regard to this
Schedule D.

Now, if you read Schedule D, and read it in that way, it seems to
me it is really immaterial to consider what is the true construction of
sec. 52; because, if you once get to that as the proper way of
reading Schedule D, if there is the hardship which it is suggested
will arise upon the true construction of sec. 52, we cannot help
the hardship.   If it is true to say, as for the purposes of argument
Mr. Finlay has so much insisted, that they cannot only tax this
interest as a subject-matter of itself, but can afterwards bring it into
the proceeds of the gains and profits of the trade, and so tax it again.
If that is true, it cannot be helped.   But I am not going to adopt
that argument as to the proposed legal construction of sec. 52.   I
took notice that the Solicitor-General, upon behalf of the Crown, has
argued, and argued strongly, that that is not the meaning of
sec. 52; and he has argued, and argued strongly, that that
hardship could not arise; and, so far as I can bind the Court, I give it
as my opinion that I accede to that argument, and that sec. 52
must be so read that hardship cannot arise, and that the people are
not to be taxed on such interest as this, first of all as a subject-matter
of itself, and then, by bringing it into the proceeds of the gains and
profits of trade, tax it again.   I agree, I say, with the argument that
that cannot be done, and if so the hardship does not apply.

Therefore for these reasons I am of opinion that the decision of the
Divisional Court was right, and that this Appeal must be dismissed.

Lord Justice Fry.—I am of the same opinion.   Schedule D of the
Act of 1853 creates a charge in respect of all interest of money.   The
£165 in question is interest of money, and, therefore, *primâ facie*, it
is chargeable.   Now, what is the short conclusion of the matter upon
the part of the Appellants?   It is attempted to remove the force of
that very short line of argument by an appeal to various considera-
tions.   One was thrown out during the course of the argument—
rather perhaps by invitation of the Court than by Mr. Finlay's own
desire—which would have raised the question whether interest,
followed as it is by the words "annuities and other annual profits and
gains", must be annual—that is to say, interest payable for the year.
But that point, not having been raised in the Court below, is not now
open for consideration, and I only advert to it for the purpose of
preventing it being said or thought that that point will be involved
in the present decision.   That point, if point there be, will be open
for discussion when the case may arise.

Then, before passing to the statute of 1853 itself, I ought to observe that the conclusion at which I have arrived from the very words I read, namely, " in respect of all interest of money ", is fortified by the first section, because that states the intention of the Legislature to charge, amongst other things, all interest of money; therefore there is a very clear expression of an intention, to my mind, that those words should be used in the ordinary signification which they bear in the English language. Again there is the forcible argument which has been addressed to us by Mr. Dicey, pointing out that all interest of money paid is a subject of deduction, and therefore it is highly probable that all interest received should be a subject of charge. Now that case is met by the Appellants inviting our consideration to several arguments. In the first place, it is said that the words upon which I have relied are for the first time introduced into the Act of 1853; they are not found in the early statute of 1842. If the point were material for consideration I would only say that my present impression is that interest of money was as much chargeable under the Act of 1842 as under the Act of 1853. In the 102nd sec., and in other sections of the Act, it appears to me clearly indicated as a subject of charge; and I think the general words of Schedule D in the earlier Act are probably adequate to charge it. But the point appears to me to be immaterial for the present consideration; either it was charged by the earlier Act of 1842, and the language of the Act of 1853 is only introduced for the purpose of making that plain which was not so plain before, or the Act of 1853 was intended to have a wider area and ambit of charge, and has effected it by the plain words to which I have referred. Again, it is said that there is gross injustice in charging the interest of money when people choose to deal in money, and however legitimate a charge that be where the recipient is not dealing in money, yet, where he deals in money and makes profits, or desires to make profits, in his trade as such dealer, the interest ought not to be charged. I had some difficulty in following the force of that argument. It seems to me comparatively immaterial what a man does with the money when he has once received it; and I think that the words " profit and gains " used in the Act of 1842 are used with reference to the profits and gains of particular investments in many cases, not necessarily to profits and gains of the trade of which those investments may be part.

But then it is said, on the other hand, that this alleged injustice does not exist, because the 52nd sec. of the Act of 1842 removes it. Whether that section does or does not remove it in all cases, it appears to me to be reasonably plain that it removes it in all cases of payment of interest by the person who is himself chargeable with it : whether it would prevent that alleged injustice, if injustice there be, arising in a case in which the payer's interest was beyond the operation of the Act, and the payee was within it, is not necessary to consider—it is enough to say that in a large number of cases the alleged injustice would be removed by the effect of sec. 52: but I fall back upon that with which I started—that the Act appears to me to create a plain charge in respect of all interest of money, and that this is interest of money, and it is impossible to escape from the

charge. The only thing I can say is that the Court below is right.

The *Master of the Rolls.*—Lord Justice Bowen agrees with this judgment.

The *Solicitor-General.*—I understand that your Lordship dismisses the Appeal with costs ?

The *Master of the Rolls.*—Yes.

---

*Friendly Societies.**

[Reprinted, by permission, from the *Quarterly Review*, April 1888.]

It is just a hundred years since the necessity of wise legislation, to further the measures of self-help adopted by the industrial classes, began to be urged by Sir George Rose and others. In 1793 the first Friendly Societies Act was passed. Its avowed object was to protect and encourage societies of good fellowship, formed for the purposes of the mutual relief and maintenance of the members in sickness, old age, and infirmity, and the relief of the widows and children of deceased members, and effecting those purposes by means of the voluntary subscriptions of the members. Later Acts have enlarged this definition, but the keynote of all legislation in this country with regard to such societies is struck by that Act. It affirmed that this protection and encouragement would be likely to be attended with very beneficial effects, by promoting the happiness of individuals and at the same time diminishing the public burthens. This prediction has not been falsified, though it has been fashionable to assert, that friendly societies by failure have caused misery and pauperism, and to overlook the vast benefits they have conferred on their members.

The facts are that the hundred years have been years of continuous progress for friendly societies, and that they are now in a sounder and healthier condition than ever before. The members have gradually learned to know more exactly what they want and the right way to obtain it; and the kindly, old, pre-scientific notions of equal rates for everybody, the young paying for the old, and the like, have gradually given way to more accurate, and therefore more really equitable, methods of working. The influence of the general annual assemblies of the affiliated orders has been great in this direction. In every respect in which self-government has had free scope, its operation has been beneficial; and the little friendly clubs

---

*1. *Reports of the Committees of the House of Commons on National Provident Insurance. Sessions* 1885, 1886 *and* 1887.

2. *Report of the Hon. E. L. Stanley to the Chief Registrar of Friendly Societies on his Inspection into the Affairs of the Royal Liver Friendly Society, with Letter of the Chief Registrar thereon,* 1886.

3. *Reports of the Chief Registrar of Friendly Societies,* 1876 *to* 1887. *Year Book of the Friendly Societies Registry Office,* 1886.

4. *The Friendly Society Movement: its Origin, Rise, and Growth; its Social, Moral and Educational Influences; the Affiliated Orders.* By the Rev. J. F. Wilkinson. London, 1886.

of a hundred years ago have developed into societies having large
accumulated capitals, the fruit of the savings and the forethought of
their members, exercised by way of provision for sickness and death.

This statement of the fact is not in accord with the prevailing
impression on the public mind.   Before and since the first committee
of investigation into the working of the Friendly Societies Acts sat,
in the year 1825, it has been matter of constant reiteration, that
friendly societies as a whole are " rotten " and " unsound."   This
impression is largely due, as we think, to a want of knowledge of the
extent of the question, and of the variety of dissimilar bodies
included under the general term " friendly society."   It will be well,
therefore, to clear the ground for the discussion of the subject by a
statement of these several varieties.   They were grouped by Her
Majesty's Friendly Societies Commissioners in their Report of 1874
into as many as seventeen distinct classes, some of which again were
divided into sub-classes, presenting marked differences in various
respects from each other, in their organization and management.

It is not necessary, for the purposes of this article, to discuss at
any great length the special characteristics of these numerous classes.
Two or three of them, however, are worth examining in some detail.
The first class named by the Commissioners is that of the affiliated
societies or orders.   Their history is very instructive.   Though it
covers nearly the whole of the hundred years over which our review
extends, the time during which they have enjoyed the recognition of
the law is little more than one-third of the hundred years.   It was the
policy of the earlier statutes to ignore them, and the Act of 1850
was the first which permitted them to register.   The Act of 1855
qualified their branches also to register, but, by a strange oversight,
did so in language, the legal construction of which rendered necessary
their registry as separate societies, and not as branches.   It appears
from the useful little work of Mr. Wilkinson that these orders
originated in an imitation of the Freemasons, and adopted a ritual
and nomenclature similar to theirs.   Thus the Odd Fellows have
their lodges, the Foresters their courts, the Shepherds their
sanctuaries, the Romans their senates, the Sons of Temperance their
divisions, the Rechabites their tents, and so forth, as designating in
each case that which in the eye of the law is a branch.   So for the
designation of the chief officer, the Odd Fellows adopt the term
Noble Grand or Grand Master; the Foresters, Chief Ranger; the
Shepherds, Chief Shepherd; the Romans, Most Excellent Consul; the
Sons of Temperance, Grand Master; the Rechabites, Chief Ruler,
and so on.   Their ceremonies on initiation, their secrets, their
lectures, their degrees, and other matters of ritual are also founded
on those of the orders of Masonry, and in their origin they were a
similar harmless festive charity.   Indeed, it may be said of all
friendly societies that eccentricity of nomenclature is their strong
point, as witness: " Who'd have thought it Lodge", " Court
Solomon's Greatly Wondering", " The Pursuers of Righteousness",
" The Reviving Stag Lodge", and a host of other strange titles
which might be quoted as adopted by these societies.

The assemblies of these bodies are described by Mr. Wilkinson as
" social meetings having a moral tendency."   Their growth into

organized benefit societies was slow and gradual. Their means of relief to members in sickness and distress were at first the casual donations of the members, afterwards converted into a fixed sum by way of periodical payment, and only long afterwards into a definite assurance earned by contributions nicely adjusted to the risks to be provided against. The like development may be observed in the history of friendly societies of all classes—from the old Guilds downwards. Members assemble in a brotherhood; one falls into distress, the others agree to relieve it by donations. In course of time it is observed that this method is precarious; that it would greatly save the self-respect of the distressed member if his relief were provided out of some fund raised by a fixed contribution made by all members alike, instead of depending upon the casual impression which his needs and his merits have made upon each member individually; while the discovery, that distress is a result of a general law, and that it is practicable to apply the lessons of experience so as to suit the remedy to the risk, and apportion the contributions accordingly, is a long step in advance. In regard to the affiliated orders, other steps remain to be taken. Their arrangements lead to a system of federation which, when fully developed, may tend to a perfect type of friendly society. In the most highly-organized orders there are three stages : the lodge, by whatever name it may be called; the district, which is an aggregation of lodges; and the order, which unites the whole. The usual arrangement was that the lodges insured sick pay, the districts death money, and the order itself insured nothing. The Friendly Societies Act of 1875, however, in defining what constitutes a branch, introduced the double test—that a branch is a number of members of a society under the control of its central body, and contributing to a fund which that body administers; yet having a fund under its own control, administered by it or its committee. The necessity of complying with this definition has made that universal which was before only partial among the orders, namely, the raising a fund under the control of the central body by a contribution from every branch. Previously, many orders had not cared to have any fund under the control of the central body other than that derived from the profit upon the sale to the lodges of the books, regalia, and other matters which the rules of the order required that they should all be supplied with by the central authorities of the order.

Several orders have accordingly raised a fund for the relief of distressed lodges and districts; but as the relief granted from that fund is casual and discretionary with the central body, it does not amount to a complete federal guarantee of the engagements of the branches. A lodge remains still responsible for its own sick claims; and if a lodge becomes insolvent, the only relief its members can obtain is such as depends upon the discretion of the central body of the order, exercised with reference to the past conduct of the lodge, and, indeed, to any other matters the central body may think material, and is not a claim of right on the part of the members. Whether any such claim of right could in safety be granted, is a question for the future to answer; if it should be answered in the affirmative, the affiliated system would then become a real federal

union, and not, as most of the orders now are, a mere gathering of bodies, each of them independent of the others.

Under the present system a lodge may be wholly insolvent, and a neighbour lodge possess a large surplus; but the members of the one have no claim on the other. So also a district may be unable to meet its claims; and as many districts raise their funds by the bad system of equal levies upon the lodges according to the number of their members, that is an event which is certain in the long run for each of such districts: while a neighbouring district, especially if it has raised its funds by the equitable method of graduated contributions, will have a large surplus; but the members of the one have not, and ought not in justice to have, any claim upon the other. It appears clear, therefore, that before any federal guarantee of the claims of members upon branches can be attempted, the orders must prescribe the rates of contribution which the members of the branches shall pay, and also supervise their management.

The larger orders have already forecast the future in these respects, and have resolved upon energetic measures to compel their lodges to adopt proper rates of contribution, and to husband their funds. In this they have had unexpected but valuable assistance from the High Court of Appeal, in the case of *Schofield* v. *Vaux*,* where it was decided that a lodge having once declared itself a branch of an order, and admitted members upon that footing, cannot secede from the order except by compliance with such rules as the order itself may make; and this, whether the branch is registered as such, or registered as a separate society, or indeed, as it would seem, if it is not registered at all.

Many of the smaller orders have also made most praiseworthy exertions to improve the condition of their lodges. Most of them originated in secessions from the larger bodies of minorities who were discontented with measures resolved upon by the majority; but even these have learned wisdom by experience, and finally adopted for themselves the very measures they would not tolerate when imposed upon them by others. A notable case of this is the National Independent Order of Odd Fellows, which, under the judicious guidance of its secretary, Mr. Cleveland, is rapidly rising to a much higher place among these organizations than it has hitherto possessed. Indeed, the working of the spirit of reform and progress is observable in nearly all the orders.

It is curious to observe how the decision in the above-mentioned case puts the finishing touch to the gradual process by which these orders have advanced to full legal recognition. Looked upon at first as wholly illegal institutions, within the mischief against which the Corresponding Societies Acts were intended to provide; capable of being used, as was thought, for all sorts of evil political purposes; at enmity with religion and with social order: such were the senseless prejudices the orders had for many years to meet and to live down. It was not till 1850 that they were admitted to registration at all; in

---

* This important case is not to be found in the authorized reports, but a report has been printed from the shorthand notes by the central body of the Manchester Unity.

1875 the Legislature devised a means of giving full legal recognition
to their contracts with their branches; and in 1886 the judicature
upholds the contract in its full force. Thus one more instance is
added to the many, which enforce the lesson legislatures are often so
slow to learn—that the test whether a contract ought to be legalized
is whether it is one the people desire to enter into, not whether it is
one the legislator thinks to be good for them. The province of the
Legislature is to enforce the contracts men make, not to tell them
what contracts they ought to make.

Even more instructive is the history of the progress of some of
these great orders towards perfection in their arrangements with
their members. The Manchester Unity of Odd Fellows has been
foremost in its efforts to secure financial soundness. Startled out of
its security, many years ago, by the pamphlets of the late Mr. F. G.
P. Neison, who showed that the society as a whole was in a deplorable
financial condition, having funds less by many millions than it
required to meet its liabilities, according to actuarial estimates, the
society entered upon the wise course of closely observing and
tabulating its own experience, under the able superintendence of its
secretary, Mr. Henry Ratcliffe; of making periodical valuations of
the assets and liabilities of every lodge; of enforcing upon all new
members the payment of properly-graduated rates of contribution;
of gradually bringing the contributions of the older members more
nearly to the mark of safety; and of requiring from all new lodges
the adoption of proper rules and rates. The result has been that
every succeeding valuation has shown a marked improvement, and
that the time is not far distant when this great order in all its lodges
will be financially sound, even from the actuarial point of view.

Another step in this direction has just been taken, namely, the
enforcement upon the districts of properly-graduated rates of contri-
bution for their insurances of burial money. The Annual Movable
Committee for 1887 has settled the question in the only right way,
the districts giving up the haphazard and unfair system of levies.

The method by which these gradual improvements have been made,
or are in progress, is that of an annual conference of delegates from
each branch, held each year in a different town, and thus spreading
all over the country the knowledge of sound principles. These
conferences usually last several days, and are conducted with great
decorum and good sense. In the larger orders the annual conference
is attended by several hundreds of individuals, the freely-elected
delegates of lodges and districts numbering hundreds of thousands of
members. At these meetings amendments of the general rules of the
order are considered and resolved upon, many of which directly or
indirectly affect the whole body of the members in all the branches.
The discussion of these questions—particularly if they are such as
involve an advanced knowledge of the requirements of actuarial
science—will often extend over several successive years, and slowly
and gradually gain ground.

In the existence of so useful an educational instrument as this
annual conference, the affiliated orders have a great advantage over
the isolated societies. It is to them we look for further improvement.
One which has not yet been perceived by the societies to be necessary,

but which we are convinced lies at the root of all sound management, is that of granting a surrender-value. When we consider for how long a time the assurance companies felt no compunction in confiscating other people's money, by the wholesale forfeiture of policies upon casual neglect to pay a premium, we need not be surprised to find that the moral sense of friendly societies is still equally undeveloped. As in the assurance companies of late years, the greater respect paid to the claims of justice has led to more careful and equitable management, so, we are persuaded, would it be the case with the friendly societies.

Such a reform as this would pave the way for the crowning achievement which lies before the orders, namely, the real and complete federation of their branches; so that the order should be able to guarantee the benefits promised by every branch. The attainment of this would so thoroughly mark the superiority of organized and centralized bodies such as the orders over the isolated and individual society, that the latter would before long disappear, and the orders only hold the field. This seems to be already the case to a great extent in the colonies: in that of Victoria, Australia, there is not a single independent society registered, but only branches of orders. For this to be desirable, however, it is necessary, first, that the orders at large should follow the examples of the Manchester Unity and Ancient Order of Foresters in the reforms that they have already adopted, and then take the further steps which we have indicated.

One drawback to the general excellence of the affiliated system must, however, be noticed. It is its expense. Sir George Young, in his report as Assistant Commissioner, points out that " the threefold " management of order, district, and lodge, costs much more than that " of a local club ; not of course more by three times, but still more. " The expense of sending delegates to the district and general meetings " is often a severe tax. The percentage of expenditure in a well- " managed lodge is higher than the average in large ordinary and local " clubs, and about equal to that in country societies. It would be still " higher, but that the secretaries are not, in general, paid the full " market value of their services." To this it may be added that the members often incur large voluntary extra expense in " regalia " and other sources of outlay not included in the expenses of management from the funds.

The space which we have devoted to this particular class of friendly society is not too much for the interest it possesses as being entirely the creature of the members, and not of the Legislature or of the enlightened patron; as being the result of a process of natural development, and as having in it the promise of the future for friendly societies, not only in this country, but in the colonies and the United States. We may now pass on to another group of societies, and point out the indebtedness of the industrial classes to many worthy and excellent men among the clergy and the wealthier orders of people for a wise initiative and for excellent management. No record of the history of friendly societies would have any pretence of completeness that did not place prominently forward this branch of the question, and show how much they owe to generous and enlightened patronage.

Among the pioneers in this course of kindly help was the Rev. J. T. Becher, Prebendary of Southwell. Early in the present century he framed a set of rules, and obtained the services of the best actuaries of that day for the construction of tables of rates, as models for use by societies; and the result was the constitution of several bodies under the name of Becher Clubs. Towards the expenses of management, subscriptions were obtained from honorary members, and it was one of the principles of the scheme that honorary members should have a place on the committee, and therefore a share in the control of the club. It was also provided that there should be a periodical valuation of the assets and liabilities, that the accounts should be rendered to the members in very full detail, and that a member desiring to leave should receive the full value of his assurance benefits.

Another clergyman to whom friendly societies owe much is the late Hon. Samuel Best, who originated what is called the deposit system. This is a combination of the friendly society and savings bank. A minimum contribution is required from every member, and the balance is retained for his benefit. If he falls sick, a portion of the sick pay is provided out of the contribution he has made to the general fund, the remainder being taken from his own private fund. When the latter is exhausted, the allowance ceases. The plan is excellently devised to meet the objection often raised by the unthinking to the system of an ordinary friendly society, that the member who escapes sickness gets nothing for his money. Under the deposit system, the fund belonging to the members who do not claim sick pay will in time reach a handsome sum, and form an abundant provision for old age; but, on the other hand, if a member suffers from prolonged sickness, he finds that his private fund has become exhausted, and the allowance from the common fund dies with it. This is, of course, no more than an application of the common proverb, "that you cannot eat your cake and have it too"; but it compares unfavourably with the system of the ordinary friendly society, which guarantees sick pay out of the contributions of the other members. Hence, though several deposit societies have been formed, they have not attracted large bodies of members, or become really popular. Mr. Best's personal influence did much to promote their success during his life, but it has not been maintained to any extent worth mentioning.

The name of the late Right Hon. W. E. Forster is another that should be remembered with respect, whenever the history of friendly societies is attempted to be traced, however superficially. He was struck with that cardinal difficulty of the question, how best to provide for old age? The ordinary friendly society cannot extract from its members a contribution larger than that required for relief during such sickness as occurs to a member while he is of the working age; whenever it holds out the promise of relief during old age, it does so only to disappoint the members at their time of greatest need; and this promise, and the consequent disappointment, have served as the text of those who condemn friendly societies. Mr. Forster, with a view of gaining the best information as to the means of meeting this difficulty, offered three prizes for the best essays on

the question, and obtained from authors connected with existing
affiliated and other societies a number of suggestions, which have
been adopted in the formation of several societies, and may tend to
help the members in old age.

In this honourable roll of men of culture and experience who
have aided the members of friendly societies with sympathy, counsel,
and active assistance, the name of Mr. John Tidd Pratt should not
be omitted. When the Act of 1827 for regulating savings banks
was passed, he was appointed under it to be the barrister for certifying
rules, and the final referee in disputes. In the next year (1828) the
law relating to friendly societies was consolidated, with many amend-
ments. It was found that the provisions of the earlier statutes—by
which the rules of an intended friendly society were to be submitted
to the justices in Quarter Sessions for their allowance and confirma-
tion, and a certificate produced to such justices, from two professional
actuaries or persons skilled in calculation, that the rates of contribu-
tion proposed to be charged would be sufficient—had altogether failed
in their intended object. While some justices took a strict view of
the requirements of the law and rejected certificates from unqualified
persons, others accepted any certificate that was tendered, and the lax
ones were the majority. The village schoolmaster was the favourite;
and his certificate, as a "person skilled in calculation", as indeed he
was, started many societies on their career. Probably it was as
valuable as, at that time of day, any certificate would have been, for
then, and for many years thereafter, the skilled actuary had no data
of any real value to go upon, and actuaries of high reputation
certified tables for societies which subsequent experience showed to be
wholly insufficient for safety. It was clear that the justices and the
skilled persons had broken down : and the Legislature wisely resolved,
that henceforth it would look after the text of the contract between
members of societies from a legal point of view, and leave the terms
of it to mutual agreement. Accordingly, all provisions for making
societies safe by Act of Parliament were abandoned, and it was
provided that societies should submit their proposed rules to a
barrister, to see that they were so framed as to carry out the
intentions of the members. To the savings bank barrister appointed
in the previous year this new duty was confided, and thus originated
the connection of Mr. Tidd Pratt with friendly societies, which lasted
more than forty years. Had he been disposed to take a merely
perfunctory view of his duties, he might have made them very easy,
but he was a man of great force of character, and he felt it
incumbent upon him to go beyond the narrow limits of his official
functions, and to assist with all the powers of a very original mind
every person who came across him with a scheme for the improvement
or development of friendly societies. The present occupant of the
office of Chief Registrar, Mr. John Malcolm Ludlow, C.B., for whose
appointment the public are indebted to the wise judgment of the late
Earl of Iddesleigh, is also a man who has made his name honoured by
many years of self-denying exertion for the good of the working
classes, and has well earned the confidence they repose in him. The
excellent Year Book which he has prepared, expounds, in the
minutest detail, the relations of the societies to the office over which he

presides, and cannot fail to be of great use to the members of societies. It is a masterpiece of clear exposition, and no officer or member who has it can justly excuse himself for the neglect of any duty or the loss of any right.

Among the societies which owe their success to the efforts of such men as we have named, are the large county societies established in many parts of the kingdom, and also numerous town and village societies. These have, in most cases, met with all the prosperity which the excellent intentions of the founders merited, but not with the popularity that should have been the consequence of it. Those for whose benefit they were intended have never joined them in the large numbers that might have been expected. The reason of this has been partly the high rates of contribution necessarily demanded by a society where a condition of its existence is actuarial soundness, as compared with those demanded by many village clubs; the uninstructed tendency of the workman to join a club in which the payments he has to make are more suitable to his means, even though the benefits it promises to him are not so secure, trusting that it may last his time, or that in the chapter of accidents it may be able to meet its claims somehow; and partly a preference for societies which he is able to manage badly for himself, over others which are managed well for him by other people. It is not to be overlooked, moreover, that the convivial and social elements in the man are more fully gratified in the small village club than in the patronized county society. The question has been revived of late years, whether societies should be allowed to be formed at all which do not give some guarantee of the soundness of the calculations upon which their proceedings are based? and Lord Greville has introduced into two successive Parliaments a Bill to prevent the establishment of any society without an actuary's certificate to its tables of rates. By the existing law a society cannot be registered which insures to its members a certain annuity or a certain superannuation, without a certificate from a qualified actuary to the tables of contribution for such insurance; and Lord Greville would extend that provision to the tables of contribution for all benefits, such as sick pay and the assurance of a sum at death, and would extend it, moreover, so as to prohibit the formation of any society without such a certificate. In former years, as we have seen, Acts have been passed prohibiting the registry of societies without some form of guarantee that their tables are sound. In the Act of 1819 provision was made, that no tables or rules connected with calculation were to be confirmed by the justices until they had been approved by two persons, at least, known to be professional actuaries or persons skilled in calculation, as fit and proper, according to the most correct calculation of which the case admitted.

It has never before, however, been contemplated, that the establishing a society without a certificate of an actuary should be an offence. One cannot help thinking that the proposers of this Draconic legislation have never formed in their own minds an idea of what it implies. No little group of men in a country village are to agree that they will share the risks of life together on terms that seem to them fair and equal, unless they can induce some actuary to

say that the terms are so. No contract having regard to the relief of possible future distress is to be entered into, unless it squares with the rules that some actuary may lay down. The people who choose to do what they will with their own and defy the actuaries are to be treated as criminals. A slate club is started; no actuary will certify it; so, send the members to prison.

We do not wish to be understood as disparaging the scientific efforts of actuaries to ascertain the real value of the risks which members of friendly societies desire to assure against, or as throwing any doubt upon the results which they have obtained, or upon the probable importance of the great series of observations—extending, we are informed, over more than ten millions of facts—which the actuary to the Central Office for the Registry of Friendly Societies has been long engaged in preparing from the quinquennial returns for the quarter of a century from 1855 to 1880. Scientific accuracy is of the highest possible importance, but in this question it is not everything. In a matter which depends so largely upon the means of the members to meet their wants, and upon the power they have to adjust the one to the other, there is more to be thought of than mere actuarial science.

Many societies have existed for long series of years, and have done much good to their members, whose tables would never have been certified by an actuary; many, indeed, have never had any tables at all. By the kindly forbearance of members who have been fortunate enough to be beyond the necessity of claiming the benefits they have assured for, by the cultivation of a sense of good fellowship, and by the formation of a sort of *esprit de corps* which forbids malingering and safeguards the funds of the society, many such societies have been prosperous and useful for generation after generation. It should be made matter of careful and anxious reflection by those who seek to level all societies up to actuarial soundness, whether they may not by such means lose other elements of soundness and usefulness still more worth the keeping.

The case of the dividing societies is one quite in point. Since the Act of 1875 provided, that a rule for, or practice of, dividing any part of the funds of a society should not disentitle it to registry, if the rules contained distinct provision for meeting all claims upon the society existing at the time of division before the division takes place, the great majority of the new friendly societies formed have been of this class. Before the passing of that Act such societies were equally popular, but they either remained unregistered or omitted from their rules all mention of the practice of division. The reason of the popularity of these societies is not far to seek: having no desire to make provision for old age, but merely for those accidental sicknesses and calamities which affect men in full work, the members see no necessity for accumulating a large fund, and prefer themselves to enjoy the fruits of their savings. They are willing, moreover, to pay a larger contribution than a man will easily be induced to pay to a permanent society, because the overplus comes back to them at the end of the year. The arrangement is well understood by the members, who get out of it all the advantages they seek, and there is no principle of sound policy upon which legal

sanction should be refused to it, merely because it has not other advantages which they do not seek.

The same observation applies to the proposal that it should be compulsory on every society to become registered. Translated into plain English this means, that every little knot of men who care to combine together for any provident purpose, to put their own money together for any purpose of mutual relief, shall be held guilty of an offence for which they are to be punishable by imprisonment, if they do not put themselves in a certain relation with a department of the State. This is a grave departure from the sound principles of our old Common Law, which left men free to act for themselves in many matters which modern theories of State socialism propose to hand over to the Government. The existing law itself goes further than there is need in this direction, for the decision of the High Court in the case of the *Padstow Total Loss Association** shows, that a mutual benefit society may in certain circumstances be deemed to be a society for purposes of profit, and if so, it becomes an illegal society if unregistered as soon as it consists of twenty members; but the mere illegality of the society would not satisfy the wishes of some of our law reformers, who would make its very existence a criminal offence by each of its members.

We need not pursue, in further detail, the other varieties of the ordinary friendly societies for insuring relief in sickness and payments at death which were enumerated by the Commissioners. One class, however, is distinguished in every possible way from all others, and should always be thought of as distinct : it would be well, indeed, if it were provided for by a separate Act of Parliament, and this we understand is proposed to be done by a Bill of which Sir John Lubbock has charge. The class referred to is that of the great collecting burial societies, the type of which in its highest development is the Royal Liver Society. This society carries on its operations throughout the United Kingdom, and has upwards of 1,100,000 members. The nature of the interest of these members in the society is widely different, however, from that of the member of an ordinary friendly society. He is insured, not for sick pay, but only for a small sum as burial money. He attends no meeting to pay his contribution, but waits till a collector calls for it. Among the members are included infants of all ages. The members, as such, have as yet had no real voice in the management.

With these societies are correlated others not registered under the Friendly Societies Acts, but doing the same business in the same way as the collecting burial societies. The most important of these industrial assurance companies is the Prudential, which has even a larger business than the Royal Liver Society, and, being a proprietary company, does not affect to give the persons insured even an illusory share in the management. The small weekly contributions of a few pence from each policyholder afford in the aggregate an enormous margin of profit for the benefit of the proprietors. On the other hand, the expenses of management are enormous. This is to a great extent unavoidable, for the services of collectors must of course be

* 20 Ch. Div. 137,

paid for. These companies, however, not being under the Friendly Societies Acts, withdraw themselves from the criticism which the registered collecting societies are exposed to, though there is reason to fear, that their manner of doing business is little, if any, better.

The admirable provision of the Friendly Societies Acts, enabling a small number of the members of a registered friendly society to apply to the Chief Registrar for an inspection into its affairs, has recently thrown a flood of light upon the manner in which the Royal Liver Society has hitherto carried on its business. We have no reason to suppose that this society is any worse than others of its class, but every ground for belief that, on the whole, it is the best of them. If that be so, we may well wonder what further disclosures, than those made in Mr. Lyulph Stanley's report, would result from an inspection into some of those others: the more so that his inspection was itself cut short by a series of very curious incidents.

So much of the story of the society as Mr. Stanley was able to elicit is very instructive. It rejoiced in the possession of two managing secretaries, of whom it may not unfairly be said that their qualifications to be managers, in the sense of knowing how to get their own way, were vastly greater than any literary qualifications they possessed for the office of secretary. In 1875, which date was adopted by Mr. Stanley as the starting point for his inquiry, these officials had the not ungenerous salary of £20 a week, or £1,040 a year each. At the annual meeting of the society held in that year, it was resolved that, in addition to their salaries, they should have a commission of $2\frac{1}{2}$ per-cent each on all new business. In 1881 their salaries were raised to £40 a week each, in addition to the commission, which by this time brought them in a further sum of £2,277 each per annum. By the time the application to the Chief Registrar was made their income amounted to £5,980 each per annum. Before the enquiry took place they resolved to bow to the storm, forego their commission, and accept fixed salaries of £3,000 each. When it closed they resigned at once their offices and the emoluments they derived from them, upon the simple stipulation that bygones should be bygones, and that no officer of the society should suffer for the side taken.

This very summary statement of the relations of the secretaries to the society suffices to show, that it was really managed in the interest of the officers, rather than in that of the members. Mr. Stanley's report, however, fills out the outline with many incidents of interest. The managing secretaries governed the society with all due constitutional form. They had a committee of management: that committee had its share of the spoil. The salary of each member was £884 a year. But, says Mr. Stanley, "during the whole of the ten "years over which my inspection has extended, the two secretaries "have been everything, the committee nothing. The very litho- "graphed forms that go out from the office, state on the face of the "resolutions of the committee, Resolved *unanimously.* For many of "the most important decisions of the office there is no minute "whatever; large payments have habitually been made on the order, "often verbal, of the secretaries, or of either of them." The committee met daily, but their main function seems to have been

drawing their salary. "The way in which the accounts and records
" of the society are kept makes it practically impossible for a member,
" if he claimed his right, under the Act of Parliament, of examining
" the books of the society, to find out anything material from them."

Mr. Stanley further reported, that the exorbitant remuneration of
the secretaries led to other mischief. Being calculated by way of
commission, it gave them " a strong motive for increasing the mass of
" the business, so as to swell the gross receipts, regardless of the fact
" that they were greatly increasing the cost of management." Before
1875 the cost of management had been less than 40 per-cent; in 1877
it was increased to over 47 per-cent, or, in other words, out of every
shilling collected from the members about $5\frac{1}{2}d.$ went in management.
The mischief was growing. " Not only was the money of the society
" spent on canvassers and travelling agents, but also an aggressive
" policy was set on foot. This policy led to underhand dealings with
" the collectors and agents of other societies, to litigation, often
" unavowed and secret, to bribery of newspapers, and the circulation
" of libels through hired writers who posed before the public as
" independent authors."*

Among the consequences that would naturally follow, as they did
follow, from this method of doing business, was a complete disregard
of the society's rules. The collectors were permitted to levy charges
upon the members, beyond those authorised by the rules, to the
extent of £2,000 a year. The minute-book was kept in a slovenly
manner. " Many important documents were not forthcoming, and
no record was kept of the most important letters of the office."
" It is obvious that in a society like the Royal Liver, there are only
" two parties who have any real power, the head office and the
" collectors. The members are scattered throughout the country.
" They have no cohesion, no power of knowing each other, and too
" small an interest to take the trouble to master the facts and vote.
" A general meeting called in Liverpool, with perhaps 1,500 persons
" present, of whom 600 or 700 will be officials and their families and
" friends, is an absurd mode of governing a society with 1,100,000
" members, £360,000 a year of premium income, and more than
" £800,000 of invested capital."† No one can doubt the justice of
this remark of Mr. Stanley.

We need hardly follow him into the details of the war carried on
by the secretaries with rival societies, or the strange expedients they
used to gain the business of societies which had failed, or the curious
history of their relations with their eminent consulting actuary.
Those who have the free use of large sums of money can obtain the
services of highly-placed professional men as auditors or actuaries, or
valuers of property, and the secretaries of the Royal Liver Society
fully appreciated their advantages in this respect, and did not
hesitate to use them.

Professional men of high position were not, however, the only
instruments used by the managers of this society for their own
purposes. When the business in hand was the controlling the
proceedings of a general meeting, they used instruments of a very

---

* Stanley, p. 7.   † Ibid, p. 9.

different kind.   In 1880, a Mr. Harper, who had previously been an
ally of the secretaries, was dissatisfied with the nature of the society's
investments, and gave notice of motion to call attention to them.
" His evidence is distinct that Mr. Liversage," one of the secretaries,
" when Mr. Harper said at the meeting he would go on with his
" motion, said ' Look round.'   He saw a lot of roughs.   One of them,
" whom he knew, told him they were there to break a fellow's head
" who was going to move a resolution.   He says further, that
" Frederick Atherton," brother to the other secretary, "was at the
" head of those roughs."*

It is hardly necessary to pursue any further this sordid story.
Under the name of a grand provident institution for assisting the
very poor to provide for that which is to their sentiments a matter of
the most urgent necessity, viz., a decent funeral when they depart
this life, we find a concern worked for the personal benefit of two or
three men and their immediate satellites, every possible device
resorted to to maintain their power, and the money, that might be
made available for increasing the benefits of the members, wasted
recklessly in the personal interest of the managers.   To the men
themselves, the prize was so rich that it is hardly to be wondered at
that they should grow unscrupulous as to the means by which they
would seek to retain its possession.   To those outside, who had some
inkling of what it was worth, it was equally attractive, and invited
attack.

The result of Mr. Stanley's inspection was the resignation of the
two managing secretaries of the society, and the appointment in their
place of the chief mover in the agitation for reform.   The expenses
were thus considerably reduced, and the rules of the society have
since been altered, so as to provide for something in the nature of
representative government; but it may well be doubted, whether
much has in fact been done to remedy the evils which seem to be
inherent in this class of society.

Is it right or even expedient that the insurances of the poorer
classes should be left in the hands of societies like these?   The
question is difficult to answer.   It is better that they should insure
their lives, even upon such costly terms as are required to meet the
expense of such management as this, than that they should not be
insured at all.   Can means be devised for granting them the benefits
of insurance on easier terms?   Yes: if they will only seek the
insurance themselves; but that is exactly what they will not do.
They have to be canvassed and coaxed to effect the insurance; to be
constantly visited, or they will not keep it up: they cannot be relied
upon for any spontaneous action in the matter.   If they would only
go to a Government office, such as the Life Assurance Department of
the Post Office, they would get better served on cheaper terms; but
they will not go, and it is difficult to see how the department can go
to them.

It has been suggested, for instance, that the letter carriers of the
Post Office should be employed as collectors of the penny subscrip-
tions of the poor for the purpose of life assurance; but a hundred

---

* Stanley, p. 10.

difficulties arise as soon as the proposition is put in practical form. The letter carrier would have to make a special visit, for the class which the burial society collector deals with is not one that has a large correspondence to receive. He would have to receive a considerable sum of money, in respect of which a system of checks would have to be devised that would be exceedingly difficult to maintain. He would have to create in his own mind a special enthusiasm for the work, which the moderate pay he gets as a letter carrier is hardly enough to engender. In other words, he would have to be paid by a commission on his receipts, and the Government department to which he is attached would have a great deal of difficulty in adjusting a number of delicate questions that would arise under this head. He would have to be instructed to use all the arts of persuasion which the burial society canvasser and collector is trained to practise, or he would not succeed in inducing the poor to insure, or to keep up their insurances. Is there any chance that the inferior officers of a government department could be drilled into the exercise of functions so far apart from the ordinary course of their duties, or, with the utmost desire to do all that could be expected of them, would succeed as well as the ordinary canvasser?

The conclusion seems to be that, if the persons for whom these small insurances are desirable could be educated to the point of seeking them for themselves without the intervention of a canvasser, and keeping them up without that of a collector, these large burial societies and industrial assurance companies could well be dispensed with; and a government office could provide the insurances better and more cheaply than they do; but while the poorer classes remain so unwilling as they are to take trouble for a future benefit, and so ready to drop a policy rather than put themselves to any inconvenience about it, the canvasser and the collector will continue to be necessary evils, and all that can be done will be to introduce into the legislation respecting such societies any provision that may tend to increase the power of the members, and limit that of the managers. This was the meaning of the several provisions of s. 30 of the Friendly Societies Act of 1875, which debarred collectors from becoming members of the committee of management of such a society, and prohibited their voting and acting at its meetings. Mr. Stanley obtained evidence to show, that these provisions of the Act are largely evaded, and it may not be impossible to devise some means of better enforcing them.

It is to be borne in mind all through the consideration of this matter, that the persons insured in such societies belong to a lower stratum of the population than the respectable artizan who forms the bulk of the members of the ordinary friendly society. It is also not to be overlooked, that illegal practices prevail among them: in particular that of the insurance of the lives of others. In any society under the Friendly Societies Acts, whether of the collecting burial society class or any other, it is illegal for a member to insure the life of another person, beyond making a provision for the necessary funeral expenses of the wife or child of the member himself; yet it is not unusual in these societies for members to insure the lives of more distant relatives, or even of persons to whom they are not

related at all. It is obvious that, apart from its illegality, a practice like this is a real danger to life. The effect of it, however, in a legal point of view, does not seem to go beyond the invalidating of the policy under the Gambling Act of last century; and it may be well worth the consideration of the Legislature to devise some specific penalty for conduct which is thus not only illegal but dangerous. Cases have occurred in the criminal courts in which the existence of policies of this kind has been an incentive to murder.

Another risk in the same direction is that arising from the insurance of the lives of children. A large proportion of the nominal " members " of these societies are infants, who are admitted in some societies from birth. These may be lawfully insured by their parents for a sum not exceeding £6. It is to be hoped that this sum is not large enough to tempt a parent to murder; but it is certainly dangerous to give the parent even the smallest monetary interest in the loss of an infant's life. It would, at any rate, have been better if the amendment introduced into the House of Lords, reducing the amount to £3, had been adopted. It was, however, objected to by the House of Commons, and not insisted upon by their lordships. Three pounds would, in most cases, be enough to cover the really necessary expenses of the burial of a young child, and whatever goes beyond that is dangerous. It is fair to state that there has never been any direct evidence of child murder for burial money existing as a system. A few flagrant instances have been discovered; but it is for the honour of human nature to hold that they are exceptional. The real danger is not so much that of actual murder, as the tendency to look upon the death of the child as a source of profit, and thus to neglect the means of keeping it in healthy life. Many of the deaths of children among the poor arise from this cause, and the existence of a money interest in the death is likely to increase rather than diminish the tendency to neglect. We think it not improbable, moreover, that the excellent provision of the Act which forbids the payment of a sum of money on the death of a child under ten years of age to any person other than its parent, or the personal representative of the parent if deceased, is too often transgressed or evaded. Even upon this point, however, it must be admitted that the Chief Registrar has not been able to obtain such evidence of malpractice as would justify prosecution in a single case in the ten years which he reviews in the reports before us. The general conclusion upon this important head is, that the evils arising from child insurance are less than might have been feared, but yet sufficiently real to justify and render necessary the maintenance and the strengthening of these precautions which the Legislature has devised to obviate them. The anomaly of infant membership has ceased in respect of societies established since 1875, as they are prohibited (except in the case of certain societies having members wholly between three and sixteen) from having members younger than sixteen years of age.

The broad distinction to be drawn between the large collecting society and the friendly society of every other class is simply this: —the one is managed by the members for their own benefit, the other exists for the benefit of those who manage it. In the latter

case, though the contributions may be ample for actuarial safety, and may indeed be excessive, the tendency will always be to increase more and more the demands upon them for the cost of management; and thus the unfortunate members may not only pay during a long series of years a sum greatly more than necessary to secure the insurance they desire, but may find that even that is not secured for them, for their partners in the unequal bargain have robbed them even of the share which was their right under the contract. The cases of the " Independent Mutual Brethren Society " and the " United Assurance Society " are illustrations of this, as are also those of the "Swansea Royal", " Integrity ", and " British Workman's " societies. The story of the first named, as told by the Chief Registrar in his reports, and with more detail by Mr. Sutton, is curious. Originally started as an ordinary small local society, it fell into the hands of people who saw their way to convert it into a society for their own benefit, rather than that of the members, and devised an ingenious system of lodges all over the country, which, not being registered as branches, gave their members no real share in the management of the concern, but looked so much like genuine branches of a self-governing unity as to induce members to join under the impression that they were so. From the very first, these people expended in the management of the society, especially in the establishment of these lodges, sums far in excess of their fund for management.

Year after year, the officers of the society were prosecuted by the Registrar for not making the required returns or valuations, as the case might be, and were content to pay the penalty, knowing that nothing more could be done to them. This did not prevent their establishing fresh " lodges ", getting more members, and spending all the money which the members were unwise enough to trust them with. A course of management such as this must come to an end in time; and at last the efforts of a few clergymen and others to make known the real character of the society, at the risk of actions for libel and heavy costs, had some success, and the influx of new members ceased. Funds were not forthcoming to meet claims; judgments were obtained against the society in many local courts in various parts of the country, only to find that nothing was left to satisfy them.

The " United Assurance Society ", originally called the " United Assurance Society of St. Patrick ", had been established more than 50 years, and when it closed its doors had nothing left out of the contributions of members, some of whom had subscribed to it for nearly the whole time of its existence, and all of whom belonged to a class of hard-working poor who could ill afford to lose their money. It had a large branch in Ireland, and extended its operations wherever the poor labouring class congregated. So little of cohesion was there between the members, that after the closing of its doors it was found impracticable to get 500 signatures, necessary for an application to the Chief Registrar to exercise the power given to him under the sections of the Act, which permit compulsory dissolution on the ground of insolvency, and enable an investigation to be made into the causes which led to the disaster.

The same sad story is to be told of the other societies named—wanton extravagance, injudicious and even fraudulent investment of funds, and a continuance of business, including the admission of new members, long after the insolvency of the society had become manifest. In one case only was a criminal prosecution instituted against the responsible managers, and that resulted in a conviction. In that instance, the assets had dwindled down till they consisted of nothing but a printing-press, which itself the society had no right to invest its funds in. In another case, the managers had ingeniously started a building society, to which they lent the funds of the friendly society, without disclosing the names of those to whom the building society advanced them. This society succeeded in inducing an industrial assurance society to undertake its liabilities, and to accept a transfer of the remnant of its assets as a consideration.

It must not be supposed that the evils inherent in this class of society have sprung up all at once. Long before the report of the Commissioners in 1874, Mr. Tidd Pratt had for years called attention to the unsatisfactory management of the collecting burial societies, and had shown in his annual reports, that they were every year expending in management as much as or more than they gave in benefits to their members, and that their accumulated assets were ridiculously small as compared with the liabilities they had incurred. Ten years after he had commenced agitating the question, the legislation of 1875 provided special checks upon this class of society, and the experience of the ten years which have since passed seems to show that these checks are insufficient. What further measures ought to be adopted?

If the State cannot enter into effectual competition with these societies, as we have seen it is hardly likely to be able to do, it is evident that their regulation and not their extinction is what should be aimed at ; for as long as they attract members who will not insure through any other medium, they fulfil a useful purpose, and the millions, to whom they have rendered assistance in time of need, far outnumber the thousands to whom their failure has caused disappointment and loss. If they can be prevented from increasing the number of the latter, they may well be left to add to the former class as many as they can. It is better that a poor man should be insured for a small sum at an unduly high rate of premium, than that he should not be insured at all. How are we to secure that he shall not be disappointed?

Something might be done by giving the proper authority power to wind up compulsorily a society of this class, when a valuation showed that its assets were insufficient to meet its liabilities. When that is the case, it is evident that the society has been ill-managed, for the contributions of members to these societies are always more than sufficient to meet the calculated risk. They, in fact, ask the members for much more than is really necessary to effect the insurances. Hence the appearance of an estimated deficiency in a valuation is proof positive against them. The authority, to whom power would be given to act in such a case, should be entitled to make such arrangement as he thought fit for the benefit of the members, by obtaining for them a reduced insurance under the

guarantee of some solvent society, or in default to distribute among them the assets of the society while it had any.

Power might also be given to the proper authority to interfere in cases where, from the annual return or otherwise, it appeared, that the society's funds were being improperly invested, or wasted in unnecessary expenses of management. In a case of this kind, it would not be unreasonable for the State to appoint a receiver of the society's funds, and thus ensure that no expenditure other than legally necessary for carrying on its business should thenceforth be incurred, and that no investment other than those contemplated by the Act and suitable for such a society should thenceforth be made. If it be thought too strong a measure to allow the Chief Registrar to take a step of this kind on his own motion, it might at least be provided that it should be competent for him to do so, when applied to by a certain number of the members of such a society.

Though the societies of this class present so unfavourable a view, and are to be looked at quite apart from the ordinary friendly benefit society, they form no exception to the general statement we have ventured to make, as to the progress of friendly societies during the last hundred years. The legislation of 1875 has served only to bring to light, and to assist in providing a remedy for, evils which existed to a much greater extent before its passing, as the reports of the Royal Commissioners and their assistants abundantly show. Previously the ruffianism, the extravagance, the dishonesty reigned unchecked; the machinery of inspection, the checks which the Act provides on the power of the collector, and the full publicity enforced as to the real facts of the society's position by the annual return and the quinquennial valuation, have made the condition of these societies very different from what it was before 1875.

Reverting from these societies to the ordinary type of friendly society, it may have been observed that we have laid little stress upon a matter which is one of the distinctive features of the Act of 1875, and which is undoubtedly of great importance, namely, the requirement once in every five years from every society of an actuarial valuation of its assets and liabilities. We are of opinion that, great as the usefulness of these valuations may hereafter become, the time has not come when a general conclusion can safely be founded upon them, that it is not from a single valuation, but from a comparison of the results of successive valuations, that inferences should be drawn as to the real condition of societies: that many elements in their solvency are necessarily omitted from the considerations of the valuer, and that only by a very slow process can trustworthy results be developed from his work.

If the requirement of compulsory valuation had been part of the law thirty years ago, as it is very much to be wished it had been, we should now be in possession of materials for forming a just judgment of the actual position of the bulk of the societies; and it would doubtless have been in every respect much better than it is. The knowledge of the actuarial conditions of soundness in friendly societies is a very important element in their management; and if that knowledge had been generally imparted to them a quarter of a century earlier than it has been, they would have had time to correct many of the mistakes they have innocently made, and would have

distributed the relief they offer more equitably among the members standing in need of it. Now that this powerful instrument has been furnished to them their progress in this direction will, doubtless, be rapid.

We have said, and as far as limits of space have allowed, have shown, that the hundred years during which friendly societies have been organized in England, have been years of uninterrupted progress towards better things in respect of their safety and their usefulness. We look forward towards the next hundred years with confidence that this progress will continue, and that year by year friendly societies will become safer and more useful. There are many, however, who are not content with the slow and gradual process of improvement from within, and are anxious to devise means by which the thrift of the working classes may be stimulated from without. Committees of the House of Commons have been sitting during the sessions of 1885, 1886, and 1887, "to inquire into the best system of national provident insurance against pauperism." It is not surprising that the committees should have been unable to agree upon any recommendation which would combine all the conditions implied in this reference. The system to be devised is to be the best; it is to be national; it is not to infringe the great principles of providence; it is to be an insurance; and not only so, it is to be a prophylactic against pauperism. The first witness before the committee of 1885, the last witness before that of 1886, and the principal witness before that of 1887, were the same, namely, Canon Blackley, who in the first instance defined a scheme which appears to him to satisfy these conditions, and in the second and third replied upon the objections that had been raised to his plan by others. The committee of 1887 has finally reported against it.

The scheme has much in it that is taking at first sight. It seeks to meet the pauperism which arises from sickness or old age by the insurance of a small weekly sum; to enforce the payment of the necessary premium at a very early period of life; and it proposes to require this payment from every individual, rich and poor, male and female. It does not provide any sum at death; and its author seems to contemplate that, though every person is to contribute the premium, no one is to claim the benefit of the insurance unless he is in such a condition of need that he would otherwise be compelled to become a pauper. It is here that we think the weakness of the proposed system mainly appears. Pauperism is a great evil, and any method that could be devised to diminish it, to any material extent, must be a great blessing; but pauperism may exist without residence in the workhouse or the receipt of outdoor relief.

Obviously, therefore, the first thing to be done in devising a plan by which the State can insure against pauperism, is to define exactly what is the pauperism that is to be insured against. Canon Blackley states his proposal in the following terms:

"I propose that every individual in the nation shall be liable, by law, after reaching the age of 18 years, to contribute, either in one sum or by instalments, £10 or thereabouts to a national sick and pension benefit society, which would secure to him or her, when prevented by sickness from earning his or her usual wages, a sum of 8s. a week until 70 years of age, and after 70 years of age a cessation of the sick pay, but a pension for life of 4s. a week."

If the premium were adequate, and the sick pay and pension were duly paid to every person who had paid the premium, the difficulty of defining what is pauperism for the purpose of such an insurance would not arise, for every purchaser, that is in the case supposed every individual of the population, would receive the insurance, whether he would otherwise have become a pauper or not; but the premium is not adequate for that purpose, and it is not intended that all who pay it shall have the benefit of it. Mr. Blackley seeks to distinguish between those who are wage-earners and those who are not, and to give the benefits of his scheme to the former class only. The result of course is that, for all persons who are not wage-earners, the payment of £10 is a tax and not an insurance; or if the possible contingency of a person now not a wage-earner becoming one before he is seventy years of age be taken into account, it is the insurance of a remote and trifling risk at a very high premium.

Canon Blackley, indeed, urges that:

"If we go to our workhouses we find that there are plenty of people there who were once rich—men who have kept their packs of hounds. We want that every one shall give security to the nation against requiring relief from pauperism, and that he shall give that which will secure him against claiming the poor rate. No man, however well provided he is at present, can say that you can give a cheaper security than would be afforded by the £10 that he will never become a pauper."

In this he seems to depart from his definition of wage-earner; but however that may be, he omits to notice that £10 would be an unduly high premium to charge where the risk of becoming a pauper is small, in other words, that he is insuring an unequal risk by equal premiums.

The confusion between an insurance and a tax, which lies at the foundation of the plan, is observable in many of its details. An insurance of two distinct benefits should be granted only in consideration of separate premiums. The manner in which the premium for sick pay ought to be calculated is very different from the manner in which the premium for a deferred annuity ought to be calculated. Accordingly, the premium for the one ought to be kept separate from that for the other. In a sick pay assurance, it is to be expected that there will be fluctuations from year to year, and that the experience of a very sickly year will be compensated for by the favourable experience of healthy years to follow. In a deferred annuity there is no such compensation to be looked for. The body which contracts to pay a deferred annuity must see that it receives the exact premium which is the calculated equivalent of the annuity, and that such premium is invested and the interest reinvested the moment it is received until the day when the annuity comes to be payable. The first thing essential is that the exact premium should be ascertained; the next, that it should be actually received. The scheme does not provide effectually for either the one requirement or the other. It mixes up the premium for the sick pay with that for the deferred annuity, and trusts to alterations in the premiums of future entrants, to be made from time to time when periodical valuations show them to be necessary, to remedy any errors. But a moment's reflection will show, that no periodical valuation can affect

the premium that ought to be paid for a deferred annuity.   Until
the deferred annuity is entered upon it is a mere matter of accumu-
lating the premium at compound interest.   The sick pay may vary
from year to year, but the deferred annuity does not come to be a
claim until fifty years after the contract has been made.   If, by
mixing up the premiums for the sick pay with those for the deferred
annuity, the latter fund is made to bear the burden of any excess of
sick pay in an unhealthy year, it suffers not merely by the actual cash
sum abstracted from it, but by all the accumulations of that sum
until the time for entering upon the annuity.   £1 paid down now
would accumulate to £8 in fifty years' time ; and therefore if £1 is
unduly abstracted from the deferred annuity fund now, that fund will
be deficient to the extent of £8 when the annuity comes to be
payable.   If £6 is the value of 4s. a week at age 70, a loss of £1
would reduce the pension to 3s. 4d.

We think that the distinction between an insurance and a tax is
very important to be borne in mind, in the consideration of this
matter.   In an insurance, if it be on sound and just principles (and
no other ought of course to be undertaken by the State), the
premium ought to be as accurately adjusted to the risk as the
conditions of the problem will allow.   If it be an insurance against
pauperism, supposing such an insurance to be at all practicable, the
present value of the sum assured would have to be multiplied by the
antecedent probability of the person insured becoming a pauper, and
the result would be the premium that ought to be charged.   If the
probability of one person becoming a pauper is less than that of
another person becoming one, he should pay a less premium.   If he
is charged the same premium, especially if he is compelled to pay it
whether he will or not, the payment becomes a tax and not an
insurance.

The reason this consideration is so important in its bearing on the
present proposal is, that the whole moral effect of the scheme depends
upon the light in which it is looked at by those who receive benefit
under it.   If they are led to suppose that they are insured persons
when the premium they are paying is not adequate to the insurance,
but is supplemented by a tax which other people are paying for their
benefit, the moral advantage supposed to be offered by the scheme as
a prophylactic against pauperism disappears altogether.   They are
paupers, and the pauperism is worse because all the old safeguards
against it, which existed in the self-respect of the labouring classes,
and their reluctance to claim relief provided out of the funds of
others, have been done away with.   The old poor law of Queen
Elizabeth, when it enacted in effect that no one should starve in this
country, but that the cost of providing for those who cannot provide
for themselves should be met by the whole community, and the
burden distributed among the members of it according to their
several abilities to bear it, had these safeguards against abuse.   The
new poor law of Canon Blackley, which is to enact that everyone
who can establish a claim on the ground of poverty, may have
4s. a week for life after attaining the age of 70, and that the cost
of such annuity shall partly be met by the contributions of those
who paid the same sum, but cannot by the conditions of the case

obtain any benefit, would enable the receiver to become a burden
upon other people, but relieve him from the shame.

The provisions for insuring the actual receipt of the premium are
the next matter to be considered.   The theory is, that the whole
amount of the single premium for sick pay up to 70 years and
for the deferred annuity thereafter, estimated loosely at £10, is to
be paid by every individual before he attains 21 years of age.   It is
suggested that a payment of 1s. 3d. a week for three years would
amount to the £10, and that every wage-earner could set aside out of
his wages that sum between the ages of 18 and 21.   If he will not
do so, it is proposed that his employer should be compelled to deduct
it from the wages when paid.   From those who are not wage-earners,
the £10 is to be collected in the same way that other taxes are
collected.   In one way or other, every individual is to be made to
pay.

With regard to the wage-earners, it is obvious that if the
employer is to pay, he will be in the position of having to pay a
youth of 20 years of age 1s. 3d. a week more than he would have to
pay a youth of 21 years of age.   This, of course, he will not
willingly do; and he can only be compelled to do it, if the labour-
market is so ill-supplied that he cannot get a sufficient stock of
workmen over 21 years of age to do his work.   As long as the labour-
market is in such a condition that men can be got to work at
minimum wages, boys will not be employed at minimum wages, *plus*
1s. 3d.   The plan in this respect may be left to work its own cure.

The demand is to be made from female wage-earners as well as
from males.   Every girl in the kingdom between the ages of 18 and
21 is to pay £3. 6s. 8d. a year.   How is she to get it?   Is female
labour so highly paid that there is a margin left for investment after
this rate?   Is not the fact the very reverse of this?   It is possible
that the framer of the scheme has contemplated a state of society in
which such an amount of saving would be barely practicable.   What
kind of female employment offers such a margin of income?   If there
be any, is it wise or right to forbid the poor girl to apply her savings
to that which is the legitimate ambition of every female—the giving
herself, not wholly unprovided, to the man of her choice, with the
little nest-egg that will help in plenishing their home?   Why should
the State insist upon her buying a deferred annuity which she does
not want, and deprive her of the chance of happiness that is within
her grasp?   May she not justly ask that the want which may fall
upon her when she is 70 should be left for the future to provide
against, when the alternative is her falling into want that will hurt
her more sorely when she is 20?   What moral obligation rests upon
her, or for the matter of that, upon anyone whatever, to provide
against the wants of the next half century when the want of the day
is pressing?   The amiable Canon who advocates the scheme shows a
sad want of appreciation of perspective.   To him the prospect of
pauperism at 70 looms as large as the actuality of starvation now;
but it is hardly likely to do so in the eye of the struggling poor.

Turning from these to the case of those who are not wage-earners,
the injustice of the scheme becomes even more apparent.   If the
wage-earner will not pay, his employer must; he who earns no wages

cannot pay, therefore his parent must. If this were a Malthusian device to impose a penalty on the father of a large family, something might be said in defence of it; but as the penalty is only to fall when the family is grown up, it may be held to be superfluous. The scheme is fallacious from every point of view: but probably the fallacy, which lies nearest the bottom of it, is that there is no other or better way of insuring against pauperism than that of buying a deferred annuity. The parent of a youth, who has not yet begun to earn his living, may be pardoned for thinking that, if he has given him the best education in his power, has apprenticed him to some trade or profession, has endeavoured to inculcate in him principles of prudence and right living, he has done something towards preventing his becoming a pauper at the age of 70, and that the proposal to tax him £10, as a further provision against that risk, is an unjust interference with his right to form and act upon the best judgment he can for his child. One of the worst features of this scheme is its socialistic phase : it assumes that all mankind have the same wants, and the same means of providing for them; indeed, it claims to be founded on Prince Bismarck's theories of State socialism. We are not sure that it would be any the better suited to English needs if that were the fact, but the evidence of Dr. Aschrott tends to show that it is not so. Prince Bismarck's scheme relates to the working-classes only, and is innocent of anything so absurd as the notion of universal insurance—it does not involve the strange expedient of a single premium—and it has nothing to say to a deferred annuity. These are the distinctive features of Canon Blackley's scheme : and not one of them appears in Prince Bismarck's. Even as regards the workmen, all that is required is that they should be members of some approved friendly society, not (except as regards accidents, and in that case only for certain specified trades) of any State institution whatever. Dr. Aschrott, indeed, said, " I do not believe that we shall ever get a compulsory insurance for old age " (q. 1726) ; and again, " I approve of all schemes which facilitate a man's securing a property " of his own for his old age; and everything that prevents a man from " using all the money he can spare in order to get a small property of " his own I would oppose (q. 1729)." We confess that we entirely agree with this witness.

Assuming, for the sake of argument, that the scheme were carried into effect, what would be the consequence upon the friendly societies ? It would cut off from them the supply of young members, for it is clear that no youth under twenty-one could afford to pay friendly society contributions in addition to his annual £3. 6s. 8d. It would thus deprive them of one material element of prosperity. With regard to the assurance of sick pay, in which at present the advantages of self-governing societies are the most apparent, it would limit their business to such additional and subsidary assurance as would be required to meet the case of these members for whom eight shillings a week in sickness is not sufficient. It would probably, therefore, reduce the bulk of societies to mere burial societies, thus destroying the societies of the better classes, and bringing all societies to the level of the worse : for, as we have seen, the real source of the evils of burial societies is the smallness of each member's interest,

giving him no substantial stake in the society's welfare and good
management. If the scheme were in other respects a sound one,
these consequences might well make us pause before adopting it: but
bad as it is, they seem to us of themselves alone conclusive against it.
It is not to socialistic Utopias of compulsory universal insurance, but
to a continuance of the sound policy of encouragement of the last
hundred years, that we look for the further progress of friendly
societies.

---

*Expenditure in Life Assurance Offices. Notes upon, and Extracts
    from, a Paper by* Mr. H. R. HARDING, *and a Table by*
    Mr. T. Y. STRACHAN, *which were submitted to the Institute*
    25 *March* 1889.

MR. HARDING'S PAPER.

A PAPER bearing the title "Life Offices:—in reference to
Expenses, New Business, Profit divided, and Competition", was
read before the Institute at its Ordinary Sessional Meeting on
25 March last, and was followed by a discussion, of which a
summary is appended hereto.

From the nature of the subject it was necessary that the
author should quote from the Board of Trade Returns of the
various life offices, and these quotations being in such a form as
to enable anyone versed in the business of life insurance to
identify in various instances the companies to which they related,
it was considered by the Council of the Institute desirable that,
acting in conformity with its practice not to give countenance to
criticisms upon the affairs of individual life offices, only an abstract
of the leading features of the paper should be given instead of
printing the paper and tables *in extenso*.

Mr. Harding, after pointing out that the subject of the expen-
diture of life offices had not been before the Institute for more
than 12 years, briefly referred to the papers written thereon by
Messrs. Deuchar, Macfadyen, Bailey, and Sprague, and then
proceeded to describe the tables computed by him for the purposes
of his paper.

" Taking up, then, the question as to whether a large or
" small new business is the more profitable to the policyholders,
" I have given in Table I the percentage of expenses to premium
" income of 40 life offices for the years 1872, 1877, 1882, and
" 1887; and the percentage of profit divided to premiums received
" on participating policies for the valuation periods ending nearest
" to the years 1872 and 1886.

"As the chief object of this paper is to enquire into the course
" of expenditure amongst ordinary life offices as it affects policy-
" holders, I have dealt with offices doing life business only; also,
" in order to deal with offices working pretty well on the same
" lines, and as the question of commission forms a material
" portion of the paper, I have excluded from consideration the
" offices not paying commission.   Further, as the expenditure in
" the early years of the existence of a life office is exceptional, I
" have dealt only with offices which were in existence several
" years prior to 1872, and which are still in existence.

"Table II shows the relative numerical position occupied by
" the offices in reference to percentage of expenses to premium
" income and percentage of profit divided for the year 1872.

"In Table III the offices are grouped together on the plan
" adopted by Mr. Macfadyen (*J.I.A.*, xviii, 335)."

In Tables III and IV the offices were grouped according to
the amount of new business annually transacted, thus:

Group (*a*) New Sums Assured, over £500,000.
  ,,   (*b*)   ,,   ,,   between £400,000 and £500,000.
  ,,   (*c*)   ,,   ,,   ,,   £300,000 ,, £400,000.
  ,,   (*d*)   ,,   ,,   ,,   £200,000 ,, £300,000.
  ,,   (*e*)   ,,   ,,   ,,   £100,000 ,, £200,000.
  ,,   (*f*)   ,,   ,,   under £100,000.

" In Table IV I have, grouping the offices in the same way,
" given the percentage of new premiums spent, after allowing $7\frac{1}{2}$
" per-cent on the renewal premium income for expenses; and the
" percentage of the renewal premium income spent, after allowing
" the whole of the new premiums for expenses.

"Tables V, VI, VII are sufficiently explained by their
" headings."

The headings of Tables V, VI, and VII were as follows:

Table V—"Average amount of New Sums Assured per
        " Office, with ratio of Expenses to Premium Income,
        " for Total Offices reported in respective years (Ordinary
        " Business)."

Table VI—The " Percentage of Commission to Premium
        " Income for each of the 40 offices and for each of the
        " years 1872, 1877, 1882, and 1887."

Table VII—" Percentage of Total Expenses (Commission
        " and Expenses of Management) to Total Premium
        " Income; and Percentage of Total Profit Divided to
        " Total Participating Premiums Paid, of all the Offices
        " dealt with."

Mr. Harding first considered, in connection with these tables, the rate of expenditure only, and came to the conclusion that " whichever way the rate of expenditure is calculated, speaking " generally, offices doing a new business of between £300,000 " and £400,000, work consistently at a less rate of expenditure " than offices doing either a larger or smaller new business— " being a very similar result to that arrived at in a footnote to " Mr. Macfadyen's paper by the Editor of the *Journal.*"

Mr. Harding next considered the question "from the standpoint " taken by Mr. Bailey, and, having regard to the rate of profit " divided as well as that of expenditure."   After referring to the results shown in his Tables I, II, and III, he said: "The " conclusion, then, I arrive at, regarding the question from both " standpoints, is that, speaking generally, no benefit is derived by "the policyholders of an office from doing a new business in " excess of £400,000 new sums assured."

"Although we arrive irresistibly at this conclusion, we see by " inspection of Tables II, III, and IV, that there are, apart from " the fact that the minimum premium offices, from the nature of " the basis of comparison, come out rather unfavourably, sufficient " variations in the relative positions of the different offices, and " groups of offices, in regard to rate of expenditure and rate of " profit divided, to indicate that in some instances other causes " are at work, besides expenditure, which affect the rate of profit " divided—chief amongst which will, no doubt, be the basis of " valuation and margin of premiums reserved; thus, we see that, " looking at both 1872 and 1886, groups (*d*) and (*e*) give a higher " percentage of profit divided than both groups (*a*) and (*c*), although " (disregarding the percentage of expenses for 1887) in group (*c*) " the percentage of expenses is smallest, and in group (*a*) less " than in group (*e*).   For this reason it appears very necessary, in " endeavouring to form any estimate of the future rate of profit " divided, to have regard to the course of expenditure over a series " of years, as well as to the immediate rate of profit divided, for " it is evident that, other things being equal, the lower the rate of " expenditure the greater the latent strength of an office, and " the greater the probability of its rate of profit divided being " maintained."

Mr. Harding incidentally referred to the "steady and continuous increase in the rate of expenditure" during the 15 years comprised in his tables, and pointed out that the actual increase in that period was over 2 per-cent on the premium income, and,

referring to his Tables VI and VII, stated that: "Comparing the
"year 1887 with 1872, 21 offices show a distinct, and in some
"cases a heavy increase, in the percentage of commission to
"premium income; whilst, with the whole of the offices dealt
"with, such percentage has increased from 4·38 in 1872 to 4·99
"in 1887."

Mr. Harding then concluded his paper with some "General
Observations." Dealing with the view taken in 1875 by
Mr. Bailey that ordinary life insurance business was not increasing
in this country, he showed that, looking at his Table V, there were
indications that this view was correct; and further, that, speaking
generally, a point had been reached in the competition "for new
" business beyond which the offices cannot get, and that even to
" keep up with the present high pressure they must, as a body,
" incur an increasing rate of expenditure."

According to Table VII, the percentage of profit divided to
participating premiums paid in valuation period had, on the whole
of the offices, increased from 24·60 to 29·07 in the 15 years, but
out of the 40 offices included in the table, 9 showed practically
stationary results, and 5 actually a decrease; and Mr. Harding
pointed out, as regards the remaining 26 offices, that it by no means
followed that an increased rate of profit divided was to be relied
upon with certainty as regards the future, and gave reasons for the
conclusion that even as regards the last 15 years the increase
shown in the rate of profit divided "must not be looked upon
altogether as arising from extra profit made by the offices."

As regards offices doing a new business of between £300,000
and £400,000, Mr. Harding, *inter alia,* suggested that it was
possible these offices received a considerable amount of almost
voluntary business, and gave reasons for this opinion.

The paper concluded as follows:

" The subject of the rate of expenditure of our life offices is no
" doubt a complicated one, as circumstances are so varying.
" Perhaps it is a matter of regret with many members of this
" Institute that it cannot, by reason of its composition and the
" terms of its charter, deal with practical questions of this nature.
" The Institute of Actuaries, as the representative and now
" authoritative body in all questions of life assurance, cannot see,
" without concern, offices drifting on a course which, to say the
" least, must affect the benefit derived from them by their policy-
" holders, and must bring at the same time discredit and damage
" upon substantial offices and the practice of life assurance.

" Doubtless, however, we shall have, at no distant date, such a
" committee of life managers as will be able to deal with questions
" of practice.   Take, for example, a most material factor in the
" rate of expenditure, namely, commission.   There are probably
" nearly as many and various scales of commission paid as there
" are life offices, and in the competition for business in some cases,
" an extraordinarily high commission is paid by offices in order to
" outbid other offices.   This most members will agree should not
" be allowed.   Personally, I think the competition in commission-
" paying has nearly reached a crisis, and that, at any rate in
" London, where the public are most highly educated in these
" matters, there are signs that the public mind is becoming
" considerably alive to the evil in the modern development of
" the practice.   The evil, however, undoubtedly exists at present,
" and it would be a great point gained if the life offices could
" agree upon a uniform practice in regard to commission-paying.

"   I was very decidedly of opinion before writing this paper, and
" the observations made in arriving at the results I have given have
" confirmed me in the opinion, that for an office to be successful,
" *as success is understood in this paper,* it is not necessary to pay an
" extravagant commission or incur very heavy expenditure.   What
" is required is that an office should keep in touch with and almost
" in advance of the public wants, and that it should be able to create
" a demand for the particular kind of policy it has to offer.   Our
" President was quite correct when he stated, on a certain occasion,
" that it was the simplification of the details relating to a life policy
" which was most needed.   It does not require much foresight to
" recognize that, in the cases of ordinary insurances, that is,
" insurances on persons in ordinary civil occupations, we shall, before
" long, have no such things as "conditions" on policies—the only
" stipulation will be the due payment of the premium ; that we
" shall see surrender-values specified, guaranteed, and forming part
" of the policy as a matter of course, and that all such items as
" ' fee for valuation ' and ' fee for registering assignment ' of
" policy will, with all offices, be things of the past.   I think it will
" be admitted that I have foundation for this belief, for, even now,
" we see that it is the offices which are moving in this direction
" that are most successful in attracting business at a moderate
" expenditure.

"   In writing this paper I have endeavoured to keep entirely
" to general arguments.   I hope that it will not be thought
" inappropriate for the Institute to give a little attention to the

" subjects to which I have referred very incompletely, as they and
" kindred subjects have attracted a good deal of public attention
" lately.   If the Institute should speak with no hesitating voice
" as to the evils of excessive expenditure only, this paper will not
" have been written uselessly."

Mr. Strachan's Table.

Bearing upon the subject of Mr. Harding's paper, a Table
prepared by Mr. T. Y. Strachan, F.I.A., from the Consolidated
Revenue Accounts, &c., of 97 offices doing life business, as
published in the Board of Trade returns from the passing of " The
Life Assurance Companies Act, 1870 " to the present time, was
printed by order of the Council of the Institute, and circulated
for use at the discussion on Mr. Harding's paper.   Mr. Strachan
explained the construction of his table at the discussion following
the reading of Mr. Harding's paper.

Mr. Strachan's Table gave for the 97 offices separately the
following particulars in connection with each of its valuation
returns to the Board of Trade:

Ratio per cent to Premiums			Propor-tion of Gross Premiums not Valued	Rate of Interest		On Profit Policies, Whole-Life—Equal Premiums	
Of Expenses	Of Cost of Capital	Of Both		Realized	Valued at	Reserve per £1 Assured and Bonus	Propor-tion of Total Business
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Discussion.

The President (Mr. Sutton) said that when this paper was
presented to the Council there was a strong feeling among the
members that any discussion on the question of comparative expenses
of management in various offices was likely, if not carefully handled, to
lead to difficulties.   But it was felt that many years had passed since
this vital question was discussed there in open conclave, and that
possibly the time had arrived when a general discussion, rather on
the principles involved in expenses of management than on the par-
ticular application in various cases, was a desirable question for the
Institute to deal with once again, and it was with the understanding
that the broad question of expenses of management should be dis-
cussed that the paper was ordered to be printed and to be read before

the Institute. Whether, when it took its final form in the *Journal* of the Institute, it would be considered by the Council desirable that the tables should be reproduced was a matter he would not deal with, but the question would certainly have the careful consideration of the Council. He hoped that care would be taken in the discussion not to give offence. He took exception to the sentence on the last page— " Perhaps it is a matter of regret with many members of this Institute " that it cannot, by reason of its composition and the terms of its " charter, deal with practical questions of this nature "—being convinced, after 25 years' experience of the Institute, that nothing would tend sooner to bring about its downfall than any attempt on the part of the Institute to dictate to life offices the manner in which they should conduct their business.

Mr. T. Y. STRACHAN said he could not help feeling that it was because this question was one that had something of the taste of forbidden fruit about it that some of their friends were anxious to have it discussed. He joined with the President in the desire that the discussion should be confined purely to general principles, and he hoped that nothing would be said that would be offensive to anyone; but that, at the same time, they would have a fair expression of opinion on the principles involved in the paper or incidentally arising out of it. There seemed to run through the paper the proposition that a large new business was not the most profitable to policyholders. He was at a loss to see why, when they were considering the question of what is a fair cost of conducting offices, they should shut their eyes to the facts that existed around them and exclude offices that did something else besides life business, and offices that did not pay commission. These were minor questions, but to his mind they were troublesome to answer. Before dealing with the broad proposition which he had already enunciated, he referred to a sort of minor proposition which was not stated, but rather inferred from the language, that bonus-giving power was reduced as the expenditure increased. He contended that the proposition was not proven by the paper. Mr. Bailey put forward in his paper the proposition that variations in bonus were due far more to expenses incurred than to any other cause, and to that extent he thought they might well go. They might well believe that expenditure was one of the largest factors: but that it was the only factor, and that directly as the expenditure increased the bonus-giving power diminished, was a proposition that they could not quite assent to, and upon the arguments submitted they were not called upon to assent to it. What he called the major proposition was that a large new business was not the most profitable to policyholders. They had in Table III a reproduction, or an imitation, of the table which Mr. Macfadyen was the first author of, and he (Mr. Strachan) proposed to call their attention to the fallacies of that table. The law Mr. Macfadyen supposed he had found was that the offices doing new business amounting to from £200,000 to £300,000 were worked at a minimum cost. Mr. Macfadyen's proposition was so contrary to his (Mr. Strachan's) first notions of business, that it startled him when he first saw it, and for a time he felt a difficulty to get figures about the new business to

know where the fallacy was. He had always been taught in business, that an increased output meant a reduced expenditure and an increased profit. He had taken out, from the best source available, the figures of all the 21 offices who appeared to do £500,000 of business. Mr. Macfadyen had only eight offices in his table. The average cost over the 21 was 15·4 per-cent for the year 1887. The average cost, according to Mr. Harding, was 15·47, and he took it that he had obtained something like the basis upon which the table was founded. But he was not prepared to admit that the cost of offices doing a business of £500,000 was $15\frac{1}{2}$ per-cent, for this reason, that under these figures he found offices that were spending $9\frac{1}{2}$ per-cent and others that were spending 41 per-cent, and he could not subscribe to the doctrine that it was unprofitable to an office to do more than £500,000 of business when it could do this at 9 or 10 per-cent, because they grouped the offices together with others that spent 40 per-cent and deduced therefrom an average of 15 per-cent. If they had proven that no office could conduct that large business except at an increased expenditure, then he could have understood the table. He did not think Mr. Macfadyen had his eyes quite shut to this question, because he pointed out that individual offices ought not, perhaps, to be judged by the table he used, but the form in which it was now put forward showed how it had taken hold of the younger members of the Institute. He submitted that the argument that extravagant expenses necessarily followed upon large business was not proven by the table when they looked beneath the surface. He came now to the rival methods of ascertaining what was the rate of cost of new business to an office. Mr. Deuchar had pointed out very carefully and ably that new business was more costly to get than old business to keep, and it was desirable not to overlook that fact. He thought he might say that Mr. Deuchar did not go further, and he did not think Mr. Sprague went further than to point out that there were at least other methods of calculating the expenses than by dividing it equally over the whole premiums received. To that extent he was prepared to go, but for general business purposes he contended that the old method of dividing the expenses over the whole premium income was the best. It was reducing, as it were, to a common denominator. There was a further method hinted at of dealing with expenses which they must not shut their eyes to, namely, that expenditure was a capital expenditure—that as long as they did not spend the whole of the capital of a company they were pretty safe, and that they might recoup themselves hereafter. If that question became a burning one they would deal with it, but in the meantime he merely mentioned it as one of the matters hinted at in the correspondence with the Board of Trade. Dealing with the table submitted by himself, which he said had been gradually built up during the last 12 to 15 years, he proceeded to explain, that in comparing the expenditure with the premiums, he had given what seemed to him very important they should have if they wanted to see what an office was spending. In putting forward the figures of the table, he said that they were not meant to be absolute tests of comparison between offices, but for broad general views they might be useful.

Mr. J. R. MACFADYEN thought the discussion opportune, in view of the excessive competition and increasing rates of commission of the present day. Nobody would deny that expenses had increased, and it would have been curious if the conclusions he had previously arrived at had been upset by the alterations made in the way of conducting business. He was glad to say they had not. He had never yet heard an argument brought forward in objection to the method of grouping as compared with the individual office method, and it was a most extraordinary thing to find that Mr. Strachan had done so. It was really the method of averages after all. Offices that were extravagant were placed in the same group with offices that were not extravagant. The one met and averaged the other, and finding that the groups always showed the same results, he thought the method much fairer than that of taking individual office experiences. Mr. Strachan seemed to think it remarkable that the more business done after a certain point the worse it was for the office. That circumstance could be accounted for to a certain extent by the fact that there were expenses which were a constant percentage—commission, for instance. Commission was one of the principal factors in the making of large offices. He could not see how the system of grouping offices was to be done away with. It was a happy idea of Mr. Strachan to employ the Consolidated Revenue Accounts. He wished that Mr. Harding, in his concluding observations, had given his own views, in place of simply taking the methods of Mr. Sprague, Mr. Bailey, and himself (Mr. Macfadyen). As for Mr. Strachan's remarks about the various plans of measuring the expenses, he (Mr. Macfadyen) had always agreed with Mr. Bailey on that subject, and the last paper he wrote was in defence of his views on the question.

Mr. AUGUSTUS HENDRIKS agreed with a great deal that Mr. Strachan had said on the question of measuring the expenditure, but, on the other hand, he supported many of the views set forth in the paper, and those of Mr. Macfadyen. If Mr. Harding had erred, it was in good company—first, in that of the last speaker, and, secondly, in that of Mr. Sprague, who he thought was the first to publicly adopt those views. It took a long while to convert some of those who were impressed with the old method of assessing expenses upon the total premium income. He thought that Mr. Strachan still held that to be the proper mode to assess them. [Mr. STRACHAN.—A proper mode.] That would be a proper mode if all the companies proceeded upon the same basis. If all started in the same year, and had transacted the same annual amount of business, it would have been right to take that view, but such was not the case. The method, therefore, although a very good one, did not present the whole of the facts, and had disadvantages, as did every other method. How was it to be expected that the expenditure in the early years of a company would be at as low a rate as in the case of an old company? If an office closed one section of its business, and started a new one, it would not assess its expenditure at the same ratio with respect to the new business and to the business which had been gathered together over a long series of years. His own views of the matter lay somewhat midway between the theory laid

down by Mr. Sprague and that of the ordinary method. He thought
that a great many of the figures brought forward by Mr. Harding
were incontrovertible. He quite admitted that when offices were
grouped together discrepancies would arise. The chairman of a
large company commented, the other day, on the question of expen-
diture in life assurance offices. The companies were divided into
groups—the first, where the rate of expenditure was from 25 per-
cent to 15 per-cent—and he thanked providence that his company
was not in that. The next division was between 15 and 10 per-cent,
and he boasted that his company was not there, and then he quietly
remarked that his office was in the next lower group, and it turned out
the rate of expenditure was 9·96 per-cent! It seemed to him that the
great evil was that companies took advantage of the theory, that a
large amount of expenditure could be incurred and charged to the
first year's business. The question was, what is the limit of this
expenditure? If a company started with a large expenditure, but
gradually and surely reduced it, then it was entitled to take rank
with the old companies; but when companies took advantage of the
idea that 100 per-cent, or even more, upon the first year's premium
could be spent, and there was no diminution in the general average
of expenditure, the members of the Institute of Actuaries, if not as a
body, then individually, ought to expose such a state of things, and
endeavour to bring it to a termination.

Mr. A. H. BAILEY mentioned that the President had quoted a
sentence from Mr. Harding's paper in which he said:—" Perhaps it
" is a matter of regret with many members of this Institute that it
" cannot, by reason of its composition and the terms of its charter,
" deal with practical questions of this nature", and had implied that he
wished to substitute rejoicing for regret. To his (Mr. Bailey's) mind
it was a matter of neither rejoicing nor regret, but he should like to
say that, by reason of its charter, the Institute could, in his opinion,
deal with practical questions of this nature. The one matter it
was to avoid was expressing an opinion on the management of any
individual office. The subject was very suitable for the Institute,
because, in his judgment, the merely practical man did not see the
bearing of it. It is necessary to understand the principles of life
assurance, and in particular the methods of valuation. In David
Jones's book there was a dogmatic statement that the only proper
way to make a valuation of the liabilities of a life assurance society
was to adopt the rate of mortality and the rate of interest that would
produce the premiums charged. He believed that at that time there
was very little thought of comparing the expenditure with the premium
income in any way whatever. Later on it was thought unimportant
to know how the premiums were arrived at, but very important to
find a table of mortality and a rate of interest which would actually
prevail in the future. Then, no doubt, there was a difficulty about the
allowance to be made for expenses. That was, as he knew, in some
instances, purely arbitrary. Latterly the practice prevailed, and at
present prevails, of dividing the premiums actually charged into two
parts—first, the amount which, according to the best judgment that
could be formed, represented what was required for the risk; secondly,
the margin which was over. The margin provided for the expenses,

the balance over being the surplus; and if the rate of mortality and the rate of interest could be accurately determined, the margin would simply represent those two items. Now as that system of valuation was generally prevalent, he submitted that the attempts to divide the expenditure between new and renewal premiums—from the point of view of the assured—was wholly fallacious and misleading. Whether the money was expended in obtaining new business or in any other way was a matter of indifference to the assured, the matter he had to look to was the rate of expenditure upon the premium income. He thought the Institute as a body should do its utmost to expose the fallacy of that division between new and renewal expenses. The three items which formed the elements in a life assurance valuation were—the rate of mortality, the rate of interest, and the cost at which the business was obtained. Of these items, the third was by a long way the most important and the greatest source of risk, the second was a very serious one at the present day, and the first was a comparatively unimportant matter.

Mr. T. G. C. BROWNE said that Mr. Strachan, in his remarks upon the table in which Mr. Harding measured the rate of profit divided with the premiums paid on participating policies, hinted that one of the elements that made such a percentage untrustworthy was the changes that had been made in the basis of valuation. Now, there was no doubt, unfortunately, that the rate of expenses had increased. But, fortunately for the cause of life insurance in this country, along with the increase that had taken place during the last 15 years in the rate of expenses, there had also been a very large increase in the reserves of most of the offices of standing. The result of the change in the valuation basis had in many cases been to absorb a great portion of what would have been otherwise called current profit during the 15 years. The process had been carried on to such a very large extent that the whole of Mr. Harding's figures in that comparison between profit divided and participating premiums was fallacious. The figures were of no moment whatever. There was one point in Mr. Strachan's table which was unfortunate, namely, that he had omitted to state the table of mortality used, although he had given the rate of interest. A very common part of the process which he had indicated, was to change from the Carlisle or some other old table at 3 per-cent to the Institute of Actuaries' Table at $3\frac{1}{2}$ per-cent. Mr. Strachan's table did not give any information whatever upon the point. He (Mr. Browne) referred to the case of an office he happened to know something about, which, before the year 1874, valued according to a modification of Davies's Equitable Table; in 1874 it adopted an H^M 3 per-cent valuation. He was not in a position to say what was thus withdrawn from current profit for the purpose of increasing the reserve, but it was a very large amount, and this could not have taken place unless the rate of mortality had been very favourable. He thought that before they could come to any satisfactory conclusion as to the effect of expenses on profits, they must have a business or several businesses under observation for a prolonged period, much longer than five years, say 15 to 20 years. During that time the reserves must be at least maintained at the same strength as at the beginning of the period, and it would rest on those who were dealing with the figures to show, that whatever

the rate of expenses had been, whether high or low, the result in profit earned—and not necessarily divided—had been of a satisfactory character.   He looked with great regret to the number of offices which were now getting into the way of spending something like 20 per-cent.   That was becoming a fashionable rate of expenditure, and, he thought, the onus of showing that such a rate of expenditure was of advantage to the assured rested with the offices who indulged in such expenditure.   He had failed to find any office that had for a prolonged period spent such a rate and had actually given satisfactory bonuses to its assured.

Mr. H. C. L. SAUNDERS reminded the meeting that offices which were called progressive were now paying commission in a different form from that hitherto customary.   That must be taken into consideration when adjudging any particular method of apportioning ratios of expenditure, because, in many cases, the payment was a fixed sum with no renewal commission.   Mr. Harding's tables showed that the commission had not seriously increased, notwithstanding that factor of commuted commissions, which had largely extended during the last 10 years.   Looking very closely into the matter of the grouping of the offices, it was seen that their circumstances did not in any way coincide.   Any change in a valuation, or any slight change in the amount of new business effected, or the expenses of the business generally, might produce an entire kaleidoscopic change in the result of the grouping.   It was a question of management, and without good management they could do nothing.   He found that the offices had £150,000,000 invested, yielding £6,000,000 interest, and that 2s. reduction in the rate of interest was a more potent factor than 1 per-cent increased expenditure.   Mr. Harding said that his conclusion was, that with a rising rate of expenditure the profits diminished.   His paper did not show this.   The last table showed that the profits had increased.   Now, if they had increased in the face of the falling interest, he thought the paper had rather gone to show that a judicious expenditure of recent years in some of the offices had been of advantage to policyholders.

Mr. GEORGE KING said that Mr. Saunders had remarked upon the increase of the rate of profit shown in Mr. Harding's tables, notwithstanding the increase in the rate of expenditure and the fall in the rate of interest, and he attributed that to the judicious expenditure of some of the companies.   He (Mr. King) thought there was a more potent cause, namely, the very largely-increased funds in hand.   It was not so much the increase in the reserve compared to the liabilities that he referred to, although that was a very important factor, but to the actual increase in the invested funds.   That had been enormous since 1870, when the accounts first began to be published, and it afforded an increasing margin of surplus interest.   He thought it was to that cause principally that the increase of the ratio of profit divided to the premiums was due.   The funds had increased in proportion to the premium income, on account of the increased age of the companies.   Of course, in addition to that, there was the increased actual reserve compared to liability. Long ago Mr. Manly showed that an office making strong reserves would in the end pay larger bonuses, and he thought that was now taking place.   To those two causes he would attribute, more than to

anything else, the increasing ratio of profit in face of the increasing
rate of expenditure and the falling rate of interest.

Mr. B. NEWBATT was anxious to support the view taken by the
President—unfortunately in opposition to the view of Mr. Bailey—
that it was not a matter of regret that the Institute was not
the best place in which to discuss a paper of this kind.   He had
all through the discussion, interesting and able as it had been,
been under the uncomfortable feeling that they dare not come face
to face with the facts of the case.   They were obliged to omit
all names of offices, all reference to offices, all means of testing
those very important questions which Mr. Saunders had hinted at,
and which lay behind the particular offices which were concerned.
What would be the issue of the discussion upon the practical
question which lay behind it?   They were airing certain theories,
but coming to no practical conclusion.   When they said that the
expenditure was increasing, say from 10 to 12 per-cent, or whatever
the particular percentages might be, they enumerated only a part of
the truth.   They were apt to overlook the equally important fact
that this increased ratio of expenditure was based upon an increased
premium income.   Between the year 1870 and the present day—the
years during which the returns to the Board of Trade had given
definite facts upon which to go—the total premium income of
the country has increased from £10,000,000 to something over
£17,000,000.   Therefore the present ratio of expenditure, whatever
it might be, was not 15 per-cent upon £10,000,000, but 15 per-cent
upon £17,000,000 or £18,000,000.   But to take a concrete case.
He used imaginary figures, but not exaggerated figures, when he said
that it was quite possible to find a case in which 20 years ago an
office spent 10 per-cent of its premium income, its premium income
then being £100,000.   The same office to-day was spending, we will
say, 12 per-cent of its premium income, and its premium income
was £200,000.   The 12 per-cent upon £200,000 represented
£24,000.   The £10,000 which gave the 10 per-cent ratio upon
£100,000 had now grown to £24,000 upon £200,000, at a ratio of
12 per-cent.   Now, when they said the increase had been from 10 to
12 per-cent, they lost sight of the enormous expenditure as compared
with that of 10 years ago.   It was not altogether the result of mere
magnitude.   There was, of course, the growth of commission, which
was a percentage, but even if all allowance was made for that, and
for the increased staff, there was then a large residuum which
had to be accounted for.   They were doing double the work
and incurring double the cost for the same result.   That was
unsatisfactory, but to a certain extent was inevitable; but had
not the time come when some limitation should be made to the
excessive prices paid to individual persons?   He now came to
what, from his point of view, was the most important matter
arising out of the discussion.   They were all agreed that, with
whatever care and skill statistics of this kind might be compiled,
it was impossible to make them of such a nature as should be
universally fair.   The question was whether the time had not
come when those comparative statistics, which were one of the
curses of modern days, should receive the reprobation at the
hands of honourable competitors which he thought they deserved.

They were made the most of by weak offices having a weak cause to support.   Put as forcibly as may be the facts and the merits of your own office, but do not do so at the expense of your friendly rivals.

Mr. G. S. CRISFORD wished to say that he joined with Mr. Newbatt entirely in deprecating any percentage comparisons, but said that he would rather have the names of the offices given than to have them merely numbered.   It enabled office No. 2, or office No. 20, which may be of the first class, to be compared with office No. 60 or No. 70, which may be of the weakest description, and he thought that anything which would enable an unscrupulous canvasser for business to take figures of the kind and show them to the world should be condemned.   If the Institute were to sanction such percentage comparisons being made public, he would rather see the name of every office put boldly, to show what there was to admire and what to object to.

The PRESIDENT thought that in the discussion that had taken place a delicate subject had, on the whole, been handled with a fair amount of discretion and judgment; and he saw no cause to regret that the paper had been presented to the Institute.   He was more than ever of opinion that the Institute of Actuaries was unfitted, in its official capacity, to attempt to dictate to offices the manner in which they should conduct their business.   He also thought that it would be, perhaps, desirable for every member of this Institute of Actuaries, who, from his very position, was supposed to be thoroughly well qualified to deal with percentages, to renew and extend his knowledge of the doctrine of averages and its proper application to the affairs of life.

Mr. HARDING and Mr. STRACHAN then briefly replied.

The following communication was addressed by Mr. Sprague to the Editor of the *Insurance Record*:

SIR,—Mr. Harding has evidently, in his paper read to the Institute last Monday, taken a great deal of trouble to compile the figures relating to a selected number of companies; and the conclusions drawn by a gentleman who has gone so thoroughly into the question, deserve to be carefully considered on their merits.

Various objections may be urged against the manner in which Mr. Harding has compiled his figures, but I will only mention one. He has (apparently) calculated the percentage which the profit divided bears to the net participating premiums after deduction of the re-assurances.   This would be correct if an office were only liable to declare a bonus on the sum it has retained at its own risk; but inasmuch as it has to declare a bonus on the full sum insured with it, the more correct course is to compare the profits with the total participating premiums, without deduction of the re-assurance premiums.

Assuming, however, that Mr. Harding's figures are correct, I will now mention one or two points in which I think they do not bear out his conclusions.   He says, " Mr. Bailey, in 1875, stated in " his paper that ordinary life insurance business was not increasing. " This, at any rate for the last three years, is confirmed by Table 5."

Mr. Harding's Table 5 contains figures relating to 17 years; and if, instead of taking three years only, we consider the whole 17, and group them as shown in the following table, we arrive at a conclusion which is the very opposite of Mr. Bailey's.

Years			New Business			Yearly Average
1871	.	.	£323,643	.	.	£323,643
1872–5	.	.	1,360,889	.	.	340,222
1876–9	.	.	1,573,353	.	.	393,338
1880–3	.	.	1,645,109	.	.	411,277
1884–7	.	.	1,711,228	.	.	427,807

These figures show clearly that there has been, on the whole, a steady increase in the amount of new business transacted by the 40 selected offices. In connection with this point, I should like to add that it is unfortunate that the Board of Trade returns do not include such particulars of new business, as would enable us to say definitely, whether the total amount of new life insurances effected in the country is increasing or not.

Mr. Harding's first conclusion is, that "offices doing a new "business of between £300,000 and £400,000 sums assured, work at "a less rate of expenditure than offices doing either a larger or "smaller amount of new business." The figures for the years 1872, 1877, and 1882, support this conclusion; but the figures for 1887 show that it is the companies that do a new business of between £400,000 and £500,000, that are worked at the least expense. It appears to follow that, in life insurance, as in so many other matters, there is an irresistible tendency to consolidation; and that, in life insurance at all events, this tendency is beneficial. We must be prepared, accordingly, to see the large offices growing still larger; and the small ones starved out of existence, if they do not read the signs of the times aright, and provide for their assured by transferring them to larger and more solid offices.

The second conclusion arrived at by Mr. Harding is thus stated: "Inspection of Tables 1 and 2 shows us, as Mr. Bailey pointed out, "that with individual offices a continued heavy expenditure is not "generally consistent with a large amount of profit divided." We are not told what rate of expenditure is to be considered "heavy", but it will probably be generally held that this term cannot be properly applied to any expenditure under 20 per-cent; and if so, the proposition will apply at most to eight of the 40 offices, those being all whose expenditure exceeds that figure. Both among these eight, and among the 32 offices whose expenditure is under 20 per-cent, we see very wide differences in the rate of profit divided, which cannot possibly be explained by the differences in the rate of expenditure. In fact, among the best of the 32 offices, the profit divided seems to have little or no relation to the rate of expenditure. This will be seen from the following table, which relates to the first 30 of the offices.

Nos. of Offices			Range of Expenditure in 1872			Average Expenditure in 1872			Average Percentage of Profit, 1872
1– 6	.	.	7·70 to 9·69	.	.	8·79	.	.	29·76
7–12	.	.	10·15 „ 11·21	.	.	10·76	.	.	35·66
13–18	.	.	11·26 „ 11·77	.	.	11·51	.	.	31·60
19–24	.	.	12·22 „ 13·31	.	.	12·83	.	.	24·03
25–30	.	.	13·56 „ 14·98	.	.	14·16	.	.	17·83

Here we see that the group containing the six most economically managed offices, whose expenditure is in every case under 10 per-cent, shows less favorable results than either of the next two groups. The second group, in which the expenditure ranges from 10·2 to 11·2 per-cent, shows the largest profit; and the third group, in which the expenditure ranges from 11¼ to 12 per-cent, shows a more favourable result than the first group. I think I am justified in drawing the conclusion that, at all events with an office whose expenditure is under 12 per-cent, we need pay little regard to the rate of expenditure when we wish to form an estimate of the future rate of profit. In such cases the most important circumstance is the comparative amount of the provision which the office has made for its liabilities. If in past years the office has been prudent, and, instead of dividing large profits prematurely, has strengthened its reserve, this will tend more than anything else to increase the future profits.

The reliance to be placed on tabular statements such as Mr. Harding's Tables 3 and 4, must depend very much upon the number of offices included in each class; and it would have been of great assistance, from this point of view, if he had indicated in his Table 1, to which of his classes (a), (b), (c), &c., each of the 40 offices belonged in 1872 and 1886. I have endeavored to determine this as regards the year 1886 for his class (f), which consists of offices whose new annual business is under £100,000; and, so far as I can ascertain, the class includes, at most, only four offices, one of which is Mr. Harding's No. 40. I submit that no trustworthy conclusion can be drawn from a class of this kind, and that it is virtually trifling with a serious subject to put forward any statistics that rest on so small a basis.

Mr. Harding's figures conclusively show that the average expenditure of the life offices is steadily increasing, but they also show —quite as conclusively—that their average profit is also increasing; and this confirms the conclusion I came to above, *that, so long as the rate of expenditure is kept within a moderate limit—say under 15 per-cent of the premiums—the profits depend, to a comparatively trifling extent, upon the rate of expenditure, and their probable future magnitude is to be estimated by wholly different considerations.* A single remark in conclusion. Mr. Harding says that there is, through force of competition, a general inclination to increase the rate of bonus. My own opinion rather is, that offices have learnt by bitter experience that such a competition is very unwise; and that, for a number of years, the competition has rather been in the direction of strengthening the reserves. This has been the tendency ever since the passing of " The Life Assurance Companies Act, 1870", and is a most beneficial result of the greater publicity enforced by that Act. The first immediate effect on an office of increasing the reserves, is of course to lessen the amount available for division as bonus; but, in the long run, the effect is to increase largely the divisible profits; and this, I believe, is the reason why the profits divided by the offices show an increase, notwithstanding their increased rate of expenditure.

<div style="text-align:center">

I am, Sir,

Your obedient Servant,

T. B. SPRAGUE.

</div>

Edinburgh,
  27 *March* 1889.

On the Further Development of Gompertz's Law. By WILLIAM MATTHEW MAKEHAM, *Fellow of the Institute of Actuaries.*

IN the article "Mortality", in the *Penny Cyclopædia,* the late Professor De Morgan gave the following description of the law of mortality propounded by Gompertz in the paper presented to the Royal Society in 1825. "As this ingenious paper", says De Morgan, "contains a deduction from a principle of high " probability, and terminates in a conclusion which accords in a " great degree with observed facts, it must always be considered " a very remarkable page in the history of the enquiry before " us. . . . There is in the human constitution a power of " resisting the effects of disease which increases from birth up to " a certain age, and diminishes from that time forwards. . . . " Mr. Gompertz assumes that the power to oppose destruction " loses equal proportions in equal times, &c., &c." Gompertz's theory of mortality, then, is based upon the supposed *physiological fact* that the living human organism is endowed with a certain recuperative power, becoming (after a few years from birth) *ever less and less efficient with the lapse of time,* which he terms "the power to oppose destruction", but which, for brevity, I will call "vital force", the truth of which supposition is evidently a question for common observation. His *law* of mortality, on the other hand, is nothing else than the assumption that this diminishing vital force *loses equal proportions in equal times,* an assumption that can only be satisfactorily tested by the careful examination of numerous trustworthy and properly-selected observations. In a paper which I read before the Institute in the year 1865 (*J.I.A.*, xii, 305), I ventured to express the opinion that the mortality prevailing among the members of our assurance societies might be expected to form one of the best means available for determining the typical or normal law of mortality; and the fact that subsequent observations on large bodies of assured lives,—not in this country alone, but also in Germany and in the United States,—have been found to admit of adjustment by means of Gompertz's law, seems to me a very powerful argument indeed in favour of the assumption of the principle which De Morgan thought to be, *a priori,* so "highly probable."

Gompertz expounds his theory in a series of articles (numbered 1 to 5) with which he commences his paper. The first four seem intended to prepare the way and lead up to the enunciation of the *law* itself, which forms the subject of the fifth article, the principal object aimed at in the four preliminary articles being, apparently,

to forestall and prevent a misconception which the author evidently thought might follow from the enunciation of the law. It must not be supposed, he says in effect, that the loss of vital force is the one necessary and sufficient cause of death, or, in other words, that the phenomenon of death would be non-existent if the vital force remained *undiminished*. For the sake of argument, he supposes, in fact, the vital force to *be* a constant quantity. In that case, he says, human beings would die all the same from natural causes, but in diminished numbers, "the deaths among " the young and old being exactly in proportion of the number " of young to the old, provided those numbers were sufficiently " great for chance to have its play "; and, the effect of age being supposed to be eliminated, it would also be purely a matter of chance whether an individual died in one year or in another. "Chance", then, in Gompertz's phraseology—rather than reduced vital force—is, properly speaking, the primary or essential element in the cause of death, and the effect of the diminution of vital force is merely to increase proportionately the chance of death in a given time. The late Duke of Wellington (*i.e.*, the great duke) at a very advanced age, died, if I recollect rightly, from a simple attack of indigestion. Probably, in the course of his long life, he had suffered and had overcome many previous attacks equally severe of the same disorder, but his reduced vital force was no doubt the cause that this particular attack (unlike the others) proved fatal. This, I think, affords a good illustration of what Gompertz means when he says, in Article 4, " It is possible that " death may be the consequence of two generally co-existing " causes; the one chance without (*i.e.*, independently of) previous " disposition to death, or deterioration; the other a deterioration, " or increased inability to withstand destruction." Increased inability to withstand destruction, according to Gompertz's view, does not alone suffice to account for the duke's death; chance, operating in this case by means of a fit of indigestion, supplies what is wanting.*

In short, it seems to me that in these preliminary articles Gompertz was endeavouring, chiefly, to guard against the erroneous idea of his theory being that, by the continual loss of vital power an individual at last reaches a state in which his

---

* Should any reader entertain a doubt of the correctness of this interpretation of Gompertz's words, I would beg to refer him to a passage in a later work by the same author, republished in vol. xvi of the *Journal* (see page 336). In showing that his law does not require that all individuals "should have ultimately the same length of life", he commences by quoting the very extract from Article 4 which I have given above.

remaining power becomes insufficient to support life, and death, therefore, *necessarily* ensues. Gompertz, I should say, was decidedly inclined to the opinion that the notion of a "limiting age" in the mortality table, that is, an age which may be *approached* to any degree of nearness, but cannot be actually *attained*, is a chimera, rather than the representation of anything actually existing in the realm of *facts*. When a man is said to die of "old age", all that is really meant is that the precise ailment which is the real cause of death is of too slight a character to require, or, perhaps, to be susceptible of specific mention.

In his very able and useful work on "Life Contingencies", which forms the second part of the *Text-Book* of the Institute, Mr. King has very judiciously reprinted Article 4 from Gompertz's paper, and I would venture to suggest that in any future edition of his work, he would be only doing justice to Gompertz, and good service to students, by reproducing the whole of the first five articles, taking care, of course, to correct the numerous misprints which disfigure the original, and to substitute for the now obsolete fluxional symbols the modern differential notation, with which students are generally much more familiar. In his remarks upon Article 4, Mr. King has assigned to Gompertz's words a meaning which (as will be seen by my last paragraph but one), it appears to me, was not intended by the author, and which seems to me also by no means to do justice to the idea which the latter really wished to convey. In suggesting that "death may be the consequence of two generally co-existing causes, the one chance, &c.", Gompertz, I take it, is not referring (as Mr. King supposes) to chance as a distinct partial force, represented by the constant A, which, together with another partial force, Bq^x, makes up the total force, μ_x; as, for instance, if A represented the risk which an individual incurs in crossing the streets, from the chance of being knocked down by a passing vehicle, and other similar accidents. Had Gompertz used the word "chance" in any such sense, I think he would have said (as he certainly *should* have said) "death may " be the consequence of *one or the other* of two generally co-existing " causes, the one chance, &c." Nevertheless, I admit that the passage is not entirely free from ambiguity, and that other recent writers, besides Mr. King, appear to have taken it in much the same sense; although, it is to be remarked, that meaning had not suggested itself to anyone until after the publication of my paper on the law of mortality, in which I proposed to express the total force of mortality by $A + Bq^x$ instead of Bq^x.

The form of the function which Gompertz deduced from the

physiological principle assumed, and which he tested by an examination of such observations as were then available, is dg^{q^x}, representing the number living at age x (or l_x according to the Institute notation) in the mortality table. If we take the values of the logarithm of this function (dg^{q^x}) at successive equal intervals, and from these deduce a series of first differences, we get the logarithms of the probabilities of living during such interval, which will form a geometrical series of which q, raised to the power of which the index represents the interval, is the ratio of progression. This expresses mathematically the fundamental law of mortality as propounded by Gompertz in his paper before mentioned. The modification of this law proposed by myself, which, for the sake of distinction, I may term my first development of Gompertz's law, supposes that it is not the logarithms of the probabilities of living which are in geometrical progression, but the first differences of such logarithms, or the second differences of the series $\log l_x$. This modification gives $dg^{q^x}s^x$ for the formula for the number living at age x.

If we take the values of $\log l_x$ for ages 18, 32, 46, 60, 74, and 88 from Woolhouse's adjustment of the H$^\mathrm{M}$ Table, and form the successive orders of differences as far as the second inclusive, we get the following results. The series in the last two columns are the logarithms of the second differences, together with *their* first order of differences, which are the values of $\log q^{14}$.

$$\log l_{18} = 4 \cdot 98787$$
$$- \cdot 04110$$
$$\log l_{32} = 4 \cdot 94677 \qquad\qquad - \cdot 01935 \quad \log = \bar{2} \cdot 28668$$
$$- \cdot 06045 \qquad\qquad\qquad\qquad\qquad\qquad\qquad + \cdot 46159$$
$$\log l_{46} = 4 \cdot 88632 \qquad\qquad - \cdot 05601 \quad \log = \bar{2} \cdot 74827$$
$$- \cdot 11646 \qquad\qquad\qquad\qquad\qquad\qquad\qquad + \cdot 55727$$
$$\log l_{60} = 4 \cdot 76986 \qquad\qquad - \cdot 20209 \quad \log = \bar{1} \cdot 30554$$
$$- \cdot 31855 \qquad\qquad\qquad\qquad\qquad\qquad\qquad + \cdot 55350$$
$$\log l_{74} = 4 \cdot 45131 \qquad\qquad - \cdot 72283 \quad \log = \bar{1} \cdot 85904$$
$$- 1 \cdot 04138$$
$$\log l_{88} = 3 \cdot 40993$$

So that, stopping at the third decimal place, the three values of $\log q^{14}$ are $\cdot 462$, $\cdot 557$, and $\cdot 554$ respectively—showing that the assumption of geometrical *second* differences accords with the facts very closely from age 32 upwards, but less so for ages under 32. From these data, it is to be inferred that a satisfactory mortality table may be obtained from that assumption by commencing, say, at age 30, and then (after completing the table from 30 upwards) working back to age 18 by the same law, and, finally, forming a complementary series in the way described in my paper published in the *Journal* in 1871 (*J.I.A.*, xvi, 344). This,

in fact, is substantially what Mr. King has done, in a very skilful and successful manner, in the construction of the Mortality Table appended to the second part of the *Text-Book*—commencing, however, with age 28, and forming the complementary series by working back to age 10.

Following Woolhouse (see *J.I.A.*, xv, 401), Mr. King gives, at page 212 of his book, a demonstration that, in applying De Morgan's law of uniform seniority, instead of substituting the same number of joint lives of equal ages, as I proposed, we may substitute a single life only, by changing the rate of interest. For the purpose of generalizing De Morgan's law, I will assume (taking the case of two lives aged x and $x+d$ respectively) $nq^z = q^x + q^{x+d}$, which gives $\quad z = x + \dfrac{\log(1+q^d) - \log n}{\log q}$. Hence $\quad l_x l_{x+d} = g^{nq^z} . s^{2z}$ (omitting constant factors, which are immaterial) $= l_z{}^n s^{(2-n)z}$. And therefore, supposing $D_{x.x+d} = l_x . l_{x+d} v^z$, it becomes equal to $l_z{}^n . (s^{2-n}v)^z$, or $l_z{}^n \epsilon^{-(\delta + \overline{2-n}.r)z}$ if we put $\epsilon^{-\delta}$ and ϵ^{-r} for v and s respectively. Now if we take $n = 1$ we have $D_{x.x+d} = l_z \epsilon^{-(\delta + r)z}$, which is Woolhouse's result. If $n = 2$, we get $D_{x.x+d} = l_z{}^2 . \epsilon^{-\delta z}$, which is the form I proposed. But if we take $\delta + (2-n)r = 0$, or $n = 2 + \dfrac{\delta}{r}$, we have $D_{x.x+d} = l_z{}^{\left(2+\frac{\delta}{r}\right)}$, and generally for m lives

$$D_{x, x+d, x+d', + \ldots} = l_z{}^{\left(m+\frac{\delta}{r}\right)},$$

z being determined from the equation

$$\left(m + \frac{\delta}{r}\right) q^z = q^x(1 + q^d + q^{d'} + \cdots)$$

So that it would be quite possible to dispense altogether with tables of annuities by forming instead a complete set of tables of the average duration of life for equal ages, and adopting Woolhouse's suggestion of lateral interpolation.

But the most advantageous way of utilizing the property last demonstrated would be to equate $\delta + \overline{2 - n}.r$ (or, for m lives, $\delta + \overline{m-n}.r$) to a certain specific rate δ', so chosen as to represent the rate most usually adopted. We thus get $n = m - \dfrac{\delta' - \delta}{r}$, and therefore $\quad D_{x, x+d, x+d', \ldots} = l_z{}^{\left(m - \frac{\delta' - \delta}{r}\right)} . \epsilon^{-\delta' z}$

A complete set of tables of annuities corresponding to $l_z{}^m \epsilon^{-\delta' z}$ for successive integer values of m, would thus enable us to find the annuity at any other rate by lateral interpolation.

Reverting now to the examination of Woolhouse's H^M Table,

if we commence at age 20 and proceed by intervals of 13 years instead of 14, we obtain the following results:

$$
\begin{array}{llll}
\log l_{20}=4\cdot98328 \\
& -\cdot04004 \\
\log l_{33}=4\cdot94324 & & -\cdot01688 & \log=\bar{2}\cdot22737 \\
& -\cdot05692 & & & +\cdot44850 \\
\log l_{46}=4\cdot88632 & & -\cdot04741 & \log=\bar{2}\cdot67587 \\
& -\cdot10433 & & & +\cdot51432 \\
\log l_{59}=4\cdot78199 & & -\cdot15495 & \log=\bar{1}\cdot19019 \\
& -\cdot25928 & & & +\cdot53349 \\
\log l_{72}=4\cdot52271 & & -\cdot52927 & \log=1\cdot72368 \\
& -\cdot78855 \\
\log l_{85}=3\cdot73416
\end{array}
$$

Here the geometrical character of the second differences, although by no means altogether wanting, is not so decided as in the case previously examined, the difference between ·514 and ·533 being six times as great as the difference between ·557 and ·554, and the continued increase in this case of the three terms suggests the inference that geometrical *third* differences might be found more suitable. The three logarithms of the third differences of $\log l_x$ are

$$
\begin{array}{l}
\bar{2}\cdot48473 \\
\quad\quad +\cdot54684 \\
\bar{1}\cdot03157 \\
\quad\quad +\cdot54167 \\
1\cdot57324
\end{array}
$$

and it is seen from their differences that a much more satisfactory result is thus obtained.

We know, from Woolhouse's investigations, that, as regards Gompertz's original law, and also that which I have designated my *first* development of that law, each of them has a different law of uniform seniority specially applicable to it, and further that such laws of uniform seniority, respectively, are not applicable to any other possible functions whatever. We cannot, therefore, expect that either of the two laws of uniform seniority in question will be available in the case of a table formed upon the assumption of geometrical *third* differences. Nevertheless, we shall find that there is another law which *is* applicable to this case, the statement of which, and its demonstration, together with some further matters connected with the subject in hand, I must reserve for another opportunity, concluding this part of my subject with the following tabular comparisons between the "actual" deaths and those "expected", by the application of what I may term my *second* development of Gompertz's law. For ages below 18, we must resort to the ingenious process by which Mr. King has contrived to render available the property of uniform seniority in his table for ages under 28.

TABLE I.

Comparison of Deaths among the Numbers under Observation.

Ages	Exposed to Risk	"Expected" Deaths	"Actual" Deaths	Error
18–22	18,493·5	119	119	± 0
23–27	68,602	482	448	− 34
28–32	133,982·5	1,034	1,051	+ 17
33–37	177,523·5	1,547	1,552	+ 5
38–42	189,208·5	1,929	1,966	+ 37
43–47	173,125·5	2,152	2,118	− 34
48–52	143,858	2,282	2,300	+ 18
53–57	110,505	2,342	2,321	− 21
58–62	78,333	2,315	2,298	− 17
63–67	50,244	2,140	2,149	+ 9
68–72	28,542·5	1,799	1,760	− 39
73–77	13,794	1,299	1,332	+ 33
78–82	5,174·5	732	729	− 3
83–87	1,370	289	280	− 9
88–92	244·5	76	66	− 10
93–	20	9	10	+ 1

TABLE II.

Comparison of Yearly Decrements among 9,743, assumed to be living at Age 18.

Age	YEARLY DECREMENTS			SUM OF DECREMENTS		
	Expected	Actual	Error	Expected	Actual	Error
18	61	59	− 2	61	59	− 2
19	61	68	+ 7	122	127	+ 5
20	62	56	− 6	184	183	− 1
21	62	67	+ 5	246	250	+ 4
22	63	59	− 4	309	309	± 0
23	63	73	+ 10	372	382	+ 10
24	64	64	± 0	436	446	+ 10
25	65	48	− 17	501	494	− 7
26	65	64	− 1	566	558	− 8
27	66	60	− 6	632	618	− 14
28	67	71	+ 4	699	689	− 10
29	68	67	− 1	767	756	− 11
30	69	74	+ 5	836	830	− 6
31	70	65	− 5	906	895	− 11
32	71	74	+ 3	977	969	− 8
33	72	73	+ 1	1,049	1,042	− 7
34	74	76	+ 2	1,123	1,118	− 5
35	75	71	− 4	1,198	1,189	− 9
36	76	75	− 1	1,274	1,264	− 10
37	78	81	+ 3	1,352	1,345	− 7
38	80	87	+ 7	1,432	1,432	± 0
39	82	88	+ 6	1,514	1,520	+ 6
40	84	81	− 3	1,598	1,601	+ 3
41	86	85	− 1	1,684	1,686	+ 2
42	88	87	− 1	1,772	1,773	+ 1
43	91	84	− 7	1,863	1,857	− 6
44	94	93	− 1	1,957	1,950	− 7

Table II—(*continued*).

Age	YEARLY DECREMENTS			SUM OF DECREMENTS		
	Expected	Actual	Error	Expected	Actual	Error
45	97	97	± 0	2,054	2,047	− 7
46	100	96	− 4	2,154	2,143	−11
47	103	107	+ 4	2,257	2,250	− 7
48	107	106	− 1	2,364	2,356	− 8
49	111	113	+ 2	2,475	2,469	− 6
50	115	120	+ 5	2,590	2,589	− 1
51	120	124	+ 4	2,710	2,713	+ 3
52	125	120	− 5	2,835	2,833	− 2
53	130	119	−11	2,965	2,952	−13
54	135	129	− 6	3,100	3,081	−19
55	141	153	+12	3,241	3,234	− 7
56	147	150	+ 3	3,388	3,384	− 4
57	154	152	− 2	3,542	3,536	− 6
58	160	156	− 4	3,702	3,692	−10
59	167	153	−14	3,869	3,845	−24
60	174	184	+10	4,043	4,029	−14
61	182	186	+ 4	4,225	4,215	−10
62	189	191	+ 2	4,414	4,406	− 8
63	197	200	+ 3	4,611	4,606	− 5
64	204	206	+ 2	4,815	4,812	− 3
65	212	215	+ 3	5,027	5,027	± 0
66	219	220	+ 1	5,246	5,247	+ 1
67	226	220	− 6	5,472	5,467	− 5
68	232	237	+ 5	5,704	5,704	± 0
69	238	246	+ 8	5,942	5,950	+ 8
70	243	213	−30	6,185	6,163	−22
71	246	222	−24	6,431	6,385	−46
72	249	268	+19	6,680	6,653	−27
73	250	243	− 7	6,930	6,896	−34
74	249	300	+51	7,179	7,196	+17
75	247	241	− 6	7,426	7,437	+11
76	243	245	+ 2	7,669	7,682	+13
77	236	224	−12	7,905	7,906	+ 1
78	227	226	− 1	8,132	8,132	± 0
79	217	219	+ 2	8,349	8,351	+ 2
80	204	196	− 8	8,553	8,547	− 6
81	189	191	+ 2	8,742	8,738	− 4
82	173	173	± 0	8,915	8,911	− 4
83	155	172	+17	9,070	9,083	+13
84	137	119	−18	9,207	9,202	− 5
85	118	117	− 1	9,325	9,319	− 6
86	100	92	− 8	9,425	9,411	−14
87	82	72	−10	9,507	9,483	−24
88	66	74	+ 8	9,573	9,557	−16
89	51	36	−15	9,624	9,593	−31
90	39	34	− 5	9,663	9,627	−36
91	28	36	+ 8	9,691	9,663	−28
92	20	36	+16	9,711	9,699	−12
93	13	29	+16	9,724	9,728	+ 4
94	8	0	− 8	9,732	9,728	− 4
95	5	5	± 0	9,737	9,733	− 4
96	3	10	+ 7	9,740	9,743	+ 3
97	2	0	− 2	9,742	9,743	+ 1
98	1	0	− 1	9,743	9,743	± 0

(*To be continued.*)

CORRESPONDENCE.

INSTITUTE OF ACTUARIES' TEXT-BOOK, PART II.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—A number of correspondents, for the most part students who have been studying for their examinations, have called my attention to sundry errata in Part II of the *Text-Book*. A table of these is appended, which it may be hoped is nearly complete. The majority of the errata are of trivial importance, and of such a kind that the context suggests at once the correction ; and in a number of instances the error occurs only in some copies of the book, other copies being correct.

I may, perhaps, be permitted to take this opportunity of pointing out and removing an ambiguity which occurs in Arts. 92 and 93 of Chap. xviii, on Policy-Values, the more so, as the matter is one of some practical importance. These articles deal with the valuation, in groups, of policies under which the premiums are payable more frequently than once a year. In Art. 92 it is assumed that the ordinary annual premium, P_x, is to be valued ; while in Art. 93 there is substituted for it $P_x^{(m)}$, namely, the premium per annum payable at intervals m times throughout the year. In consequence of this change in the premiums valued, there is a change in the nature of the policy, which, however, is not brought out in the articles in question. It would be well, therefore, to add an explanatory sentence to each of the articles as follows :

To Art. 92 add :

" When valuation in groups is resorted to, then, according to " the method of this article, the assumption is tacitly made that " the premiums are really yearly premiums payable by instal- " ments, so that those instalments for the current year, unpaid " at the time of death, will fall to be deducted from the sum " assured on settlement of the claim."

To Art. 93 add :

" According to this method, the periodical premiums are " really premiums in themselves, and not merely instalments of " yearly premiums as in Art. 92."

The rationale of the Addendum to Art. 92 may be thus explained:

Considering a little more minutely the question discussed in Art. 92, we may first ascertain what net premium must, under such conditions, be charged. The premium not being all payable at the beginning of the year, but by m equal instalments throughout the year, there will be a loss of interest on all the instalments except the first ; but because the unpaid instalments for the year of death are to be deducted on settlement of the claim, there will, in the end, be as many full years' premiums paid as if premiums were payable annually

in advance. The first instalment will not be deferred at all; the second will be deferred $\frac{1}{m}$ of a year; the third, $\frac{2}{m}$ of a year; and so on; and the last, $\frac{m-1}{m}$ of a year; and it will come to the same thing on the average, if the whole premium be deferred, as regards interest but not as regards mortality, by $\frac{m-1}{2m}$ of a year. Therefore, if P'_x by the total premium per annum, we must have

$$v^{\frac{m-1}{2m}} \times P'_x(1+a_x) = A_x;$$

whence

$$P'_x = (1+i)^{\frac{m-1}{2m}} \times P_x.$$

Passing now to the policy-value: At any time the annuity for finding the value of the future premiums (leaving for the moment out of account the unpaid instalments for the current insurance year), will be the same as if the premiums were payable annually in advance, except that, because of the conditions of the case, the premiums must be discounted for $\frac{m-1}{2m}$ of a year more. That is, the value of the premiums payable annually being $P_x(\frac{1}{2}+a_{x+n})$, the value of the premiums by instalments must be $P'_x v^{\frac{m-1}{2m}}(\frac{1}{2}+a_{x+n})$; and this becomes, when we write P'_x in terms of P_x, $P_x(\frac{1}{2}+a_{x+n})$, as before. To this, however, we must now add the actual amount of unpaid instalments for the current insurance year, as these are certain to be received, and in their case the operation of discount is insignificant. On the average, the actual amount of outstanding instalments is $\frac{m-1}{2m}P'_x$, or, what is practically the same thing, $\frac{m-1}{2m}P_x$. Therefore, the total value of the future premiums is $P_x(\frac{1}{2}+a_{x+n}) + \frac{m-1}{2m}P_x, = P_x\left(\frac{2m-1}{2m}+a_{x+n}\right)$, as in Art. 92.

I remain, &c.,

London,   GEORGE KING.
18 *May* 1889.

ERRATA.

In the first of the following columns is given the number of the page, and in the second that of the line. The letters *t* and *b* denote that the lines are to be counted from the top or bottom of the page, respectively. The third column gives the error, and the fourth the correction.

Page	Line		
9	11 t	·64496	·64541
,,	16 t	·64496	·64541
15	11 b	*After the algebraical expression insert* times	
25	12 b	d_{+2}	d_{+1}
33	16 t	$n|\hat{e}$	$n|\hat{e}_x$
37	3 b	*After* respectively, *insert* and, after reduction, replacing *m* by *t*,	
46	9 t	*Reverse the signs of the two expressions*	
,,	16 t	*The expression should be preceded by* $+$ *instead of* $-$	
50	14 b	greater	less
61	23 t	*After* to be	*Insert* approximately
16	13 t	Art. 40	Art. 38
24	14 b	A^3	A^3_x
31	13 t	Art. 48	Art. 49
34	1 t	$a_{\overline{xxx\ldots m}}$	$a_{\overline{xxx\ldots(m)}}$
,,	17 t	$a_{\overline{wxyz}}$	$a_{\overline{wxyz}}^{\frac{3}{}}$
36	6 b	A *numerator*	$\lfloor n A$
44	Heading	Chapter VI	Chapter VII.
46	4 t	$= v^n \lceil$	$= \Sigma v^n \lceil$
59	3 t	equation 36	equation 37
70	1 b	(14)	*Delete* . . . (14)
71	2 t	*After* &c.	*Insert* . . . (14)
,,	4 t	$\dfrac{d \log D_x}{D_x}$	$\dfrac{d \log D_x}{dx}$
84	8 b	$A_{x\overline{n}|}^{-}$	$\overline{A}_{x\overline{n}|}$
85	10 t	2 Geo. II.	11 Geo. II.
87	10 b	$a_x =$	$\mathring{a}_x =$
,,	8 b	$\dfrac{i^2}{8}$	$\dfrac{i^2}{16}$
,,	7 b	$\dfrac{i - i^2}{12}$	$\dfrac{i - \frac{1}{4}i^2}{12}$
,,	5 b	$A_x \dfrac{i - i^2}{12}$	$A_x \dfrac{i - \frac{1}{4}i^2}{12}$
91	4 t	$a_x =$	$\mathring{a}_x =$
92	14 t	$\mathring{a}^{(m)}$	$\mathring{a}_x^{(m)}$
96	10 t	$A_{x\overline{1}|}^{1}$	$\overline{A}_{x\overline{1}|}^{1}$
,,	14 t	$A_{\overline{x+1}:\overline{1}|}^{1}$	$\overline{A}_{\overline{x+1}:\overline{1}|}^{1}$
200	5 b	a_{xy}	a_{yz}
212	11 b	$\log {}_t p_x$	${}_t p_x$ *Delete* log
217	16 t	$\bar{a}_{12}$	$\bar{a}_{12:20}$

19	2 t	$\log v$	$\log v^t$
22	4 t	*The expression should be preceded by $+$ instead of $-$*	
25	3 t	$l_{x+n-\frac{1}{2}}$ numerator	$l_{y+n-\frac{1}{2}}$
26	1 t	values	value
31	6 t	$\bar{a}_{xy}$	$\bar{a}_{xz}$
,,	13 t	$\bar{a}_{xy}$	$\bar{a}_{xz}$
37	1 b	$\overline{\mathrm{A}}^2_{xyz}$	$\mathrm{A}^2_{\underset{1}{xyz}}$
39	5 b	6 (Col. 1)	56
52	2 b	formula 14	formula 13
68	19 t	$\mathrm{A}^3_{\underset{1}{3:45:60}}$	$\mathrm{A}^3_{\underset{1}{30:45:60}}$
,,	1 b	(x)	(w)
69	3 t	$^t\bar{a}_x.dt$	$^t\bar{a}_w.dt$
82	1 b	end of t years	end of mt years
84	1 t	$\mathrm{N}_{x+t-1}+\mathrm{N}_{x+2t-1}+\&\mathrm{c}.+\mathrm{N}_{x+mt-1}$	$\pi(\mathrm{N}_{x+t-1}+\mathrm{N}_{x+2t-1}+\&\mathrm{c}.+\mathrm{N}_{x+mt-}$
94	1 t	*Delete from the top of the page to the end of Art. 70, and substitut the following:—*	

$\pi w \dfrac{\mathrm{M}'_{x+1}}{\mathrm{D}'_x}$; and in respect of the third, $\pi w^2 \dfrac{\mathrm{M}'_{x+2}}{\mathrm{D}'_x}$, and so on, whe

we write w for $\dfrac{1}{1+j}$; and the total value of the return will b

$$\pi \frac{\mathrm{M}'_x + w\mathrm{M}'_{x+1} + w^2\mathrm{M}'_{x+2} + \&\mathrm{c}.}{\mathrm{D}'_x}.$$

We therefore have

$$\text{Benefit Side} = \frac{\mathrm{M}_x}{\mathrm{D}_x} + \pi \frac{\mathrm{M}'_x + w\mathrm{M}'_{x+1} + w^2\mathrm{M}'_{x+2} + \&\mathrm{c}.}{\mathrm{D}'_x}$$

$$\text{Payment Side} = \pi \frac{\mathrm{N}_{x-1}}{\mathrm{D}_x}$$

Whence
$$\pi = \frac{\mathrm{M}_x \div \mathrm{D}_x}{\dfrac{\mathrm{N}_{x-1}}{\mathrm{D}_x} - \dfrac{\mathrm{M}'_x + w\mathrm{M}'_{x+1} + w^2\mathrm{M}'_{x+2} + \&\mathrm{c}.}{\mathrm{D}'_x}} \quad . \quad . \quad (45)$$

70. In practice J will never be such a rate of interest as is to be foun in ordinary tables, and therefore in formula 44 the value of A'_x wi always have to be specially computed, and the most convenient cours will be to use a formula of approximate summation. Any of th formulas of approximate summation given in Chap. xxiv may b employed. In the case of formula 45, the negative term in th denominator can be computed by constructing the commutatio column of M'_x. This term may, however, be put in the for $\dfrac{\mathrm{C}'_x + \mathrm{C}'_{x+1}(1+a_1) + \mathrm{C}'_{x+2}(1+a_2) + \&\mathrm{c}.}{\mathrm{D}'_x}$, to the calculation of whic Lubbock's formula may be applied.

107	14 t	R*i* denominator	$\mathrm{R}_x i$

298	9 t	$\dfrac{R'_x}{D'_x}$	$\dfrac{M'_x + wM'_{x+1} + w^2 M'_{x+2} + \&c.}{D'_x}$
,,	11 t	*Instead of equation 54, write*	

$$\pi = \frac{\dfrac{M_x}{D_x} + c\,\dfrac{M'_x + wM'_{x+1} + w^2 M'_{x+2} + \&c.}{D'_x}}{\dfrac{N_{x-1}}{D_x} - (1+\kappa)\,\dfrac{M'_x + wM'_{x+1} + w^2 M'_{x+2} + \&c.}{D'_x}}$$

312	8 t	*The whole of the expression in each of these lines should have ..	
,,	10 t	*for a factor, and not merely the first term. Therefore, insert .*	
		necessary brackets.	
,,	13 t	$\dfrac{f}{1-\overline{A}_y}$	$\dfrac{\overline{A}_x f}{1-\overline{A}_y}$
313	2 b	$\dfrac{f}{1-\overline{A}_y}$	$\dfrac{\overline{A}_x f}{1-\overline{A}_y}$
318	4 t	Col. 10	Col. 9
319	2 b	·00155 (Col. 9)	·00115
323	6 t	*Number omitted*	*Insert* (7)
,,	8 t	*Ditto*	*Insert* (8)
330	5 t	$P_{x+n} =$	$P_{x+n} + d =$
336	20 t	$\dfrac{P'}{1\cdot05}$	$\dfrac{P}{1\cdot05}$
347	1 t	formula 31	formula 32
351	13 b	Art. 80	Art. 82
356	11 b	formula 55	formula 57
362	8 b	5485·600	5489·602
,,	6 b	253·024	253·208
,,	5 b	5253·024	5253·208
,,	4 b	236·861	236·394
,,	3 b	253·024	253·208
,,	2 b	489·885	489·602
,,	1 b	5253·024	5253·208
364	11 t	P'	P'_1
367	5 t	*Numerator* of second expression — (*minus*)	+ (*plus*)
371	14 b	$A_{x(i}$	$A_{x(i)}$
380	4 b	Houghton	Haughton
389	9 b	of p_x	of $\log p_x$
393	Table	(*Age* 14) ·99990	$\overline{1}\cdot99990$
398	Table	(*Age* 93) 1·471668	1·471667
405	8 t	Art. 66	Art. 67

410	4 t	$\log rp_x$	$\log rp_{xy}$
414	5 b	*Cancel Art. 94, as the method does not apply to De Morgan's form of Commutation Columns.*	
416	11 b	$C^1_{x:y-1}=C^1_{xy}-C^1_{xy}\times \Delta l_{y-\frac12}$	$C^1_{x:y-1}=C^1_{xy}-C_x\times \Delta l_{y-\frac12}$
,,	10 b	$=C^1_{xy}+C^1_{xy}\times d_{y-\frac12}$	$=C^1_{2y}+C_x\times d_{y-\frac12}$
,,	8 b	$C^{\frac12}_{x-1:y}=C^1_{xy}-C^1_{xy}\times \Delta d_{x-1}$	$C^{\frac12}_{x-1:y}=C^1_{xy}-v^{y+1}l_{y+\frac12}\times \Delta d_{y-1}$
,,	7 b	} *Cancel these two lines*	
,,	6 b	}	
,,	5 b	C^1_{xy}	C_x, or $v^{y+1}l_{y+\frac12}$
,,	3 b	or $(1+i)^{\frac12}d_{y-\frac12}$	*Delete* or $(1+i)^{\frac12}d_{y-\frac12}$
417	8 t	$[t]\{\log \Pi_{xy}-\log A^{\frac12}_{x+1:\overline{y+1}}\}$	$[t]\{\log A^{\frac12}_{x+1:\overline{y+1}}-\log \Pi_{xy}\}$
,,	11 t	$\beta-\alpha$	$\alpha-\beta$
,,	15 t	$\beta_{-1}-\alpha_{-1}$	$\alpha_{-1}-\beta_{-1}$
430	1 b	kx	$kx^{(m)}$
434	3 b	23	20
441	14 b	Art. 20	Art. 17
453	3 b	$n=\frac12$	$n=-\frac12$
460	15 b	Art. 25	Art. 22
,,	6 b	Art. 28	Art. 25
462	13 t	$\Delta^3 0^{3'}$	$\Delta^3 0^3$
463	2 t	Art. 29	Art. 26
468	5 b	$u_{(3n-1)}$	$u_{(3n-1)t}$
470	12 b	u_{mn-1}	$u_{(m-1)n}$
,,	6 b	$480n$	$480n^3$
471	5 t	McLachlan	McLauchlan
473	5 b	19·9095	19·9051
474	21 t	$\times = n$	$\times n =$
478	6 b	$\frac13$	$\frac1{|3}$
479	9 b	*After* $\dfrac{\Delta^3 u_x}{n^3}$	*Insert* approximately
482	2 t	n_{5n}	u_{5n}
,,	6 t	-03724	$-\cdot03724$
,,	9 t	v^t	$\log v^t$
,,	7 b	·0004	·0007
483	13 t	·0003	·0004
487	3 b	·0613	·0614
490	11 t	·0088	·0086
491	7 t	(*Against* No. 27) $-\dfrac{n^2-1}{12}\cdot\dfrac{du_0}{dx}$	$+\dfrac{n^2-1}{12}\cdot\dfrac{du_0}{dx}$

ERRATA IN THE TABLES.

Page	Age	Column	Error	Correction
498	0	$\Delta \operatorname{Colog} p_{\varkappa}$	0·96353	$\bar{1}$·96353
,,	13	,,	·00000	0·00000
505	61	$R_{\varkappa}$	73314·37	77314·37
547	90	$A_{\varkappa}$	·84512	·85412

THE LEGAL STAMP DUTY ON RE-ASSURANCE POLICIES, EFFECTED BY WAY OF GUARANTEE ON A COPY OF THE ORIGINAL POLICY.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—Up to the present it has been, I believe, the general practice to stamp re-assurances in the same way as direct policies, the result being that the Government receive double stamp duty on the amount re-assured.

This society has recently obtained an adjudication of the proper stamp which should be placed on re-assurances, effected by way of guarantee on a copy of the original policy, and it would appear that a sixpenny stamp is legally sufficient for this purpose, whatever be the amount of the policy. It would also appear that the copy policy does not require to be authenticated with a shilling stamp.

The means adopted of getting the duty assessed were as follows:

The original policy, kindly lent us by the re-assuring office, was lodged at Somerset House, together with our guarantee endorsed upon a copy of it. We were then required to stamp our guarantee with a sixpenny stamp, and supply the authorities with a copy of the original policy, endorsed with a copy of our guarantee, and after some delay our guarantee was returned to us marked—

<table>
<tr><td>2,926.</td><td>Commissioners
of Inland Revenue</td><td></td></tr>
<tr><td></td><td>(26)</td><td>(6)</td></tr>
<tr><td>13 Jun, 9.</td><td>Adjudged duly
stamped.</td><td></td></tr>
</table>

(89)

I did not understand that any special form or style of guarantee would be insisted on, the view taken by Somerset House being, it seemed, that a re-assurance in this form was of the nature of an indemnity.

I thought of raising the question as to the stamp duty on an ordinary policy granted by one office to another and bearing a re-assurance endorsement, but did not do so at the moment, thinking

such a course might perhaps upset the whole negotiation.   I propose
to do so at a later date.

<div align="center">

I am, Sir,
Your obedient servant,

E. COLQUHOUN.
</div>

Legal and General Life Assurance Office,
10, *Fleet Street, London, E.C.*
24 *July* 1889.

———

[It appears probable that if the agreement by the re-assuring
company were under seal a 10s. stamp would be necessary.   The
agreement above referred to would be under hand merely.—
Ed. *J.I.A.*]

———

THE INSTITUTE OF ACTUARIES.

———

SYLLABUS OF PRIZES OFFERED BY Mr. SPRAGUE

FOR ESSAYS UPON THE FOLLOWING SUBJECT.

"*An examination of the published experience of one or more Life Offices
or groups of Life Offices, for the purpose of determining, with the
greatest accuracy obtainable,—(1) The Rate of Mortality among
recently-selected Lives; (2) The rate at which that Mortality
increases during the* 10 *years after Insurance; and (3) Whether
such rate of increase depends upon the rate of discontinuance or
not.*"

———

The Council of the Institute of Actuaries have the pleasure to
announce that Mr. Sprague has kindly placed at their disposal the
sum of £70, which sum is to be applied in giving two prizes, namely,
£50 to the author of the best essay upon this subject, and £20 to the
author of the second best essay, or is to be otherwise divided between
two or more essayists, if and as the Adjudicators may recommend.

Without restricting candidates in their treatment of the subject,
their attention is called to the following points:

(A) It is anticipated that each body of statistics will show a
rate of mortality to some extent peculiar to itself; and
it is therefore desirable that every published experience of
a life office, or of a combination of life offices, should be
critically examined, and the different results compared.

(B) Among the various publications which supply statistics on
the subject are the following:

(*a*) Containing full statistics as to the discontinuances.

1. *Equitable* Experience—"Tables showing the total
number of persons assured in the Equitable Society,
&c.—London, 1834."
2. *Amicable* Experience—"Tables of Mortality deduced
from the experience of the Amicable Society, by
Thomas Galloway—London, 1841."

3. *Institute of Actuaries* (or 20 British Offices) Experience —"The Mortality experience of Life Insurance Companies, collected by the Institute of Actuaries—London, 1869." C. & E. Layton.

4. Experience of American Offices—"System and Tables of Life Insurance. A treatise developed from the experience and records of 30 American Life Offices, under the direction of a Committee of Actuaries, by Levi W. Meech—Norwich, Conn., 1881. For additional copies, address the Actuary in Charge."

5. Experience of *Gotha* Life Office—"Mittheilungen aus der Geschäfts-und Sterblichkeits-Statistik der Lebensversicherungsbank für Deutschland zu Gotha, 1881." Hermann Böhlau, Weimar.

6. Experience of German Offices—"Deutsche Sterblichkeits-Tafeln aus den Erfahrungen von dreiundzwanzig Lebensversicherungs-Gesellschaften—Berlin, 1883." Ernst Siegfried Mittler und Sohn.

(*b*) Containing partial statistics as to discontinuances.

7. Experience of the *Mutual* of *New York*—"Report of the Mortuary Experience of the Mutual Life Insurance Company of New York, by W. H. C. Bartlett, LL.D.—New York, 1875."

8. Experience of the *Connecticut Mutual*—"Mortality Experience of the Connecticut Mutual Life Insurance Company—Hartford, Conn., 1884."

(*c*) Containing no statistics as to discontinuances.

9. Scottish Amicable Experience—"Tables of the Mortality Experience of the Scottish Amicable Life Assurance Society—Glasgow, 1861."

10. Experience of *Mutual Benefit* of Newark—"Mortuary Experience of the Mutual Benefit Life Insurance Company,—Newark, N.J., 1880."

(C) It is desirable that full consideration should be given to the various questions involved in the subject, to which no published experience affords materials for an answer, indicating at the same time the general outlines of the methods to be pursued in the future collection and compilation of any experience, in order that these deficiencies may be supplied.

CONDITIONS OF THE COMPETITION.

1. That the Competition be open to all Members of the Institute.
2. That the Essays be sent in to the President of the Institute before or on 31 March 1890.

    (*If Essays are written in a foreign language, they should be accompanied by a literal English translation.*)

3. That the names of the Competitors be sent in under seal, with a Motto corresponding to one to be prefixed to the Essay—the Motto and Essay not to be in the handwriting of the Competitor.
4. That the Essay or Essays to which a Prize may be awarded be the property of the Institute.
5. That a Prize or Prizes be only awarded if the Adjudicators shall consider an Essay or Essays to be worthy of the distinction.
6. That unsuccessful Essays be returned, on application at the Institute, with the corresponding envelopes unopened.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXVIII.—PART III.

JANUARY 1890.

CONTENTS OF NO. CLV.

JOURNAL

INSTITUTE OF ACTUARIES.

Opening Address by the President, William Sutton, Esq., M.A.

[Delivered 25 Nov. 1889.]

By the desire of the Council I have undertaken to open the
Session with an address. While fully appreciating the honour
conferred upon me, I do not hesitate to say that personally I
should have been pleased to decline it, on the ground that a large
number of valuable papers on subjects suitable for discussion at
the Institute's meetings had been submitted to and approved by
the Council, and were only waiting their turn for presentation to
the members. I am sure you will agree with me that under these
circumstances I should be fully justified in declining the proposed
honour; but, unfortunately, these circumstances have no founda-
tion in fact. At the present time the Council have but a single
paper before them for reading at the sessional meetings—a state
of things eminently unsatisfactory, and by no means reflecting
credit upon the industry and ability of the rising generation of
actuaries. It is the more noteworthy when taken in conjunction
with the efforts made by the Council in recent years to assist the
younger members towards obtaining that coveted distinction,
" Fellow of the Institute of Actuaries, by Examination." When
these efforts were first commenced there were not wanting those
who held that they were somewhat unnecessary, and that up to

that time, at all events, on the principle, I suppose, of the
"survival of the fittest" there had been no dearth of young
actuaries.   Owing, to a great extent, to the educational assistance
afforded by the Council, the supply of young actuaries has greatly
increased of late years, and it is therefore the more remarkable
that so few of them appear anxious to "stand out from the ruck"
and to obtain distinction by writing papers for the Institute.   I
cannot believe that the Institute's examination tests are so severe
as to destroy the intellectual vigour of the successful candidates or
to eradicate that reasonable ambition which is one of the main-
springs of a professional career.   It is true that it is not so easy
now as formerly to find an interesting subject to write upon, but
I will not pay the younger members the doubtful compliment of
treating this as an excuse.   Nor, assuredly, can it be said that the
papers submitted in the past by young members have been
subjected to a criticism so harsh that the young members of the
present day shrink from such an ordeal.   Speaking, now, from an
experience of many years as a member of the Council, I can say
most positively that nothing gives greater satisfaction to the
Council than the announcement of a paper from a new hand
seeking to win his spurs ; and speaking from a still longer
experience as a regular attendant at the Institute's meetings, I
am sure papers of this kind have always been received in a manner
calculated to give pleasure and encouragement to their authors.

I know that from time to time there has existed a feeling
among the young members that to write papers for the Institute
was a mistake, tending as it did, in their opinion, rather to obtain
for the writer a reputation as a mere theoretical actuary than as an
actuary well qualified to preside over the fortunes of a successful
life office, with, of course, the corresponding personal advantages
such a position is calculated to carry.   Admitting, as I do, to the
utmost the force of the human-nature element which constitutes
such a material portion of this argument, I say put it to the test
of facts.   Look round among the young actuaries who have of
recent years succeeded in the manner in which you long for
success.   I believe I shall be absolutely correct in saying that in
hardly a single case of recent years in London has a member of the
Institute been rewarded in the manner here contemplated who
has not, in one way or another, done something of this kind to
bring him to the front—in nearly every case by writing at least
one paper for the Institute.

To all young actuaries, ambitious as to their future, I would

say in all seriousness never forget, *experto crede,* that in strictly
carrying out the Institute's Baconian motto:

> " I hold every man a debtor to his profession, from the which as men
> " of course do seek to receive countenance and profit, so ought they of duty
> " to endeavour themselves by way of amends to be a help and ornament
> " thereunto ",

you will be carrying out your first duty to the Institute of which
you are a member, and that the chances are in the proportion of
something like ∞ to 0, that in so doing you are at the same time
laying the foundation of your own professional success.

I pass now to other matters.

Through the courtesy of Mr. Izrael C. Pierson, Actuary of the
Washington Life Insurance Company of New York, I received a
short time ago the *History of the Foundation of the Actuarial
Society of America*—a title which explains itself. I am sure I am
speaking on behalf of the Council and of all the other members of
this Institute, when wishing every success to the recently-started
" Actuarial Society of America." I am glad to say that there
are members still with us who assisted at the foundation of our
own Institute of Actuaries, and who, if their memories are equal
to the strain, will vividly call to mind the trials and difficulties
surrounding its establishment. The Institute has now outgrown
all these, and is at the present time in as flourishing a condition
as its most ardent well-wishers would desire; and in the interests
of all connected with the profession of actuary, it is a matter for
sincere congratulation that our actuarial brethren on the other
side of the Atlantic have at last seen their way to constitute
themselves into an organization with, probably in the future, a
constitution not widely differing from that of our own Institute.
It will, I am sure, be of interest to our members if I read to them
an extract from the proceedings of the first meeting. I cannot
quote from these proceedings as much as I should like to quote,
but the following will certainly justify quotation. It is taken
from the speech of Mr. Sheppard Homans, the Chairman.

> " In my judgment we must be very careful as to the nature
> " and scope of that organization. The success of the Institute of
> " Actuaries of London is largely owing to the fact that they have
> " left out what may be called politics. Any reader of the *Journal
> " of the Institute* must be struck with the absence of any criticism
> " of individual companies or individual plans of business, and I
> " think therein lies the secret of their success and of their
> " usefulness. But England is somewhat different from this

" country. The English actuaries, as a general rule, are rather
" more executive than professional. In this country it is different.
" There are very few actuaries who are at the same time executive
" officers of companies, and there is a feeling, more or less well
" founded on the part of leading officers and leading companies,
" that there is danger of what may be called politics, in the
" organization of an Institute of Actuaries in this country.
" * * * * We want to draw the line sharply between
" ourselves as business men and ourselves as individual members
" of a profession where there is much to learn, for we are on the
" threshold, as it were, of the science of life insurance, and our
" object should be the general good, and the avoidance of anything
" that looks like a criticism of individual companies or individual
" plans." I will only add to this that, in my opinion, should the
Actuarial Society of America be conducted on the same lines as
those on which the Institute of Actuaries has flourished for more
than 40 years, it, too, will assuredly flourish, and be hereafter of
incalculable good to the business of life insurance.

The word "politics", so happily used by Mr. Sheppard
Homans—who is, by-the-bye, one of our own Corresponding
Members—in the speech from which I have just quoted, leads me
insensibly to the main feature of this night's address. Were your
President for the time being in any way mixed up with these
before-mentioned "politics", he would certainly not venture upon
the remarks he is about to make; but as he is in no way so
connected, he has formed the opinion, rightly or wrongly, that he
might reasonably engage your attention, for the remainder of this
address, with some remarks upon the present aspect of life
insurance as a business in this country.

It is needless to say that, in any remarks I may have to make
upon this subject, I am not speaking from the knowledge of detail
acquired by one still actively engaged in the business, but rather as
one who some years ago was actively engaged in the business, and
has now to occupy the part of a looker-on, who remembers the
business as it used to be conducted, but has no exact technical
knowledge of the minutiæ which form part of the business as now
carried on.

Well, now, the first thing that strikes me in connection with
the business of life insurance in this country is, that the same
forms of return are required to be made to the Board of Trade
under the Life Assurance Companies Act of 1870 as were prescribed
when that Act was passed. It is true that section 9 of "The Life
Assurance Companies Act, 1870", provides that :

" The Board of Trade, upon the application of or with
" the consent of a company, may alter the forms contained in
" the Schedules to this Act, for the purpose of adapting them
" to the circumstances of such company, or of better carrying
" into effect the objects of this Act."

As far as I can gather, the " objects " of the Act are left to be
inferred from the Act itself, and I am probably safe in taking it
for granted that no serious departures from the forms contained
in the schedules have been applied for by any companies and
agreed to by the Board of Trade, or proposed by the Board of
Trade and agreed to by any companies ; and that to all intents
and purposes the schedules are the same as on the passing of the
Act in August 1870—more than 19 years ago. As regards
Schedules 1–4, as they stand, it is worth pointing out that no
separation is made of with and without-profit business ; no sepa-
ration of commission and expenses between new and old business ;
no information as to number of policies issued, or becoming claims,
or lapsing, or surrendered ; and the corresponding information as
to sums assured, premiums payable, &c. These are all matters,
particulars as to which could easily be furnished.

The fact that no alteration in the schedules has taken place
cannot be said to be in consequence of the perfect manner in
which they were originally drawn. Even in 1872 attention was
publicly drawn to certain imperfections. In that most valuable
little work *Life Insurance in* 1872,—a work far too much over-
looked at the present time—it is pointed out that the returns
made in conformity with the schedules gave no particulars as to
new business, and that the annual return schedules gave no
information as to total sums assured or the number of policies.
As regards the former, it is described as an " important omission
which, it is to be hoped, will before long be supplied."

Again, in 1874, we have the same writer saying : " The returns
" under the Act do not at present enable us to ascertain to what
" extent policies are taken credit for as assets in any particular
" account, and it is therefore at present impossible to apply this
" test of solvency without having further information than the
" Parliamentary returns give. It appears, therefore, that for the
" effectual detection of insolvency in doubtful cases, legislation is
" required. The Act should be amended, so as to require all
" companies to make a separate return of the policies which are
" reckoned as assets in their balance sheets."—(Paper read by
Mr. T. B. Sprague at Belfast Meeting of British Association in 1874.)

Any number of excellent authorities could be brought in evidence as to the imperfection of existing requirements; but yet, nearly 20 years have passed and nothing has been done. Surely it is a matter for serious consideration on the part of those who manage the first-class offices, whether the time has not come when they should move in the matter, if on the ground of self-interest alone. People in this country are, as a rule, slow to move, even in matters of this kind, where they are so deeply interested; and, in my opinion, it is the duty of the managers of the first-class offices to move, and that speedily, otherwise it may be that disasters will occur, and then will follow far more stringent legislation than the first-class offices would like, or is, perhaps, on the whole, desirable.

To my mind there is no doubt whatever that in the competition for business which goes on, most improper and misleading use is frequently made of the figures culled from the Board of Trade returns, and I do hope that a combined effort will soon be made by those concerned to have matters placed upon a proper basis, by alteration in the form of the Board of Trade returns.

The schedules are not the only portion of the Act which, in my opinion, require amendment. Within my own knowledge, in this present year, the provisions of the Act dealing with insolvent companies have completely failed to prevent enormous hardship being inflicted upon policyholders of a company now in liquidation—the failure arising from the circumstances of the case being of a character not present at the time to the mind of those who drafted the bill.

The author of *Life Insurance in* 1872, speaking of the advantages of publicity to the companies themselves, hazarded the following opinion : " Among other benefits that will probably result to the " companies, is a reduction in their rate of expenditure, and " particularly in the rate of commission paid by some of them. " When these particulars have to be published to the world, they " are more closely scrutinized by directors; and there is reason " to believe that already the Act has brought about a reduction " in the rate of commission allowed to agents by certain offices." To what extent this prognostication has proved correct, as judged by the returns made by offices to the Board of Trade up to the present time, I leave those of you who are familiar with the subject to decide. The only test I have applied myself is as follows : I have selected some 33 offices, all doing life insurance business only, and all paying commission, whose first annual

accounts were abstracted by Mr. Sprague and published in *Life Insurance in* 1872, and have compared the figures there given with those lodged at the Board of Trade for the year 1888. The results are as follows:

	Premiums	Com-mission	Percentage of Premiums	Expenses of Man-agement	Percentage of Premiums	Total Expenses	Percentage of Premiums
	£	£		£		£	
Life Insurance in 1872	5,116,109	198,510	3·88	399,022	7·80	597,532	11·68
For year 1888, Board of Trade Returns ...	7,078,111	307,707	4·35	671,237	9·48	978,944	13·83

From these figures it will be seen that, treating these 33 offices as one, the percentage of premium income spent has increased some 2·15, and that whereas commission in 1872 constituted 3·88 per-cent of the premiums, in 1888 it constituted 4·35 per-cent of the premiums, although the amount spent in commission in 1888 constituted about $31\frac{1}{2}$ per-cent of the total expenses as compared with some $33\frac{1}{4}$ per-cent in 1872. If there has been, as I am rather inclined to think, some general change in the meantime in the manner of paying commission, this may possibly affect any comparison. But, to judge from the figures as a whole, it would appear that between 1872 and the present time there has been an *absolute* increase in the percentage of premiums (with *and* without profit) spent on total expenses of over 2 per-cent—equivalent to a *relative* increase of nearly $18\frac{1}{2}$ per-cent—and that the item of commission has hardly increased in the same proportion as the other expenses. To put matters another way, had these 33 offices been able and content to keep up their premium income at the amount at which it stood in *Life Insurance in* 1872, and at the same total percentage for commission and management—that is, 11·68 per-cent—they have collectively incurred an additional expenditure of nearly 20 per-cent in respect of the increased premium income represented by the difference between the total premium income in 1888 and that of *Life Insurance in* 1872. From the facts already alluded to in relation to the form of the Board of Trade returns, it is not possible to critically analyze these figures in the manner I should desire, but I may say that 24 of the 33 offices show an increase in the rate of total expenses, while the remaining nine point the other way. Further, out of the 33 offices, only seven show a decrease in the premium income, and it

is fair to add that not one of these seven is in the list of nine whose total expense ratio shows a decrease. This subject of expense ratios is, as you are all aware, a matter which is daily being discussed among those engaged in the business of life insurance, and, as you are also all aware, is one that has many sides to it.

I do not propose to draw any elaborate conclusions from the few figures I have quoted, but I think I may fairly draw the inference that what I may term the office expenses, as distinguished from commission, have, comparing the state of things in 1888 with the state of things in 1872, considerably increased. This conclusion is, perhaps, one which will be generally admitted. No doubt, directors' fees have been increased, and, following on this, the salaries of the managers and other officials at the head offices, but these would hardly account for any large portion of the increased expenditure. No doubt, too, the staff at the head offices has shown a tendency to grow with the business, but I do not think this would much affect matters. As far as I can judge, the increased expenditure arises from the fact that, instead of doing as was done years ago,—having a head office only, and expecting all business to come to that head office,—the plan now generally in vogue is to have branch offices with all their accompaniments. In other words, the head offices are fetching the business instead of having it brought to them, and they are doing this, to a great extent, by means of smaller offices carried on at least as expensively as the head offices themselves. Canvassers for business, I believe I am not incorrect in assuming, are now employed by the great bulk of the offices, or at all events by their branches, and I cannot help saying that these canvassers, agents, or whatever their name, have only to go a stage further, and fetch the premiums, and then the resemblance of ordinary life insurance business to that which goes by the name of industrial business will be complete as regards what is done, and, possibly, not so widely different, all things considered, as regards the cost of doing it.

To anyone thinking the matter over it must, at all events to begin with, appear somewhat noteworthy that in these days of cheap and rapid locomotion, of advertizing, and of postal and other facilities of communication, the direct dealings between intending policyholders and the head offices would appear to grow less and less; and that whereas in other businesses there is considered to be a great tendency for producer and consumer to come into direct communication with one another, thus ousting the middleman, in the business of life insurance the middleman, in the form of branch

offices, district inspectors, agents, canvassers, &c., &c., seems, on the contrary, to have greater influence and be more flourishing than ever. It is the more noteworthy when we remember that, as regards policyholders, the transactions are, speaking generally, all for cash down, and that the middleman does not, in this case, give a large selection of various makers' goods to choose from, and the benefit of his own unbiassed and skilled judgment in advising his customers—"nothing like leather" being, from the nature of the case, his trade motto. Nor do the middlemen, in the case of life insurance, have the *raison d'être* they have in many other affairs of life—they do not, for instance, buy and hold large stocks of goods to be sold in small quantities to retail consumers. The explanation of their present thriving existence must be sought in other directions, to which I can only briefly refer to-night.

In the first place, I am not at all sure that life insurance companies as a rule, at the present time, give any encouragement to intending policyholders to deal direct with the head office. The head office might see difficulties in the way under the established system of middleman. As I said some time ago, I am speaking, in many respects, as one not familiar with the present practice in respect of details; but I should be glad to know that I am wrong in this matter.

Next, although great strides have been made towards the removal or modification of many irritating and prohibitory restrictions formerly in vogue in connection with policy conditions, speaking generally, there appears still room for considerable improvement.

Again, whatever the cause, life insurance has never yet become popular. The disasters, moreover, which have occurred within the last 20 years are not forgotten, and the unfavourable recollection may again and again hereafter be renewed by the occurrence of similar disasters.

The magic of personal influence, therefore, appears to be, in great measure, necessary to obtain business, and this must, I imagine, be the main reason for the prosperous existence of the middleman in life insurance business. The head office manufactures the article, but has to look to its middleman to get the orders.

In a recent discussion at the Institute upon this very subject, one of its most prominent and eminent members, alluding to the increased cost of new business arising from competition, described this competition as resembling the old game of "pull devil, pull baker." There is another competition involved as to which I should like to hear the opinion of the middleman—the

agents, branch offices, &c.   Besides the actuary, manager, or other head official, there is another official who has, on more than one occasion of late, managed to make himself heard—and that is the medical officer.   Indeed, not content with carrying out his own duties, he has been putting on the uniform of the actuary, and, if I may be permitted to say so, "rushing in where actuaries fear to tread."   Taking life insurance as a business, where, in the nature of things, bad debts are sure to be made from time to time, the medical officer can, no doubt, be of use in many clear cases where it is apparent bad debts are very likely to occur; but medical science has by no means arrived at that stage of perfection where, even assuming every justice has been done to it by the operator, its conclusions are to be implicitly relied upon. I can, therefore, well enter into the feelings of the middleman when he finds that, after taking infinite trouble to get a proposal from an individual, the proposal is ultimately accepted with "five years added", or an addition to the premium of one, two, or three per-cent on the sum assured, or whatever form the decision takes, or is declined altogether.   Here, in my humble opinion, is one of the weak spots of the present practice.   I am fully aware of the arguments on the other side, but I wish to look at it from the broad point of view of trading for profit.   The fact of medical examination alone probably deters thousands upon thousands of persons of quite as good health really as those who propose, from proposing at all.   This renders the middleman's task more difficult, and his remuneration comparatively more expensive; for ordinary life insurance business has hardly as yet taken the form of another kind of life insurance business, where policies are taken out, then soon dropped, and again taken out and again soon dropped, with distressing frequency.   In other words, ordinary life insurance business, as generally carried on at the present time, reminds one of a company owning coal-fields of varying quality, that persistently continues to work only the pit yielding the best class of coal, long after that pit has got so deep and intricate that the coal, by the time it is brought to the pit's mouth, has cost so much that, excellent coal though it be, customers can with immense difficulty be found to purchase it, so that, after reckoning with the middleman, the margin of profit becomes "small by degrees and beautifully less."   It is as true now as it ever was, that

> "To follow foolish precedents, and wink"
> "With both one's eyes, is easier than to think."

I should, myself, very much like, for instance, to see the experiment

tried, in ordinary life insurance business, of constituting a without-profit branch, at such higher premiums as may be considered necessary, where the sum assured per individual should be limited to a moderate amount, the assurance to be a deferred assurance instead of immediate, and the medical examination taking a much less important and less formidable shape than it does, on paper at all events, at the present time, in consideration of the assurance not taking effect immediately.

In connection with this question of expenses, we are constantly being reminded that companies doing life assurance business are trading companies. Let me read to you what was said on this point in 1874: "While it is desirable that " insolvent companies should be exposed, and their operations " stopped, it would be even more desirable to prevent, if prac-" ticable, such mismanagement as will lead to insolvency in the " future. It is, however, of course, out of the question to attempt " to regulate by Act of Parliament the rate of expenditure at " which a company shall conduct its business; and it is contrary " to the spirit of British legislation to interfere in any way with " the internal management of trading companies. In the United " States much more is done in this way than would be tolerated " in this country. The law there even prescribes the particular " kinds of securities in which the funds of the life offices shall be " invested. Each company is required to make returns of its " liabilities, of a much more minute character than is required " here; and a reserve, of not less than a fixed amount, must be " made by a company for its liabilities under each policy it has " issued. There is, in each of the States, a public official, whose " duty it is to see that the provisions of the law are complied " with; and if he finds that any company's assets fall short of " the legal standard, he is authorized (or rather bound) to put the " law in action against it, and have it wound up. This law " certainly has the effect of preventing the scandals in America " which we have seen in England; but it may be doubted whether " this advantage is not too dearly purchased. The effect of sub-" jecting any business to such minute regulations, is to impede " its natural healthy development; and it is better to leave the " details of the business to be settled by the parties interested. " This was the principle of the excellent Act passed in 1870. " The companies were left at full liberty to manage their affairs as " they thought best; but the secrecy that formerly attached to " the proceedings of so many, good and bad alike, is now " effectually dispelled. The companies are not yet compelled to

" make a proper reserve for their liabilities, yet, if they make an
" insufficient reserve, they can no longer conceal the fact.   The
" public are thus enabled to judge of the solvency of the company
" they have trusted, and to understand the full meaning and
" value of the contracts they have entered into with it.   This
" principle, it appears to me, may with advantage be carried a step
" further.   Let each company be required, before issuing any
" policy, to state the principles on which its reserve is calculated,
" and let it be a part of the contract, that the reserve for each
" policy issued shall never be less than a certain specified amount.
" This would enable intending insurers to discover beforehand the
" nature of the security offered them; and would probably do
" more than anything else to prevent future insolvencies."—
(Mr. T. B. Sprague, "On the Causes of Insolvency in Life Insurance
Companies." Belfast meeting of the British Association, 1874.)

I have made this quotation at some length, from a desire to
do full justice to the writer, and because I wish to state that there
are many parts of it with which I entirely agree.   At the same
time there are points raised which might well be considered in
some detail.

"Trading companies" means, I presume, companies trading
for profit.   Who constitute these companies?   Without-profit
policyholders may fairly be treated, in all cases, as not forming
a part of the "trading company", but rather as customers, so
that we are left with the shareholders and the with-profit policy-
holders or with the latter only, according to whether the "trading
company" is "mutual" or "proprietary."

The next question is, who finds the capital to trade with for
" profit"?   It may safely be answered, as regards "proprietary"
companies, that the capital at risk in the early years is that of the
shareholders, and that when the company becomes a going concern
and is prudently managed, this capital slowly but surely ceases to
become "at risk", what is "at risk" being the shareholders'
portion of profit.   When this stage is arrived at, it may be fairly
said, I think, that the capital to trade with for profit is, roughly
speaking, the excess margin of premiums paid by the with-
profit policyholders, and the capital, representing the value of
the company's liabilities in respect of its future engagements, as,
speaking generally, the paid-up shareholders' capital represents
a small portion of the total trading capital.   And as regards
"mutual" companies, it is the like, *mutatis mutandis.*   Now,
with regard to the long quotation I have just read, I should,
in the first place, desire to point out that I should be disposed to

extend the application of the arguments. At the time that paper was written the failure to pay the sum assured was practically the only kind of insolvency in view, and the fact that many paid for a share of profits also, as well as for the sum assured, was not dealt with. Taking advantage of the good done, or presumably so, by the Act of 1870, I should be disposed, under the altered condition of things, to deal with the subject of insolvency from the additional point of view of those who had contracted with a company in such terms that, not only did they pay a certain premium like others for a specified sum assured, but also an additional premium of considerable amount to be returned to them in the form of benefit out of the profits.

When the famous discussion took place, some years ago, on the expenditure incurred by offices in obtaining new business (and I would venture to remark, *en passant,* that I have never been very clear as to how much of it was to be attributed to the inelasticity—that is the proper word, I think—of the much-abused net-premium method of valuation, how much to an affectionate regard for the welfare of the young offices, and how much to a desire to justify an increased rate of expenditure), one of the arguments brought forward by those who took exception to the increased rate of expenditure was, that existing with-profit policyholders were likely to suffer by the introduction of new partners into the concern at a heavy initial cost; and to this argument it was suggested in reply, " Suppose that the " expenses attaching to the first year's insurance are so heavy " as, together with the current risk, to absorb the whole of " the first year's premiums. All that would be necessary, in " order to guard against the assured suffering any diminution of " profit in consequence of a large increase of new business and a " large increase in the total rate of expenditure of the new office, " is to provide that the *first premium paid by each new member* " *shall be wholly disregarded in all calculations for the purpose* " *of ascertaining the reserve to be made for the liabilities, the* " *surrender value, and the division of profits."*—(*Journal of the Institute of Actuaries,* xix, 10.) A most excellent provision, and when combined (not a very easy matter, I admit) with the same author's suggestion already quoted, namely, " Let each company be " required, before issuing any policy, to state the principles on " which its reserve is calculated, and let it be a part of the con- " tract that the reserve for each policy issued shall never be less " than a certain specified amount", would, to use the writer's own words, " enable intending insurers to discover beforehand

" the nature of the security offered them; and would probably
" do more than anything else to prevent future insolvencies"—
insolvencies, I would suggest, both as regards payment of the
original sum assured and as regards profits. Following on this, I
would venture the opinion that, were these suggestions carried out,
they would have a very wholesome effect upon the present state of
affairs. Imagine the feelings of a person proposing to insure his
life, say, for £10,000 at a with-profit annual premium of £300, upon
studying the terms of the proposed contract as here indicated. A
sensible man would know that there must be some initial expense,
and that probably, and reasonably enough, no doubt, from a
trading point of view, someone would also draw a commission on
the transaction; but I imagine he would be interested further in
the proposed fate of his first £300. Even treating companies as
trading concerns, he would reasonably apply a pretty well-known
maxim in trade: " a reduction in price on taking a quantity",
and may feel somewhat staggered at finding that it does not
apply, and he may possibly come to the conclusion that thrift in
the form of life insurance is an expensive article. When
the discussion about expenditure, which I have referred to,
took place, it is worthy of note that reference was, in the early
stages, always made to the " current risk" in the form of the
company's liability to pay the sum assured if death took place in
the first year; but in the later stages of the discussion it did not
occupy such a prominent position—in fact, matters soon drifted
to the stage when it was to a great extent ignored, and for
" sweet simplicity's sake" the argument proceeded rather on the
assumption that the whole premium was to go in initial expenses;
and I would venture to throw out the suggestion that this,
or something like it, is possibly what we are to consider
is meant by " Life Assurance at less than cost price." For
sound trade finance, I should say this method is inferior to that
of the traditional pie-man putting occasionally silver coins in
his pies. Matters move apace. The assumption of the whole of
the first year's premium being expended in initial expenses,
originally started in somewhat academic fashion, has now become
a standard for the purpose of comparison. Unfortunately, there
is good reason to believe that what was originally an assumption
has now become a common practice. For myself, I never can
call to mind that discussion of 12 to 14 years ago without
thinking of Shakespeare's *King John :*

> " How oft the sight of means to do ill deeds "
> " Makes ill deeds done!"

Future " leading actuaries and insurance experts" may see their
way to argue the question of expenses on the supposition of the first
two years' premiums being spent in expenses, and so much per-
cent of third and subsequent premiums.   No doubt figures can
do a great deal on paper ; but a good many important " if's " are
often involved in the process, and do not get their proper weight
assigned to them.

Now, referring to *Life Insurance in* 1872, and taking matters
as they are now, it does appear to me that, just as then, it was
claimed as one of the great benefits of the Act of 1870, that
" fraudulent amalgamations would be prevented in future", the
time may soon arrive, if the present state of things continues, when
the cry will be for an Act to prohibit companies from " obtaining
" new business on terms calculated to damage the interests of
" existing policyholders."   In other words, you may have brought
about the first step towards " State interference."   Life insurance
companies, no doubt, are, to some extent, trading companies ; but
they have been specially legislated for in the past, and may be
still more stringently legislated for hereafter.

We have been recently told, by an authority in these matters,
that under 15 per-cent of the premium income (with *and* without
profit, remember) is a moderate limit for the rate of expenditure.
If the question were asked, why under 15 per-cent, most of us
could, no doubt, supply readily one answer ; but somebody else of
equal authority might be disposed to say 16 or 17 per-cent is a
moderate limit—where *is* the line going to be drawn ?   Who is
reaping or going to reap the benefit ?   Is it the policyholders, or
is it the management ? and here I use the word " management "
in a wide sense.

We have also been told that there is an " irresistible tendency
" to consolidation, and that, in life insurance at all events, this
" tendency is beneficial."   The so-called consolidation, as I have
already pointed out, seems to me to mean a big office with a lot of
little offices, and as regards the beneficial tendency I would ask,
who is the beneficiary ?   We are also told that the small offices
will be " starved out of existence, if they do not read the signs of
" the times aright, and provide for their assured by transferring
" them to larger and more solid offices."

The wish is often father to the thought.   I can testify that
" to be starved out of existence" is not necessarily a painful process
for an office, looked at from a pecuniary point of view; and I
do not admit that the words "larger" and "more solid" necessarily

go together. Moreover, just as little railways have been found to combine against being "starved out of existence" by the big railways, we may, hereafter, find the many little life offices see their way to combine against the few big ones. But to come to the point. There is surely nothing of a disgraceful, or even objectionable, nature necessarily implied when an office "gives up the ghost", and transfers its liabilities to another office upon proper terms to the policyholders of both offices. Companies are corporate bodies, but there is no law requiring them to live for ever. I have a strong opinion that ordinary "trading companies"—whether doing life insurance business or other business—in many respects resemble individual human beings—

> "They have their exits and their entrances";

and I fail entirely to see why a life insurance company, when it can, from inanition, no longer trade at a reasonable profit—

> "His youthful hose, well saved, becomes a world too wide
> For his shrunk shank "—

in other words, when the indispensable permanent expenses form an unreasonably large strain upon its resources, should not honourably transfer its engagements to a younger and more flourishing concern.

It is, I hope, unnecessary for me to remind you that I have been able, within the limits of this address, to touch upon only a few of the many features of the business of life insurance as now carried on in this country. I hope sincerely that my remarks will be received by you in as kindly a spirit as they have been made. No one can occupy the post I have at present the honour to hold without becoming, sooner or later, if not already, fully aware of the enormous interests involved in the good conduct and well-being of the life offices of this country, and without becoming anxious to see the high reputation they have earned, speaking collectively, in the past, preserved, and, if possible, added to in the future. I cannot, however, refrain from saying that, in my opinion, the indications of the last few years are not altogether satisfactory. I have already endeavoured, to the best of my ability, to refer to some of the causes which have led to this. It is for you to form your own opinion as to the accuracy of the views I have, I am afraid only roughly, enunciated.

If I have "read the signs of the times aright", the conclusion I come to is that—quoting with suitable alteration a well-known phrase—

> Something is wanting in the state of Denmark.

On the Further Development of Gompertz's Law.   By WILLIAM
MATTHEW MAKEHAM, *Fellow of the Institute of Actuaries.*

[*Continued from page* 159.]

ALTHOUGH, as I have said, observations upon assured lives
(in my opinion) afford, upon the whole, by far the best examples
of the typical characteristics of the normal law of mortality, yet
it is evident that all such observations are nevertheless affected
to a considerable extent, first, by the selection exercised by the
office in regard to the admission of lives, and, secondly, by the
selection exercised by the policyholders in the option which they
exclusively enjoy of terminating the contract of assurance when-
ever their convenience or caprice may induce them to avail
themselves of the privilege.   While the form of selection last
mentioned is probably much the more important of the two in its
effect upon the *aggregate* mortality, yet, for obvious reasons, its
effect is of minor importance considered as a "disturbance" of
the normal law which expresses the relation connecting rates of
mortality at successive ages with each other.   The number of
persons *withdrawing* from time to time at particular ages must
necessarily be, more or less, proportional to the number existing
at those ages among the entire body, but there can be no con-
nection whatever between the number of persons *admitted* from
time to time and the corresponding existing number.   Hence,
although the effect of withdrawal upon the *aggregate* rate of
mortality is considerable, its operation upon the mortality table is
throughout pretty uniform, and consequently it does not *distort*
the table to anything like the same extent as the effect of
selection in regard to admissions.   For instance, at the earlier
ages, say under 30, the lives recently admitted necessarily
constitute nearly the whole of the lives upon which the observa-
tions at these ages are based; but this is evidently not the case at
the middle and higher ages, the *proportion* of lives newly admitted
being at this part of the table comparatively small.

It is this "artificial" disturbance, caused by the *unequal*
effect of selection from new admissions, that necessitates the
introduction of the complementary series in Mr. King's table.

KING'S Hᴹ TABLE	
Age	Complementary Series
10	2,906
11	2,652 −254
12	2,363 −289
13	2,052 −311
14	1,731 −321
15	1,416 −315
16	1,119 −297
17	852 −267
18	622 −230
19	435 −187
20	289 −146
21	182 −107
22	108 −74
23	60 −48
24	31 −29
25	15 −16
26	7 −8
27	3 −4
28	1 −2
29	0 −1

This complementary series, which, with its differences, I here reproduce, presents much the same characteristic features as the ordinary mortality table. The differences are of the same sign throughout, and, although they increase (numerically) for a few terms at the commencement, they soon begin to decrease, and finally vanish, as the effects of the disturbance become gradually less and less. This is precisely what occurs in the mortality table itself from and after the age of 70, or thereabouts.

In a normally-constituted body of lives, there will necessarily be always a certain number in various stages of deterioration,—some even actually at the point of death,—and the effect of the selection upon admission is evidently to exclude, more or less completely, the persons so affected. If the lives thus excluded could be subsequently traced, and observations made as to the mortality prevailing among them, then, by incorporating such observations with those of the general body, we should be dealing with a body of lives *normally* constituted, and the total mortality found to be prevalent would represent, more or less accurately, the *normal* law of mortality. But this course is quite impracticable, and we can only form some idea of the effect of selection upon the earlier decrements of the mortality table by deducing the complementary series as Mr. King has done. The effect of subtracting the "complementary" from the "normal" series is, of course, to reduce the normal decrement at each age by the corresponding decrement of the complementary series, and the deaths so deducted are supposed to represent those which have been withdrawn from observation by the process of selection. It is true the "disturbance" dealt with by Mr. King does not represent the *entire* effect of selection upon admission, but only the *excess* thereof at the earlier over that of the other ages. Nevertheless, the principle is the same; and the fact that the decrements of the complementary series are all of the same sign is entirely consistent with, and therefore to that extent confirmatory of, the suggested explanation of the phenomenon.

So much, then, for the *artificial* disturbance arising from the effect of selection upon admission. It remains now to investigate the still more interesting and important case of a disturbance due, not to *artificial*, but to *natural* causes, as exemplified in the H$^{M.5)}$ Table of the Institute observations.

Although the abnormally high mortality so apparent in the observations of John Finlaison on Male Annuitant Life, which occurs at about the age of 23 (and at a few years immediately preceding and following that age), could scarcely have failed to attract the notice of all who had given those observations even the most cursory examination, I believe I was the first directly to call attention to it at a meeting of the members of the Institute (see *J.I.A.*, xii, 305). In doing so, I ventured the opinion that as this particular feature, if observable at all in the old " Experience" and in the " Friendly Societies'" observations, was so utterly insignificant in amount as to warrant the conclusion that it forms no part of the normal law, and, therefore, that the proper mode of dealing with it is to treat it as a " disturbance", by which the normal law is, in certain cases only, affected. The explanation of the phenomenon, which then seemed (and which still seems) to me the most natural and simple, is that those who, at the period of early manhood, are tempted to lead irregular lives are, from the nature of the case, almost entirely excluded from the observations on the members of "Assurance" and " Friendly" societies; whereas, the Tontine Nominees who formed the body upon which Mr. Finlaison's observations were based—being for the most part young men of independent means—would naturally comprise the full complement, or the usual proportion of persons of irregular habits. Not that I at all intend to imply that the classes from which assurance and friendly societies are chiefly recruited do not also comprise the full complement of such " eccentrics", but only that the individuals in question will, in quite exceptional cases only, be found enrolled as members of provident societies.

While the H^{M} Table fully bears out the old "Experience" observations in this particular, it is somewhat remarkable that the H$^{M.5)}$ Table as completely contradicts them. Although, probably, no one anticipated such a result as this,—certainly, *I* did not,— there can, nevertheless, be no doubt of the *fact*, and it only remains for us to endeavour to explain or account for the seeming contradiction. The abnormal mortality referred to culminates in the H$^{M.5}$ Table at the age of 24. Now, the whole

of the deaths at this age must have occurred among individuals who were admitted at ages *under* 19. But it is well-known that assurances are not effected from provident motives at such ages as these. In all probability, therefore, the whole of these assurances were effected in connection with life interests in property, and, if so, the individuals in question must have been circumstanced almost identically the same with those included in John Finlaison's observations. Ought we, then, to be surprised if they exhibit also the same characteristic in regard to mortality, bearing, of course, in mind that they were admitted before the ages at which the tendency to irregular habits is usually developed. If I am right in my conjecture, this further conclusion evidently follows, namely, that if these few exceptional cases had been excluded from the II^{M} observations, the mere shadow of an excessive mortality at the same ages which the latter exhibit (in Woolhouse's Table) would, in all probability, have disappeared altogether.

From Woolhouse's $\text{II}^{\text{M}5}$ Table we get :

$$
\begin{aligned}
\log l_{32} &= 3\cdot93116 \\
&-\cdot06660 \\
\log l_{46} &= 3\cdot86456 &&-\cdot05632 &&\log = 2\cdot75066 \\
&-\cdot12292 &&&&+\cdot55385 \\
\log l_{60} &= 3\cdot74164 &&-\cdot20161 &&\log = \bar{1}\cdot30451 \\
&-\cdot32453 &&&&\phantom{\log = \bar{1}\cdot30451}+\cdot55462 \\
\log l_{74} &= 3\cdot41711 &&-\cdot72299 &&\log = \bar{1}\cdot85913 \\
&-1\cdot04752 \\
\log l_{88} &= 2\cdot36959
\end{aligned}
$$

These figures show that the natural disturbance which we have been considering has no effect upon the mortality table at ages above 32. I have, therefore, commenced with that age in deducing the normal series of the table, and forming the complementary series in the way previously explained, the following results have been obtained. The first column contains the age; the second, the unadjusted complementary series; the third and fourth columns contain the same series adjusted by the formula $ds^x y^{q^x}$, with its differences.

If the adjusted series in the third column be carried back a few years below the age of 18, its terms become entirely insignificant, and we should have a series beginning practically with 0, and after increasing to a maximum at age 20, finally

18	129	129·0	+14·6
19	151	143·6	+ 7·8
20	162	151·4	− 1·0
21	157	150·4	−10·4
22	140	140·0	−18·6
23	118	121·4	−21·1
24	93	97·3	−25·7
25	69	71·6	−23·6
26	48	48·0	−19·0
27	33	29·0	−13·3
28	20	15·7	− 8·2
29	10	7·5	− 4·4
30	4	3·1	− 2·0
31	1	1·1	− ·8
32	0	·3	− ·2
33	0	·1	− ·1
34	0	·0	

vanishing again at a few years after age 30. In a series of this kind the differences are, of course, all positive for ages below the maximum term, and negative for all higher ages, the sum of the positive terms, and the sum of the negative terms, being, moreover, exactly equal. The effect, therefore, of deducting the " complementary " from the " normal " series in a disturbance of this kind is merely to transfer a certain number of deaths from the ages at which, according to the normal law, they *should* have occurred to other ages not far removed. This, I take it, is the way in which a disturbance from " natural " causes may generally be supposed to act, as it would be difficult to conceive a natural disturbance which should have the effect of transferring a certain number of deaths from one part of the mortality table to another part separated from it by any considerable number of years.

To make my meaning a little clearer, as to the way in which I conceive these natural disturbances to operate, I will suppose the mortality table to represent a body of individuals all born at the same time, and, therefore, all attaining the several stated ages simultaneously. Now, suppose that, in a given year, the winter should happen to be one of extraordinary severity, the effect of which would be, no doubt, to cut off prematurely a certain number of individuals suffering at the time from serious ailments, or who are " sick unto death ", without producing any bad effect upon those who are in robust health. It is evident that in the case supposed the result upon the mortality table would simply be to swell the number of deaths at the particular age at which this excessive cold is supposed to occur, and to lessen to precisely the same extent the number of deaths at the ages immediately following, inasmuch as a certain number of those who would otherwise have died at the latter ages have been *prematurely* removed by death. And precisely the opposite effects may be supposed to result from the supposition of an unusually *mild* winter. Of course it will be understood that I am not suggesting these suppositions as being in any way explanatory or descriptive of the real *nature* of the operating cause of what I have termed a natural disturbance, for it is evident that the variations of the seasons could not possibly affect particular ages in the way supposed, but that I have made them solely with the view of illustrating what I conceive to be its *mode of action*, or the way in which the observed effects may be supposed to be produced upon the decrements of the table.

By an unfortunate omission in the " *Mortality Experience* ",

published by the Institute, the various tables setting forth the numbers exposed to risk, &c., at each age do not include the data upon which the $H^{M.5}$ Table is based, so that I am unable to give a tabular comparison of " Actual " and " Expected " deaths. In lieu of this, I have given in parallel columns the annual decrements among 97,250 persons living at age 18, first by Mr. King's H^{M} Table, and next by my adjusted $H^{M(5)}$ Table, which will serve to exhibit the great pliability of the formula, as the principle of adjustment, it will be remembered, is identically the same in both cases.

Table showing the adjusted Yearly Decrements among 97,250, assumed to be living at Age 18, according to the H^M and $H^{M(5)}$ Observations.

Age	DECREMENTS		Age	DECREMENTS		Age	DECREMENTS	
	H^M	$H^{M.5}$		H^M	$H^{M(5)}$		H^M	$H^{M(5)}$
18	466	609	46	988	1,020	74	2,493	2,313
19	509	675	47	1,023	1,049	75	2,464	2,283
20	549	762	48	1,063	1,083	76	2,417	2,237
21	583	856	49	1,104	1,120	77	2,348	2,173
22	610	936	50	1,147	1,158	78	2,260	2,090
23	632	994	51	1,196	1,199	79	2,151	1,988
24	648	1,010	52	1,246	1,244	80	2,022	1,870
25	659	991	53	1,299	1,290	81	1,877	1,735
26	665	946	54	1,356	1,339	82	1,716	1,588
27	674	892	55	1,417	1,392	83	1,543	1,430
28	679	844	56	1,478	1,448	84	1,364	1,266
29	688	812	57	1,545	1,506	85	1,183	1,099
30	693	791	58	1,616	1,566	86	1,004	934
31	702	785	59	1,686	1,630	87	832	777
32	711	787	60	1,759	1,693	88	672	630
33	721	793	61	1,834	1,758	89	528	497
34	731	801	62	1,910	1,825	90	403	380
35	744	810	63	1,988	1,892	91	297	281
36	758	821	64	2,064	1,958	92	209	200
37	772	832	65	2,138	2,022	93	144	138
38	788	847	66	2,209	2,084	94	93	90
39	808	863	67	2,278	2,141	95	58	57
40	825	878	68	2,339	2,194	96	34	34
41	848	897	69	2,393	2,240	97	18	18
42	873	917	70	2,439	2,278	98	10	10
43	897	940	71	2,473	2,306	99	5	5
44	926	965	72	2,496	2,321	100	3	3
45	956	989	73	2,502	2,325	101	1	1

I have already observed that nothing could be more satisfactory, in my opinion, than Mr. King's adjustment of the H^M Table by what I have termed the first development of Gompertz's law—*i.e.*, by assuming geometrical second differences;

and although, for the purpose of illustration, I have, in the first portion of this article, treated the same data upon the assumption of geometrical *third* differences, I think the course followed by Mr. King is decidedly the more appropriate to this particular case. By using geometrical third differences, it is true, we get a greater grasp for the application of the normal series, inasmuch as the whole of the table, from age 18 upwards, is thereby included, whereas the assumption of geometrical second differences necessitates our commencing with the higher age of 28; but this very fact forces me to the conclusion that the latter assumption is more in accord with the facts of the case. Giving due weight to the considerations set forth at the commencement of this the second portion of the present article, it is impossible to think that the distortion in the mortality table resulting necessarily from the unequal pressure, at the earlier ages, of the effect of selection upon admission, can really be confined to ages under 18, or, in other words, that the normal series can possibly be represented by the actual mortality at and after an age so little advanced as this. It will be understood, then, that I have thrown out the suggestion of geometrical *third* differences, not as in any way superseding the method of geometrical *second* differences, whenever the latter may be found applicable to the particular case in hand, but merely as a substitute which may be available in certain cases where the other is found to be unsuitable.

The essential factor or radix, as it may be termed, of the several functions which express, respectively, Gompertz's law and its successive developments, is the superposed exponential g^{q^x}. Thus the form of the function

1. Whose logs
2. The first differences of whose logs
3. The second differences of whose logs
4. The third differences of whose logs

are in geometrical progression, is

$$g^{q^x}$$
$$dg^{q^x}$$
$$ds^x g^{q^x}$$
$$ds^x w^{x^2} g^{q^x}$$

and so on.

Or these relations may be expressed in another way, which I think preferable, namely,

1. Gompertz's law, as propounded by him,
2. The *first* development of Gompertz's law
3. The *second* development of Gompertz's law

assumes that $cl_x =$

$$g^{q^x}$$
$$g^{q^x} . s^x$$
$$g^{q^x} . s^x . w^{x^2}$$

c^{-1} being equivalent to the constant d of Gompertz's formula.

If we take $cl_x = y^{q^x}.s^x.w^{x^2}$, or, striking out the constant factor c,

$$l_x = y^{q^x}.s^x.w^{x^2}$$

$$l_{x+d} = y^{q^{x+d}}.s^{x+d}.w^{(x+d)^2}$$

$$= y^{q^{x+d}}.(sw^{2d})^x.w^{x^2} \ldots \text{(striking out constant factors)}$$

$$l_x.l_{x+d} = y^{q^x+q^{x+d}}.(sw^d)^{2x}.w^{2x^2}$$

and $$l^2_z = y^{2q^z}.s^{2z}.w^{2z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [1]$$

Putting $2q^z = q^x + q^{x+d}$, which gives

$$z = x + \frac{\log(1+q^d) - \log 2}{\log q} = x + d' \text{ (suppose)},$$

we get $\; l_x.l_{x+d} = y^{2q^z}.(sw^d)^{2z}.w^{2z^2-2d'z)} \ldots \begin{cases} \text{(again striking out} \\ \text{constant factors)} \end{cases}$

$$= y^{2q^z}.(sw^{d-2d'})^{2z}.w^{2z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

$$= l^2_z.w^{2(d-2d').z} \quad . \quad . \quad . \quad . \text{ from } [1] \text{ and } [2]$$

$$= l^2_z.w'^z \ldots \text{ (putting } w' = w^{2(d-2d')}).$$

Hence, in a mortality table constructed according to the second development of Gompertz's law, we should require, for the computation of annuities on joint lives, a set of tables for equal ages at successive rates of interest.

<center>(*To be concluded.*)</center>

---

*On a Table of Coefficients arising out of a given Mortality Table,
for Finding Annuity-Values at any Rate of Interest that may
be required. By* THOMAS J. SEARLE, *A.I.A., late Assistant
Secretary of the Employers' Liability Assurance Corporation,
Limited.*

1. I HAVE long had an idea that the ordinary formula for the value of an annuity,

$$a_x = r.p_x + r^2.{}_2p_x + v^3.{}_3p_x + \&c.,$$

might be transformed into some such form as the following:

$$a_x = A + Bi + Ci^2 + Di^3 + \&c.;$$

where the coefficients would represent the condensed effect of the mortality, and the powers of i would make the calculation easy and rapidly convergent. I have lately succeeded in making the proposed transformation.

2. The terms, however, instead of being all $+$, are alternately $+$ and $-$, which is of no consequence; and the coefficients, A, B, C, &c., increase so rapidly as to be unmanageable, which is more serious. But this latter defect can be completely remedied by proceeding by powers of $10i$, instead of by powers of i.

3. The working formula, therefore, is

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c. \quad . \quad . \quad (1)$$

4. The first term is appropriately called e_x, because it is, in fact, the *curtate expectation*, which may be easily seen by making $i = 0$, when all the other terms vanish; and, as is well known, the annuity becomes the *curtate expectation*.

5. I propose, first, to show a specimen of a table of the coefficients, with some examples of its use, and then to explain by what means the table was calculated.

Use of the Table.

6. The following extract from the Table will serve our immediate purpose.

Coefficients of $(10i)^n$ *in the Formula* $a_x = e_x - f_x(10i) + \&c.$

H^M *Mortality.*   *Any Rate of Interest.*

n	Age 45	Age 46	Age 47	Age 48	Age 49
10	·9288	·7397	·5870	·4640	·3653
9	1·9809	1·6033	1·2932	1·0394	·8322
8	3·9712	3·2680	2·6807	2·1916	1·7854
7	7·4299	6·2200	5·1915	4·3196	3·5823
6	12·8581	10·9567	9·3109	7·8896	6·6651
5	20·3496	17·6633	15·2935	13·2073	11·3744
4	29·0164	25·6769	22·6717	19·9721	17·5508
3	36·5252	32·9858	29·7330	26·7477	24·0110
2	39·4162	36·3754	33·5177	30·8343	28·3159
1	34·8427	32·9148	31·0588	29·2724	27·5524
0	23·2915	22·5790	21·8750	21·1789	20·4892
Totals	210·6105	190·9829	173·2130	157·1169	142·5240

7. As the terms are alternately + and − , and proceed by powers of $10i$, the easiest mode of applying the table will be to commence with the coefficient of the highest power of $10i$, multiply this by $10i$, and subtract the product from the next coefficient, multiply the remainder by $10i$, and repeat the process until we arrive at, and use up, the coefficient of $(10i)^0$, (that is, e_x,) and obtain the required annuity.   The table is presented in the form best adapted for the suggested calculation.

8. Let it be required to obtain the value of an annuity, at age 45, by the H^M Mortality at 4 per-cent interest.

The following table contains the whole of the calculation :

·9288	·0000	·9288
1·9809	·3715	1·6094
3·9712	·6438	3·3274
7·4299	1·3310	6·0989
12·8581	2·4396	10·4185
20·3496	4·1674	16·1822
29·0164	6·4729	22·5435
36·5252	9·0174	27·5078
39·4162	11·0031	28·4131
34·8427	11·3652	23·4775
23·2915	9·3910	13·9005
210·6105	56·2029	154·4076

9. The figures in the first column are the given coefficients, copied from the table ; those in the third column are obtained by subtracting those in the second from those in the first ; the figures in the second column are obtained by multiplying those on the previous line of the third column by $10i$, in this case ·4.   Thus, there being nothing to deduct from ·9288, it is carried out into the third column and multiplied by ·4, the product, ·3715, being placed in the second column, and subtracted from the next coefficient, 1·9809, leaving 1·6094 ; this again is multiplied by ·4 and the product subtracted from the next coefficient, and so on, until we come to 13·9005, which is the required annuity.

10. The totals are added only as a check upon the process : the total of the first column shows that the figures have been correctly copied from the table ; the difference of the first and second totals being equal to the third shows that all the sub-tractions have been correctly made, and a comparison of the second total with the third total, *minus* the final value, proves the

correctness of all the multiplications by ·4. There is thus very
little risk of error.

11. Let it be now required to find the value of the same
annuity at 3 per-cent interest. This is the calculation:

·9288	·0000	·9288
1·9809	·2786	1·7023
3·9712	·5107	3·4605
7·4299	1·0382	6·3917
12·8581	1·9175	10·9406
20·3496	3·2822	17·0674
29·0164	5·1202	23·8962
36·5252	7·1689	29·3563
39·4162	8·8069	30·6093
34·8427	9·1828	25·6599
23·2915	7·6980	15·5935
210·6105	45·0040	165·6065

We start from exactly the same figures and go through exactly
the same process, the only difference being that in the one case
we use ·4 for a multiplier and obtain a 4 per-cent annuity, and
in the other we use ·3 for a multiplier and obtain a 3 per-cent
annuity.

12. Both the results just obtained are true to the last place of
decimals with those in the Institute Tables. We might proceed
in the same way to obtain the annuity at any rate for which the
annuity-values are not published, say $3\frac{1}{3}$ per-cent, or any other
rate; but it will be unnecessary to show any further example, as
those already given will suffice.

CONSTRUCTION OF THE TABLE.

13. For the construction of the table we shall need the
following formulas:

$$r = (1+i)^{-1} = 1 - i + i^2 - i^3 + \&c. \quad \dots \quad (2)$$

$$a_{\omega-2} = r \cdot p_{\omega-2}$$
$$\therefore l_{\omega-2} \cdot a_{\omega-2} = r \cdot l_{\omega-1}$$
$$= l_{\omega-1} - l_{\omega-1} \cdot i + l_{\omega-1} \cdot i^2 - l_{\omega-1} \cdot i^3 + \&c. \quad \dots \quad (3)$$

$$a_x = r \cdot p_x (1 + a_{x+1})$$
$$\therefore l_x \cdot a_x = r \cdot l_{x+1} (1 + a_{x+1})$$
$$= (l_{x+1} + l_{x+1} \cdot a_{x+1})(1 - i + i^2 - i^3 + \&c.) \quad \dots \quad (4)$$

14. Let it now be assumed that

$$l_x a_x = E_x - i \cdot F_x + i^2 G_x - i^3 H_x + \&c. \quad . \quad . \quad . \quad (5)$$

$$l_{x+1} \cdot a_{x+1} = E_{x+1} - i \cdot F_{x+1} + i^2 G_{x+1} - i^3 \cdot H_{x+1} + \&c. \quad . \quad (6)$$

where E_x, F_x, G_x, &c., and E_{x+1}, F_{x+1}, G_{x+1}, &c., are coefficients at present undetermined. It will follow, from (4) and (6), that

$$
\begin{aligned}
l_x a_x &= (l_{x+1} + l_{x+1} \cdot a_{x+1})(1 - i + i^2 - i^3 + \&c.) \\
&= (l_{x+1} + E_{x+1} - i \cdot F_{x+1} + i^2 \cdot G_{x+1} - i^3 \cdot H_{x+1} + \&c.) \\
&\quad \times (1 - i + i^2 - i^3 + \&c.) \\
&= (l_{x+1} + E_{x+1}) - i \cdot F_{x+1} + i^2 \cdot G_{x+1} - i^3 \cdot H_{x+1} + \&c. \\
&\quad - i(l_{x+1} + E_{x+1}) + i^2 \cdot F_{x+1} - i^3 \cdot G_{x+1} + \&c. \\
&\quad\quad + i^2(l_{x+1} + E_{x+1}) - i^3 \cdot F_{x+1} + \&c. \\
&\quad\quad\quad - i^3(l_{x+1} + E_{x+1}) + \&c.
\end{aligned}
$$

which should be, from (5),

$$= E_x - i \cdot F_x + i^2 \cdot G_x - i^3 \cdot H_x + \&c.;$$

and this will be so if we equate the coefficients as follows:

$$
\begin{aligned}
E_x &= l_{x+1} + E_{x+1} \\
F_x &= l_{x+1} + E_{x+1} + F_{x+1} \\
G_x &= l_{x+1} + E_{x+1} + F_{x+1} + G_{x+1} \\
H_x &= l_{x+1} + E_{x+1} + F_{x+1} + G_{x+1} + H_{x+1} \\
\&c. &= \&c.
\end{aligned}
$$

or, reducing,

$$
\left.
\begin{aligned}
E_x &= l_{x+1} + E_{x+1} \\
F_x &= E_x + F_{x+1} \\
G_x &= F_x + G_{x+1} \\
H_x &= G_x + H_{x+1} \\
\&c. &= \&c.
\end{aligned}
\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)
$$

15. When, therefore, the coefficients E_x, F_x, &c., are given for any age, together with the values of l_x, we can readily find similar coefficients for the next younger age. But we have already shown, in formula (3), that for age $\omega - 2$, all these coefficients $= l_{\omega-1}$, and commencing with that age, which is the oldest age that has any annuity-value, we can proceed backwards, step by step, throughout the mortality table to the youngest age.

16. Next let us put the series into one advancing by powers of $10i$, assuming that

$$l_x a_x = E'_x - (10i)F'_x + (10i)^2 G'_x - (10i)^3 H'_x + \&c. \quad . \quad . \quad (8)$$

it is evident that this can be done by making

$$\left.\begin{array}{l} E'_x = E_x \\[4pt] F'_x = \dfrac{F_x}{10} \\[4pt] G'_x = \dfrac{G_x}{100} \\[4pt] H'_x = \dfrac{H_x}{1,000} \\[4pt] \&c. = \&c. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

17. We finally wish to transform it into our proposed original formula,

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c. \quad . \quad . \quad (1)$$

which is only dividing the coefficients throughout by l_x, or, putting it symbolically, making

$$\left.\begin{array}{l} e_x = \dfrac{E'_x}{l_x} \\[6pt] f_x = \dfrac{F'_x}{l_x}, \text{ and so on} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

18. We are now prepared to commence the calculation of the coefficients. The l_x values, and the coefficients deduced from them, E_x, F_x, &c., were tabulated as follows:

Age	l_x	E_x	F_x	G_x	H_x
97	9	...	...	...	...
96	49	9	9	9	9
95	135	58	67	76	85
94	274	193	260	336	421
93	469	467	727	1,063	1,484
92	723	936	1,663	2,726	4,210
91	1,052	1,659	3,322	6,048	10,258
90	1,460	2,711	6,033	12,081	22,339
89	1,955	4,171	10,204	22,285	44,624
88	2,570	6,126	16,330	38,615	83,239
87	3,343	8,696	25,026	63,641	146,880

19. The l_x values having been inserted, the value of l_{97} is entered in all the columns opposite age 96, and the table is then

completed horizontally, age by age, by the use of the formulas (7):
that is to say, 49 is added to the 9 in the E column, and the sum
58 set down in the same column; this 58 is added to the 9 in
the F column, and so on.   But we shall not have proceeded far
before we shall discover that, instead of completing the table
horizontally, we can obtain precisely the same figures by completing
it perpendicularly by constant summation.   This might have
been demonstrated algebraically in paragraph 14, by rearranging
formulas (7), but I have preferred to set it out as it came to me.

20. Besides the obviously superior convenience of the perpen-
dicular formation, it has two other important advantages, namely,
that it enables the correctness of the summations in any column
to be proved by first making a direct total of the previous column,
and that it facilitates the succeeding stage of the operation.

21. This next stage is the dividing of the F column by 10, of
the G column by 100, and so on, which is done by inserting, or
supposing to be inserted, decimal points, giving one decimal place
in the F column, two in the G column, until we find that we have
decimal places enough, and commence to drop the last file of
figures at the summation of each successive column.   The summa-
tions and cuttings off will be repeated until the figures get smaller
and smaller, and ultimately all vanish.   Those at the older ages
will vanish first, and those at the younger ages last.

22. Intending to work for five places of decimals in the final
results, I retained two places in the successive summations, but, as
I have since found that four places in the results are fully sufficient,
I might have spared myself very many figures.   For ages where
the divisor, l_x, is less than 10,000, it is necessary to retain more
decimals in the summations than it is for the bulk of the table, in
order to get an equal number of decimals in the results.

23. The mode of cutting off the last file of figures, at the same
time that the summations were made, is worth noting.   A specimen
of it is subjoined:

9·	1
67·	8
260	34
727	106
1,663·	273
3,322	605
6,033	1,208
10,204·	2,229
16,330	3,862
25,026	6,364

Instead of increasing the next figure where the figure cut off made
a carrying, it was increased where the total, from the top, of the
figures cut off made a carrying or an additional carrying, a dot
was placed to mark each place of increase, and the figures cut off
were entirely disregarded, and the dots added in instead as 1
each.   This, it will be found, has the same effect as if we had
summed the figures in full and cut them down afterwards.   The
point is important, owing to the accumulating power of any error
—I feared that my results would be vitiated by my having increased
when cutting off an exact 5: that they have not been so affected
must be due to my having worked with too many decimals; in
future an increase should only be made for every alternate 5.

24. After finishing the summations, and before commencing
the divisions by l_x, the whole of the figures obtained were cross-cast,
and the total of the cross-castings was proved with the totals of the
whole of the columns.   This entirely precluded the possibility of
an error in the summations; and gave, besides, a valuable check
upon the divisions, by enabling me to prove the correctness of the
final results against their own totals, the totals of the final results
being themselves also useful, as mentioned in paragraph 10.

25. The divisions were made by taking the reciprocals of l_x,
and then by mental multiplication (see *J.I.A.*, x, 326).   The
totals were divided by ordinary long division.

The Completed Table.

26. The complete table of coefficients for the H^M mortality is
appended hereto.   There are, for the younger ages, a large
number of coefficients, but I thought it best to present all the
coefficients that are necessary for the calculation of an annuity, at
any rate up to 10 per-cent, although, for smaller rates of interest,
many of the coefficients may be disregarded, as they have no
effect whatever on the resulting annuity.

27. At 10 per-cent interest, $10i = 1$, and the working formula

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c. \quad . \quad . \quad (1)$$

becomes

$$a_x = e_x - f_x + g_x - h_x + \&c.$$
$$= (e_x + g_x + \&c.) - (f_x + h_x + \&c.) \quad . \quad . \quad . \quad (11)$$

where we see that *every* coefficient is brought into full play.

28. But at 5 per-cent, for instance, $10i = \cdot5$, and we may consider the following scheme:

17	6·5536
16	3·2768
15	1·6384
14	·8192
13	·4096
12	·2048
11	·1024
10	·0512
9	·0256
8	·0128
7	·0064
6	·0032
5	·0016
4	·0008
3	·0004
2	·0002
1	·0001
0	·00005

From this it appears that if the coefficient numbered 17 is anything less than 6·5536, its continued multiplication by ·5 will cause it to disappear without having any effect on the resulting annuity; and the same result will ensue if any of the coefficients, according to its number, is less than the figures stated against that number. All such coefficients, and consequently all those of higher powers of $10i$, may be disregarded.

29. Applying this rule, we now show how many coefficients come into use for the H^M mortality at interval ages.

Age	Number of Coefficients when Interest does not exceed		
	10 per-cent	5 per-cent	3 per-cent
10	31	18	13
20	28	16	12
30	25	15	11
40	22	13	10
50	19	12	9
60	16	10	8
70	13	8	7
80	10	7	6
90	7	5	4
96	4	3	3

30. In the final tables the coefficients which are not wanted when the rate does not exceed 5 per-cent are printed in *italic* figures, and the totals are shown both without and with those figures.

31. It will also be noticed that all the coefficients are given to four places of decimals, with the exception of the leading coefficient, e_x, the values of which are stated to five places. The reason of this is, that although two values may each be strictly correct to four places, their difference will sometimes be wrong to the extent of ·0001, and I have, therefore, given five places for the coefficients which do not stand to be multiplied by $10i$.

32. With this precaution, even at the youngest age, where any accumulation of error—if there were any—would certainly show itself, the four-figure coefficients give annuity values strictly correct to four places, and I have every confidence that it will be found to be so throughout the table, with an occasional difference for very high rates of interest, of not exceeding ·0005, such differences being probably due to the cause explained in the last paragraph.

33. The checks adopted in the construction of the table were quite sufficient to secure its correctness, but in order to study the question of decimal places, and to put the new table to its most extreme test, a special table of a_x at 10 per-cent interest was calculated by a direct process. The value of v at 10 per-cent is $\dfrac{10}{11}$, and the formula used for this purpose was

$$l_x a_x = \frac{10}{11} (l_{x+1} + l_{x+1} \cdot a_{x+1}).$$

The 10 per-cent annuities from the table of coefficients, for comparison with those thus calculated, are very readily got by the formula (11) in paragraph 27, namely:

$$a_x = (e_x + g_x + \&c.) - (f_x + h_x + \&c.),$$

and this formula includes the effect of every coefficient to its last place of decimals, and thus thoroughly and finally proves the accuracy of the whole table.

34. The value to the actuary of tables in this form becomes evident when an annuity is wanted for a given age by an ordinary table of mortality, but at an unusual rate of interest, but their value is much more apparent when an annuity is wanted at an ordinary rate, but by an unusual, though valuable, table of mortality

—such, for instance, as that of the Peerage families, by Messrs. Bailey and Day, or some other table for which annuities are calculated at 3 per-cent, or not at all.

35. Fully to meet this want would require the publication, in book form, of tables of coefficients for all standard tables of mortality which are not obsolete. I am intending to apply the process to other mortality tables than the H^M; whether I can undertake the larger work indicated will depend upon whether the new tables succeed in finding favour with the profession, and upon a preliminary enquiry as to how many tables of mortality are entitled to be included in the canon of standard tables.

36. It would be possible to obtain the coefficients for joint lives of equal ages by using the squares of the l_x values, and it seems that the series would converge more rapidly, and the coefficients be fewer in number than for single lives; but I have not, at present, leisure to pursue the enquiry further in that direction.

POSTSCRIPT.

37. Since the above was submitted to the Institute, Mr. Charles D. Higham has kindly pointed out that tables in this form will find one possible application in the "Apportionment of a Fund between the Life Tenant and the Reversioner." Mr. Baden (*J.I.A.*, xvi, 269) and Mr. Sprague (*J.I.A.*, xviii, 77) have recommended an apportionment which necessitates the use of an annuity at the exact rate of interest realized by the fund, and Mr. Sprague, in illustrating the subject, calculates, by interpolation, annuities at the rates of 3·157, 3·333, and 3·529 per-cent.

38. If it be required to obtain the value of a_{60} to three places of decimals at 3·157 per-cent, according to the H^M Table, the following would be the calculation:

·054	·000	·000	·054
·144	{ ·016 ·001 }	·017	·127
·357	{ ·038 ·001 ·001 }	·010	·317
·829	{ ·095 ·003 ·002 }	·100	·729
1·783	{ ·219 ·007 ·004 }	·230	1·553
3·510	{ ·466 ·016 ·008 ·001 }	·491	3·019
6·215	{ ·906 ·030 ·015 ·002 }	·953	5·262
9·669	{ 1·579 ·053 ·026 ·004 }	1·662	8·007
12·736	{ 2·402 ·080 ·040 ·006 }	2·528	10·208
13·3297	{ 3·0624 ·1021 ·0510 ·0071 }	3·2226	10·1071
48·6267	9·2436	9·2436	39·3831

39. It will be noticed that an additional column is introduced into the calculation, in order to provide for the multiplication by the 4 digits of $10i = ·3157$. This example will enable an opinion to be formed as to the comparative facility of the method by interpolation, as indicated at *J.I.A.*, xviii, 86, and of that by the new table. The results, however, are not comparable, the value just found, 10·107, being that according to the H^M Table, while Mr. Sprague's result is based upon the Carlisle Table.

Coefficients of $(10i)^n$ in the Formula,

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M *Any Rate of Interest.*

n	Age 10	Age 11	Age 12	Age 13	Age 14	n
30	·0001	·0001	·0001	...	...	30
29	·0003	·0002	·0002	·0001	·0001	29
28	·0009	·0007	·0005	·0003	·0003	28
27	·0024	·0018	·0013	·0010	·0007	27
26	·0064	·0048	·0035	·0026	·0019	26
25	·0165	·0124	·0093	·0069	·0052	25
24	·0414	·0314	·0237	·0179	·0134	24
23	·1012	·0775	·0591	·0450	·0341	23
22	·2408	·1861	·1433	·1099	·0841	22
21	·5563	·4338	·3371	·2611	·2015	21
20	1·2458	·9806	·7691	·6012	·4685	20
19	2·7005	2·1455	1·6987	1·3406	1·0548	19
18	5·6550	4·5356	3·6256	2·8890	2·2952	18
17	11·4166	9·2449	7·4619	6·0042	4·8177	17
16	22·1698	18·1282	14·7762	12·0079	9·7313	16
15	41·3044	34·1096	28·0809	23·0505	18·8705	15
14	73·6194	61·4082	51·0683	42·3501	35·0291	14
13	125·1208	105·4357	88·5884	74·2307	62·0446	13
12	202·0171	172·0070	146·0412	123·6699	104·4741	12
11	308·5326	265·4838	227·8187	195·0038	166·5316	11
10	443·4968	385·7392	331·6228	289·5769	250·0149	10
9	596·4771	524·5139	460·0705	402·6104	351·5915	9
8	745·3331	662·7888	587·9653	520·4392	459·7593	8
7	857·9201	771·6917	692·5364	620·2033	554·3952	7
6	900·0971	819·1816	743·9187	674·2439	610·0383	6
5	849·2945	782·3078	719·1296	659·8416	604·4752	5
4	708·1999	660·4720	614·7838	571·2864	530·0946	4
3	509·6427	481·4092	453·9376	427·3723	401·8379	3
2	305·9235	292·8264	279·8397	267·0589	254·5722	2
1	145·3194	141·0313	136·6730	132·2889	127·9225	1
0	49·79094	49·03612	48·23253	47·39317	46·53175	0
Totals without the Italics	6895·67614	6236·81542	5635·54553	5082·62787	4587·94455	Totals without the Italics
Gross Totals	6906·24374	6245·22592	5612·21703	5093·90767	4596·92205	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ *in the Formula,*

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M　　　　　　　　　　　　　　　　　　　*Any Rate of Interest.*

n	Age 15	Age 16	Age 17	Age 18	Age 19	n
29	·0001	...	...	...	...	29
28	·0002	·0001	·0001	·0001	...	28
27	·0005	·0004	·0003	·0002	·0001	27
26	·0014	·0010	·0008	·0006	·0004	26
25	·0038	·0028	·0021	·0015	·0011	25
24	·0100	·0075	·0056	·0041	·0031	24
23	·0257	·0194	·0146	·0109	·0081	23
22	·0641	·0487	·0369	·0279	·0211	22
21	·1551	·1190	·0911	·0696	·0530	21
20	·3641	·2821	·2181	·1682	·1294	20
19	·8276	·6476	·5055	·3937	·3060	19
18	1·8185	1·4371	1·1331	·8915	·7000	18
17	3·8550	3·0772	2·4509	1·9481	1·5455	17
16	7·8659	6·3431	5·1041	4·0991	3·2863	16
15	15·4102	12·5558	10·2089	8·2856	6·7135	15
14	28·9045	23·7988	19·5566	16·0424	13·1395	14
13	51·7401	43·0573	35·7649	29·6586	24·5596	13
12	88·0641	74·0849	62·2152	52·1663	43·6827	12
11	141·9190	120·7175	102·5137	86·9296	73·6252	11
10	215·4808	185·3668	159·2154	136·5719	117·0200	10
9	306·4619	266·6836	231·7372	201·1263	174·3879	9
8	405·4394	356·9886	313·9166	275·7108	242·0004	8
7	494·7577	440·9123	392·4624	349·0909	310·1272	7
6	551·1126	497·2409	448·1643	403·5980	363·2500	6
5	552·9929	505·3169	461·3317	420·8879	383·8203	5
4	491·2675	454·8310	420·7777	389·0683	359·6162	4
3	377·4230	354·1949	332·1984	311·4551	291·9735	3
2	212·4495	230·7509	219·5249	208·8073	198·6271	2
1	123·6107	119·3874	115·2825	111·3208	107·5252	1
0	45·66060	44·79207	43·93798	43·10922	42·31679	0
Totals without the Italics	4140·56040	3737·02277	3373·91248	3047·86812	2755·70139	Totals without the Italics
Gross Totals	4147·68650	3742·66567	3378·37158	3051·38452	2758·46919	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ *in the Formula,*

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M   *Any Rate of Interest.*

n	Age 20	Age 21	Age 22	Age 23	Age 24	n
27	·0001	·0001	·0001	...	...	27
26	·0003	·0002	·0002	·0001	·0001	26
25	·0008	·0006	·0004	·0003	·0002	25
24	·0023	·0017	·0012	·0009	·0007	24
23	·0061	·0045	·0033	·0025	·0018	23
22	·0159	·0119	·0089	·0067	·0049	22
21	·0403	·0306	·0231	·0174	·0131	21
20	·0994	·0761	·0581	·0443	·0336	20
19	·2374	·1837	·1418	·1091	·0836	19
18	·5486	·4289	·3345	·2601	·2015	18
17	1·2239	·9670	·7621	·5989	·4692	17
16	2·6301	2·1003	1·6732	1·3292	1·0527	16
15	5·4308	4·3840	3·5307	2·8359	2·2712	15
14	10·7452	8·7702	7·1419	5·8011	4·6987	14
13	20·3080	16·7611	13·8044	11·3415	9·2927	13
12	36·5300	30·4949	25·4059	21·1168	17·5061	12
11	62·2811	52·5983	44·3372	37·2906	31·2863	11
10	100·1566	85·5928	73·0176	62·1594	52·7914	10
9	151·0557	130·6620	112·8363	97·2508	83·6317	9
8	212·2068	185·8446	162·5119	141·8479	123·5530	8
7	275·3843	244·2557	216·3490	191·3059	168·8328	7
6	326·7451	293·6168	263·5244	236·1503	211·2406	6
5	349·8657	318·6466	289·9029	263·3860	238·9043	5
4	332·3583	306·9326	283·1961	260·9769	240·1505	4
3	273·6831	256·4099	240·0534	224·5080	209·7027	3
2	188·9601	179·7085	170·8203	162·2368	153·9210	2
1	103·8904	100·3695	96·9388	93·5689	90·2415	1
0	41·56131	40·82603	40·10241	39·37880	38·61698	0
Totals without the Italics	2491·16251	2255·87353	2043·47324	1851·15560	1676·67148	Totals without the Italics
Gross Totals	2495·96771	2259·67913	2046·48014	1853·52510	1678·53288	Gross Totals

n				Age 96	Age 95	n
4	...	...	...	...	·0001	4
3	...	...	...	·0002	·0006	3
2	...	...	...	·0018	·0056	2
1	...	...	...	·0184	·0496	1
0	...	...	...	·18367	·42963	0
Totals without the Italics	...	...	...	·20387	·48543	Totals without the Italics
Gross Totals	...	...	...	·20407	·48553	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ in the Formula,

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M *Any Rate of Interest.*

n	Age 25	Age 26	Age 27	Age 28	Age 29	n
26	·0001	...	...	...	...	26
25	·0002	·0001	·0001	·0001	...	25
24	·0005	·0003	·0002	·0002	·0001	24
23	·0013	·0010	·0007	·0005	·0004	23
22	·0037	·0027	·0020	·0015	·0011	22
21	·0098	·0073	·0054	·0040	·0030	21
20	·0254	·0191	·0144	·0107	·0080	20
19	·0639	·0487	·0369	·0279	·0210	19
18	·1556	·1198	·0919	·0703	·0536	18
17	·3663	·2851	·2212	·1710	·1318	17
16	·8311	·6541	·5132	·4014	·3130	16
15	1·8134	1·4435	1·1456	·9065	·7154	15
14	3·7917	3·0557	2·4535	1·9644	1·5684	14
13	7·5925	6·1861	5·0263	4·0729	3·2914	13
12	14·4736	11·9346	9·8149	8·0510	6·5870	12
11	26·1810	21·8534	18·1950	15·1122	12·5209	11
10	44·7252	37·8005	31·8714	26·8106	22·5012	10
9	71·7528	61·4217	52·1594	44·7081	38·0194	9
8	107·3827	93·1305	80·5991	69·6137	60·0039	8
7	148·6961	130·7024	114·6605	100·4009	87·7507	7
6	188·6024	168·0842	149·5281	132·7955	117·7349	6
5	216·3255	195·5576	176·4947	159·0487	143·1091	5
4	220·6452	202·4263	185·4430	169·6587	155·0119	4
3	195·6094	182·2310	169·5560	157·5854	146·2961	3
2	145·8653	138·0854	130·5866	123·3843	116·4762	2
1	86·9541	83·7186	80·5410	77·4350	74·4025	1
0	37·90529	37·15828	36·40836	35·66148	34·91908	0
Totals without the Italics	1518·31919	1373·34628	1243·63786	1126·30288	1020·19268	Totals without the Italics
Gross Totals	1519·77709	1375·92798	1245·66946	1127·89698	1021·44008	Gross Totals

n	Age 94	Age 93	Age 92	Age 91	Age 90	n
6	...	...	...	...	·0001	6
5	...	·0001	·0001	·0002	·0004	5
4	·0002	·0004	·0009	·0016	·0027	4
3	·0015	·0032	·0058	·0098	·0153	3
2	·0123	·0227	·0377	·0575	·0827	2
1	·0949	·1551	·2300	·3158	·4132	1
0	·70438	·99574	1·29461	1·57700	1·85685	0
Totals without the Italics	·81308	1·17674	1·56901	1·96170	2·37075	Totals without the Italics
Gross Totals	·81328	1·17724	1·56911	1·96190	2·37125	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ in the Formula,

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M　　　　　　　　　　　　　　　　　　　　*Any Rate of Interest.*

n	Age 30	Age 31	Age 32	Age 33	Age 34	n
24	·0001	·0001	...	...	...	24
23	·0003	·0002	·0001	·0001	·0001	23
22	·0008	·0006	·0004	·0003	·0002	22
21	·0022	·0016	·0012	·0008	·0006	21
20	·0059	·0044	·0032	·0024	·0017	20
19	·0158	·0118	·0088	·0065	·0048	19
18	·0407	·0308	·0232	·0174	·0131	18
17	·1013	·0776	·0592	·0450	·0341	17
16	·2433	·1885	·1455	·1120	·0858	16
15	·5625	·4411	·3447	·2685	·2084	15
14	1·2485	·9909	·7840	·6183	·4859	14
13	2·6524	2·1314	1·7075	1·3637	1·0856	13
12	5·3748	4·3739	3·5490	2·8711	2·3155	12
11	10·3477	8·5297	7·0117	5·7475	4·6974	11
10	18·5392	15·7349	13·1078	10·8900	9·0222	10
9	32·2588	27·3085	23·0611	19·4249	16·3191	9
8	51·6125	44·3000	37·9363	32·4096	27·6200	8
7	76·5462	66·6409	57·8942	50·1843	43·4019	7
6	101·1986	92·0531	81·1647	71·1193	62·7122	6
5	128·5636	115·3107	103·2418	92·2669	82·3019	5
4	141·4336	128·8631	117·2270	106·4688	96·5347	4
3	135·6569	125·6419	116·2112	107·3388	98·9997	3
2	109·8525	103·5077	97·1237	91·5932	86·0090	2
1	71·4416	68·5530	65·7287	62·9677	60·2689	1
0	34·18059	33·41661	32·71351	31·98084	31·21802	0
Totals without the Italics	924·20749	837·38631	757·97824	686·92664	622·53612	Totals without the Italics
Gross Totals	925·18039	838·14301	759·34854	687·99794	623·37082	Gross Totals

n	Age 89	Age 88	Age 87	Age 86	Age 85	n
7	...	...	·0001	·0001	·0002	7
6	·0001	·0002	·0003	·0006	·0008	6
5	·0008	·0012	·0019	·0028	·0010	5
4	·0043	·0065	·0094	·0131	·0178	4
3	·0228	·0324	·0439	·0578	·0741	3
2	·1140	·1503	·1904	·2351	·2842	2
1	·5219	·6354	·7486	·8652	·9847	1
0	2·13350	2·38366	2·60126	2·81022	3·01051	0
Totals without the Italics	2·79650	3·20826	3·59546	3·98422	4·37531	Totals without the Italics
Gross Totals	2·79740	3·20966	3·59586	3·98492	4·37631	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ *in the Formula,*

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M   *Any Rate of Interest.*

n	Age 35	Age 36	Age 37	Age 38	Age 39	n
22	·0001	·0001	·0001	·0001	...	22
21	·0004	·0003	·0002	·0002	·0001	21
20	·0013	·0009	·0007	·0005	·0003	20
19	·0036	·0026	·0019	·0014	·0010	19
18	·0097	·0072	·0053	·0039	·0029	18
17	·0257	·0193	·0145	·0106	·0080	17
16	·0656	·0499	·0378	·0286	·0215	16
15	·1611	·1242	·0953	·0729	·0556	15
14	·3806	·2971	·2311	·1791	·1383	14
13	·8613	·6811	·5368	·1217	·3300	13
12	1·8616	1·4919	1·1918	·9489	·7530	12
11	3·8277	3·1096	2·5186	2·0334	1·6361	11
10	7·4537	6·1102	5·0436	4·1306	3·3723	10
9	13·6734	11·4257	9·5215	7·9125	6·5561	9
8	23·1794	19·9092	16·8389	14·2048	11·9499	8
7	37·4192	32·2371	27·6849	23·7178	20·2674	7
6	54·9493	48·0436	41·9449	36·4867	31·6867	6
5	73·2715	65·1050	57·7355	51·0974	45·1262	5
4	87·3778	78·9531	71·2175	64·1256	57·6309	4
3	91·1742	83·8429	76·9874	70·5849	64·6094	3
2	80·6681	75·5677	70·7052	66·0739	61·6635	2
1	57·6342	55·0658	52·5660	50·1344	47·7679	1
0	30·51602	29·78613	29·05993	28·33757	27·61747	0
Totals without the Italics	564·19742	511·35903	463·52253	420·21017	380·63717	Totals without the Italics
Gross Totals	564·81552	511·86063	163·90943	420·50747	381·19487	Gross Totals

n	Age 84	Age 83	Age 82	Age 81	Age 80	n
9	...	...	...	...	·0001	9
8	·0001	·0001	·0001	·0002	·0003	8
7	·0003	·0004	·0006	·0008	·0011	7
6	·0012	·0018	·0025	·0035	·0048	6
5	·0055	·0076	·0103	·0138	·0183	5
4	·0235	·0308	·0398	·0511	·0653	4
3	·0932	·1159	·1433	·1761	·2157	3
2	·3387	·4004	·4718	·5545	·6510	2
1	1·1101	1·2468	1·4002	1·5722	1·7667	1
0	3·21291	3·42993	3·67087	3·93269	4·21917	0
Totals without the Italics	1·78391	5·23143	5·73627	6·30389	6·94097	Totals without the Italics
Gross Totals	4·78551	5·23373	5·73917	6·30489	6·94247	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ in the Formula,
$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M *Any Rate of Interest.*

n	Age 40	Age 41	Age 42	Age 43	Age 44	n
21	*·0001*	*·0001*	...	...	...	21
20	*·0002*	*·0002*	*·0001*	*·0001*	*·0001*	20
19	*·0007*	*·0005*	*·0004*	*·0003*	*·0002*	19
18	*·0021*	*·0015*	*·0011*	*·0008*	*·0006*	18
17	*·0059*	*·0044*	*·0032*	*·0023*	*·0017*	17
16	*·0161*	*·0120*	*·0089*	*·0066*	*·0048*	16
15	*·0422*	*·0319*	*·0239*	*·0179*	*·0134*	15
14	*·1063*	*·0814*	*·0621*	*·0472*	*·0357*	14
13	*·2573*	*·1998*	*·1545*	*·1190*	*·0913*	13
12	·5953	·4689	·3679	·2875	·2238	12
11	1·3124	1·0488	·8349	·6622	·5232	11
10	2·7443	2·2257	1·7986	1·4483	1·1619	10
9	5·4157	4·4593	3·6593	2·9926	2·4390	9
8	10·0242	8·3833	6·9887	5·8073	4·8102	8
7	17·2729	14·6790	12·4376	10·5066	8·8489	7
6	27·4510	23·7191	20·4379	17·5613	15·0481	6
5	39·7640	34·9551	30·6501	26·8066	23·3862	5
4	51·6910	46·2641	41·3125	36·8062	32·7178	4
3	59·0383	53·8466	49·0135	44·5215	40·3676	3
2	57·4661	53·4707	49·6696	46·0607	42·6448	2
1	45·4645	43·2200	41·0324	38·9044	36·8411	1
0	26·89873	26·17883	25·45627	24·73249	24·01078	0
Totals without the Italics	345·13843	312·91943	283·65927	257·10069	233·02338	Totals without the Italics
Gross Totals	345·56933	313·25123	283·91347	257·29489	233·17118	Gross Totals

n	Age 79	Age 78	Age 77	Age 76	Age 75	n
10	...	...	*·0001*	*·0001*	*·0001*	10
9	*·0001*	*·0001*	*·0002*	*·0003*	*·0005*	9
8	*·0004*	*·0006*	*·0009*	*·0012*	*·0017*	8
7	*·0017*	*·0024*	*·0033*	*·0046*	*·0062*	7
6	·0066	·0080	·0120	·0161	·0213	6
5	·0242	·0317	·0411	·0532	·0683	5
4	·0829	·1047	·1314	·1641	·2039	4
3	·2633	·3199	·3870	·4663	·5599	3
2	·7627	·8912	1·0376	1·2046	1·3949	2
1	1·9841	2·2240	2·4863	2·7736	3·0884	1
0	4·52471	4·84399	5·17377	5·51705	5·87603	0
Totals without the Italics	7·64851	8·42439	9·26917	10·19195	11·21273	Totals without the Italics
Gross Totals	7·65071	8·42749	9·27367	10·20115	11·22123	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ *in the Formula,*

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^{M} *Any Rate of Interest.*

n	Age 45	Age 46	Age 47	Age 48	Age 49	n
19	·0001	·0001	·0001	...	...	19
18	·0004	·0003	·0002	·0001	·0001	18
17	·0012	·0009	·0006	·0004	·0003	17
16	·0035	·0026	·0019	·0013	·0010	16
15	·0099	·0073	·0054	·0039	·0029	15
14	·0268	·0201	·0150	·0111	·0082	14
13	·0697	·0530	·0402	·0303	·0227	13
12	·1735	·1339	·1030	·0788	·0601	12
11	·4118	·3228	·2521	·1961	·1519	11
10	·9288	·7397	·5870	·4640	·3653	10
9	1·9809	1·6033	1·2932	1·0394	·8322	9
8	3·9712	3·2680	2·6807	2·1916	1·7854	8
7	7·4299	6·2200	5·1915	4·3196	3·5523	7
6	12·8581	10·9567	9·3109	7·8896	6·6651	6
5	20·3496	17·6633	15·2935	13·2073	11·3744	5
4	29·0164	25·6769	22·6717	19·9721	17·5508	4
3	36·5252	32·9858	29·7330	26·7477	24·0110	3
2	39·4162	36·3754	33·5177	30·8343	28·3159	2
1	34·8127	32·9148	31·0588	29·2724	27·5524	1
0	23·29151	22·57809	21·87499	21·17890	20·48019	0
Totals without the Italics	211·02231	191·30569	173·46509	157·31300	142·67589	Totals without the Italics
Gross Totals	211·30741	191·52389	173·63149	157·13890	142·77119	Gross Totals

n	Age 74	Age 73	Age 72	Age 71	Age 70	n
12	...	...	...	...	·0001	12
11	...	·0001	·0001	·0001	·0002	11
10	·0002	·0002	·0004	·0005	·0008	10
9	·0007	·0009	·0013	·0019	·0026	9
8	·0024	·0034	·0047	·0064	·0088	8
7	·0085	·0115	·0154	·0207	·0275	7
6	·0281	·0369	·0483	·0629	·0815	6
5	·0873	·1113	·1413	·1788	·2253	5
4	·2523	·3114	·3836	·4711	·5767	4
3	·6700	·8003	·9546	1·1363	1·3490	3
2	1·6109	1·8586	2·1127	2·4668	2·8339	2
1	3·4316	3·8121	4·2339	4·6984	5·2057	1
0	6·24897	6·64832	7·07515	7·52563	7·99541	0
Totals without the Italics	12·33767	13·59012	14·99195	16·56063	18·29501	Totals without the Italics
Gross Totals	12·34097	13·59502	15·00145	16·56953	18·30751	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ in the Formula,

$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M

Any Rate of Interest.

n	Age **50**	Age **51**	Age **52**	Age **53**	Age **54**	n
18	*·0001*	...	...	...	...	18
17	*·0002*	*·0002*	*·0001*	*·0001*	*·0001*	17
16	*·0007*	*·0005*	*·0003*	*·0002*	*·0002*	16
15	*·0021*	*·0015*	*·0011*	*·0008*	*·0005*	15
14	*·0061*	*·0044*	*·0032*	*·0023*	*·0017*	14
13	*·0170*	*·0126*	*·0093*	*·0068*	*·0050*	13
12	*·0456*	*·0344*	*·0259*	*·0193*	*·0144*	12
11	·1171	*·0899*	*·0687*	*·0522*	*·0395*	11
10	·2865	·2237	·1738	·1345	·1036	10
9	·6638	·5273	·4171	·3285	·2575	9
8	1·4492	1·1718	·9437	·7569	·6047	8
7	2·9609	2·4385	2·0007	1·6353	1·3316	7
6	5·6131	4·7115	3·9409	3·2848	2·7284	6
5	9·7680	8·3630	7·1372	6·0716	5·1487	5
4	15·3388	13·4476	11·7212	10·1872	8·8285	4
3	21·5067	19·2177	17·1287	15·2284	13·5051	3
2	25·9558	23·7447	21·6756	19·7449	17·9488	2
1	25·8976	24·3047	22·7716	21·3003	19·8922	1
0	19·80586	19·12690	18·45114	17·78073	17·11770	0
Totals without the Italics	129·40836	117·27740	106·36164	96·45313	87·46680	Totals without the Italics
Gross Totals	129·48016	117·42090	106·47024	96·53483	87·52820	Gross Totals

n	Age **69**	Age **68**	Age **67**	Age **66**	Age **65**	n
13	...	...	...	*·0001*	*·0001*	13
12	*·0001*	*·0001*	*·0002*	*·0003*	*·0003*	12
11	*·0003*	*·0005*	*·0006*	*·0009*	*·0013*	11
10	*·0011*	*·0015*	*·0022*	*·0030*	*·0042*	10
9	*·0037*	*·0051*	*·0070*	*·0095*	*·0129*	9
8	*·0119*	·0161	·0215	·0287	·0380	8
7	·0365	·0480	·0628	·0817	·1057	7
6	·1051	·1348	·1719	·2182	·2755	6
5	·2827	·3534	·4387	·5426	·6681	5
4	·7033	·8541	1·0325	1·2431	1·4907	4
3	1·5964	1·8823	2·2105	2·5866	3·0161	3
2	3·2469	3·7087	4·2214	4·7900	5·4190	2
1	5·7552	6·3463	6·9774	7·6514	8·3704	1
0	8·47961	8·97499	9·47733	9·98942	10·51214	0
Totals without the Italics	20·20571	22·31839	24·61403	27·13172	29·89564	Totals without the Italics
Gross Totals	20·22281	22·32559	24·62403	27·14552	29·91444	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Coefficients of $(10i)^n$ *in the Formula,*
$$a_x = e_x - f_x(10i) + g_x(10i)^2 - h_x(10i)^3 + \&c.$$

H^M   *Any Rate of Interest.*

n	Age 55	Age 56	Age 57	Age 58	Age 59	n
16	·0001	·0001	·0001	...	...	16
15	·0004	·0003	·0002	·0001	·0001	15
14	·0012	·0009	·0006	·0004	·0003	14
13	·0036	·0026	·0019	·0014	·0010	13
12	·0106	·0078	·0057	·0042	·0030	12
11	·0298	·0223	·0166	·0123	·0091	11
10	·0794	·0604	·0460	·0347	·0261	10
9	·2010	·1562	·1208	·0930	·0713	9
8	·4810	·3810	·3005	·2359	·1844	8
7	1·0800	·8726	·7021	·5627	·4490	7
6	2·2581	1·8621	1·5298	1·2521	1·0207	6
5	4·3517	3·6660	3·0778	2·5751	2·1468	5
4	7·6286	6·5722	5·6450	4·8335	4·1254	4
3	11·9460	10·5396	9·2742	8·1386	7·1219	3
2	16·2808	14·7361	13·3086	11·9921	10·7803	2
1	18·5465	17·2633	16·0419	14·8810	13·7788	1
0	16·46233	15·81603	15·17930	14·55240	13·93513	0
Totals without the Italics	79·31543	71·92553	65·18000	59·11640	53·61373	Totals without the Italics
Gross Totals	79·36113	71·95953	65·25110	59·16950	53·65333	Gross Totals

n	Age 64	Age 63	Age 62	Age 61	Age 60	n
15	...	...	...	...	·0001	15
14	...	·0001	·0001	·0001	·0002	14
13	·0002	·0002	·0003	·0005	·0007	13
12	·0005	·0008	·0011	·0015	·0022	12
11	·0018	·0024	·0035	·0048	·0066	11
10	·0057	·0079	·0107	·0145	·0195	10
9	·0174	·0233	·0311	·0412	·0543	9
8	·0501	·0657	·0856	·1111	·1435	8
7	·1361	·1743	·2223	·2822	·3567	7
6	·3463	·4334	·5401	·6704	·8289	6
5	·8192	1·0005	1·2175	1·4762	1·7834	5
4	1·7810	2·1205	2·5164	2·9767	3·5098	4
3	3·5054	4·0622	4·6939	5·4087	6·2150	3
2	6·1137	6·8799	7·7241	8·6518	9·6686	2
1	9·1368	9·9538	10·8249	11·7519	12·7361	1
0	11·04693	11·59529	12·15904	12·73744	13·32975	0
Totals without the Italics	32·93553	36·28559	40·01494	44·10764	48·62605	Totals without the Italics
Gross Totals	32·96113	36·32029	40·03064	44·12904	48·65535	Gross Totals

NOTE.—For Rates not exceeding 5 per-cent, the italic figures do not come into use.

Discussion.

The CHAIRMAN (Mr. M. N. Adler) regretted that the President had to vacate the chair, but was glad to be able to convey Mr. Sutton's views on the paper to the members. Mr. Sutton, in a memorandum he had written, said: "This paper, whatever its " intrinsic worth may ultimately prove to be, is of interest as being " the first contribution of its author; and as it clearly indicates " that great labour and no small amount of technical skill have been " brought to bear upon the subject, we may, with confidence, look " forward to other valuable contributions from Mr. Searle. Whether " the plan set out in the paper is likely to prove of general utility, " time will show; but the author, none the less, deserves our thanks " for the paper he has read to-night." He (Mr. Adler) considered that the paper showed a considerable amount of ingenuity and mathematical acumen, but he thought that where the coefficients were not already calculated, there would be a good deal of time involved in calculating them, and then working out the annuities. In the cases where the coefficients were calculated, he believed the method would be of practical utility. As regards the calculation of isolated annuity-values, Mr. King had, in the *Text-Book*, given a complete list of formulas, some of which only required the use of four or five terms, and possibly many actuaries would prefer to employ one of these formulas: that, however, was a question open to practical trial. The author thought that the series would prove convergent as regards joint-life annuities. He hoped that such would prove to be the case. A similar attempt to solve the problem set by Mr. Searle was made years ago by Mr. Meikle (*J.I.A.*, iii, 325). He gave a method of obtaining the value of a life annuity at one rate of interest from the value at another given rate. He used the method of finite differences. Of course, that was a different process from that followed in the present paper, but it dealt with the same question. He thought the paper eminently fitted for discussion.

Mr. GEORGE KING asked Mr. Searle if he had noticed that the coefficients he had obtained were merely a practical illustration of a very general principle. In fact, it was an example of Taylor's theorem of the differential calculus, that every function can be expanded in terms of the powers of the variable and the differential coefficients of the function. He would show how Mr. Searle's coefficients were derived directly from Taylor's theorem. Taking Maclaurin's modification of Taylor's theorem, they had the formula, where $a_{(i)}$ represents the annuity at rate of interest i:

$$a_{(i)} = a_{(o)} + i \frac{da_{(o)}}{di} + \frac{i^2}{\underline{2}} \frac{d^2 a_{(o)}}{di^2} + \&c.$$

But $$\frac{da}{di} = \frac{da}{dv} \cdot \frac{dv}{di} = -v^2 \frac{da}{dv},$$

which is equal to $-v(\mathrm{I}a)$ (see p. 340 of the *Text-Book*, Part II),

where (Ia) represents an annuity commencing at 1 and increasing 1 per annum.   Therefore

$$a_{(i)} = a_{(o)} - i \times v_{(o)}(Ia)_{(o)} + \frac{i^2}{\lfloor 2} \cdot \frac{d^2 a_{(o)}}{di^2} + \&c.$$

$$= e - i(Ie) + \frac{i^2}{\lfloor 2} \cdot \frac{d^2 a_{(o)}}{di^2} + \&c.$$

There they had at once the principle upon which, without its being apparent in the paper, Mr. Searle's coefficients had proceeded.   It would be noticed that, naturally, as he had shown, the second term became negative.   It would have been so also with regard to the fourth term, whereas the first and third would be found to be positive. They had in (Ie) the form of the summation, which was shown numerically in the paper, and which the author hit upon as a second means of calculating his tables; and, moreover, the symbol $\frac{d^2 a_{(o)}}{di^2}$ indicated a second summation, and so on.   He (Mr. King) had not had time to work out the actual mathematical forms of the second differential coefficient and of those of higher orders.   They would become somewhat complicated, but still it was quite within the power of mathematics to give them a symbolical value in terms of expectations and increasing expectations.   That was, however, unnecessary, because Mr. Searle had shown how, without getting the mathematical form of those coefficients, their numerical values could be found. The paper, therefore, gave an interesting illustration of the application of Maclaurin's theorem to the doctrine of life contingencies.   There were in the paper some ingenious equations, by means of which the coefficients are found, and the advantageous transformation of writing $10i$ instead of i.   The investigation as to the number of coefficients required was interesting and useful.   The author had devised various ways of making certain of the accuracy of his figures.   He (Mr. King) did not think the coefficients were required to be worked out so as to give four-place annuity tables.   It was his growing conviction, as his experience extended, that they generally went to far too great minuteness in decimal places.   It was not inexactitude he would plead for, but simply the ignoring of the very insignificant fractions to which they were apt to pay too great attention.   Investigation would show that three decimals in annuity-values were the very utmost the original data were capable of yielding, and that the fourth figure, however elegant it might appear, was really useless.   It might be said, perhaps, that for contingent assurances, where the difference between the two annuity-values was taken, an advantage was obtained by having four figures in the annuity-value.   He tested that by taking two tables founded upon the same mortality experience but slightly different in their graduation, and found that, as regards the difference between the annuities, even the second decimal place was altered by a very slight change in the graduation. That showed clearly that even contingent assurances were not calculated more accurately by using four-figure annuities than three.   He thought Mr. Searle should limit his coefficients to three figures, and so not only reduce their magnitude but also the number of them.

Whether tables of this kind will be useful in practice was a question he felt some hesitation in answering, because what was useful to one computer would perhaps not be useful to another. They became used to their own systems, and though better ones were introduced, they could sometimes work more quickly with the old ones. Having the coefficients already calculated, the work for a single-life annuity seemed decidedly shorter than if any of the formulas of approximate summation were used, and the work might be entrusted to less experienced hands, seeing that the checks were more complete. In practice it was very seldom indeed that an annuity on a single life was wanted which was not given in a prepared table. Annuity-values for two or more joint lives were frequently required, and he was afraid Mr. Searle's tables would not be useful for the purpose. It was not sufficient to have tables for joint lives of equal ages, as Mr. Searle suggested. They wanted values for every possible combination of ages, and if it were possible to calculate the coefficients for every combination of ages, the results would be so bulky as to be absolutely useless, and they must fall back upon formulas of approximate summation, which gave results with little labour and great exactitude. Possibly, when the tables were printed they might be found to be more useful than appeared at the moment. It would be hardly necessary to calculate the coefficients for such tables as the Carlisle, where so many annuity-values were tabulated. He would suggest that, if more coefficients were calculated, those for the latest Government annuity experience would be useful. Another table would, for general statistical purposes, become extremely useful in the near future, namely, that which he might call the English Life Table No. 4,* which appeared in a recent number of the *Journal* (*J.I.A.*, xxvii, 503).

Mr. R. P. Hardy confessed to have always had great sympathy for studies in the by-paths of the profession. From a very early age, he endeavoured always to view a function in every possible light of interpretation that it would bear, and had found great advantage therefrom. He thought they could not have too many representations of those elementary problems, and hoped, if Mr. Searle knew of any others, he would not hesitate to bring them forward. They must be careful not to do him an injustice. No doubt he was aware that any series could be referred to some elementary form of summation; but that was not what he wanted to teach. What he said was: " I have " discovered an exceedingly pretty way of looking at this subject, " which may be very useful, and I throw it down on the table for the " common benefit." He thought that was the spirit in which they ought to receive those communications. Mr. King's remark about the number of places of decimals in an annuity-value was quite right, but they should never encourage young men to work in a slovenly way. Let them, when they get into the sere and yellow leaf, curtail and cut down their work, but never let them learn to work in a

---

* Mr. Noel Humphreys' Table, which is referred to by Mr. King, can only be looked upon as an approximation to the English Life Table No. 4. The final table appears in the *Supplement to the 45th Report of the Registrar-General*, pp. vii, viii, and will shortly be reprinted in the *Journal*.—Ed. *J.I.A.*

slipshod fashion. Slovenly habits of working brought about slovenly
habits of mind, and those eventually brought trouble. With regard
to the practical use of Mr. Searle's paper, he did not think they
had anything to do with that to-night. They should study it and
apply it to such uses as their needs might require. He himself would
probably calculate a single or joint-life annuity-value by Woolhouse's
method, but not because that method was the better, but one became
accustomed to work in a particular way, and found a difficulty in
altering one's habits of life.

Mr. T. G. Ackland said that it must have been evident to
Mr. Searle that in arriving at his initial equation, setting forth the
coefficients, he was not making anything in the nature of a discovery.
It was, as Mr. King had pointed out, an elementary principle of the
differential calculus that a function can be expanded in terms of the
ascending powers of the variable, with coefficients afterwards to be
determined. Mr. Searle had approached the matter in his own way,
and worked it out in an interesting manner, and he (Mr. Ackland)
should not be surprised to find on further investigation that there
was more utility than appeared on the surface in the formulas and
methods which he had investigated. It was somewhat singular that
the coefficients which Mr. Searle arrived at should be identical with
functions which Dr. Farr obtained by successive summations of the
column l_x (*J.I.A.*, ix, 188) and which he used for the purpose of
population investigation. These columnar values were also referred to
in Part II of the *Text-Book*. Mr. Searle, in cutting off the decimal
places, had adopted the process of putting down dots to indicate
the carriage. There was another method which was suggested by
Mr. Peter Gray, by which, if they inserted a 5 at the top of the
column, and added that in with the other values, the figures which it
was desired to reject might be thrown out without correction, thus
obtaining the same result as by the use of the ingenious dots Mr. Searle
had employed. As to the method of mental multiplication referred
to by Mr. Searle, and which was originally brought under their notice
by Lieutenant-Colonel Oakes (*J.I.A.*, x, 326), he would ask whether
it was to be trusted to give correct results, and whether one could,
in practice, carry such a succession of figures in one's mind, and put
down the result with accuracy, because, after all, mental processes
must be tested by the correctness of the final result. Mr. King had
referred to the effect of graduation on the decimal places of the
annuity-value. It would be remembered that Mr. Sutton, many years
ago, worked out the question very elaborately by deducing the annuity-
values according to different methods of graduation, and showed that
the resulting annuity-values were, as a rule, remarkably close together.
As regards the extensions of his methods to joint lives, Mr. Searle
had indicated that the values would have to be limited to the cases of
two lives of equal age, and to include every combination of ages the
coefficients would be too voluminous. The practical utility of these
tables had been referred to, and his conviction was that to work
out an annuity at a single age they would use one of the methods of
approximate valuation already referred to, or, preferably perhaps, the
summation formula of Lubbock, by which, having the commutation
column D_x and its differences, they could readily obtain the annuity-

value, but it might be that in some cases the method suggested by
Mr. Searle would be equally useful.

The CHAIRMAN thought that Mr. King was far from wishing to
encourage any want of labour or accuracy on the part of the student,
but in life contingencies, where inaccuracy was known to prevail in
the fundamental data, nothing was gained by extraordinary accuracy
at a particular stage. This view was confirmed by Mr. Woolhouse,
in his recent paper on his method of getting out a rough estimate
valuation (*J.I.A.*, xxvii. 433), also by an interesting paper sent years
ago by Professor de Morgan to the *Journal* (*J.I.A.*, x, 247), as to
the uselessness of retaining in extensive valuations fractions of a £;
yet valuations dealing with millions were sometimes brought out to
the third decimal place. He was glad to find that allusion had been
made to the mental process employed in calculations, and he would,
speaking of Professor de Morgan, call attention to his *Arithmetic.*
The appendix to that work was extremely interesting, containing
many hints for quick reckoning, and suggestions as to mental processes
to shorten labour, and its perusal would prove of the utmost profit.

Mr. SEARLE, in reply, said he highly appreciated the way in which
Mr. King had studied his paper from beginning to end, and, in fact,
had given almost a lecture upon it. There was great value in many
of the suggestions that had been made, particularly the references to
other information bearing upon the same subject. The mental
multiplication was not a very easy process, but if there were numerous
calculations to be made the method would be found satisfactory. Not
having an arithmometer he was obliged to adopt the system. As
regards a comparison with other methods of getting isolated annuity-
values he was inclined to prefer his own. He claimed these three
advantages for his method: that his system gave an exact result and
not an approximation only; that it was an extremely simple process;
and that the resulting figures were checked by summations.

Life Assurance in Ireland in 1725.

THE following will be interesting to the readers of the *Journal,*
as indicating the mode of transacting life assurance business more
than 150 years ago. We are able to print from a copy of the
original document by the kindness of Mr. A. H. Bailey. It will
be remembered that a copy of a life assurance policy granted in
the year 1721, appeared in the *Journal* some years ago (*J.I.A.,*
xxii, 218).

" INSTRUCTIONS FOR JNO. PORTER, ESQ^R, OF THE CITY OF
DUBLIN, FOR ASSURING LIVES IN IRELAND.

" *November,* 1725.

" 1st. All Persons whose lives are to be assured must first appear
        before you, and then you are to take a convenient time
        to Enquire after their State of Health, & manner of Life,

either by Persons in their Neighbourhood, or by such other means, as you can best Inform yourself.

" 2ⁿᵈ. You are to be particularly carefull, that the Person who appears, is really the person, whose Life is to be assured.

" 3ʳᵈ. The Person whose life is to be assured must be Warranted not to exceed the Age mentioned in yᵉ Label, ortherwise, the Policy will be void.

" 4ᵗʰ. It is allways to be Enquired, whither the person whose life is to be Assured hath had the smallpox.

" 5ᵗʰ. If a Woman's life is offered, whither she be married or not, because Child bearing Women & Persons not having the smallpox must pay a higher premium.

" 6ᵗʰ. If the Person, for whose Benefitt, the Assurance is made, is unknown to you, you are to learn if possible, the reason why the Assurance is made, for unless there are good reasons, why the Assurance is made, the person assured, may be in a worse state of health than you apprehend.

" When you have Satisfied your self in the foregoing Enquiry, and are of Opinion, that an Assurance may be made, you are to take 5/- for the Policy and for the Premium, as followeth, vizᵗ.: For any person, in a Good State of health, having had the smallpox, and not exceeding 50 years of Age, nor under 10, 5 Guineas Pʳ Cent.

" For Persons of the like Ages not having had the smallpox & child bearing Women, 6 Guineas Pʳ Cent. You are not to assure more than £500 on any one Life nor for more than one Year at a time, nor for less than Six months, and in the latter case the assured must pay 3 gˢ. & ½ per Cent. according to the Conditions of the Persons before described. If any Person whose life is assured, goes out of the Kingdom of Ireland, without leave first had for their so doing, the Policy is void.

" You may give leave for any person assured to go from Ireland to Great Brittain, & back, upon their paying you ½ a guinea, for the same, which leave & Additionall Premium is to be endorsed on the Policy.

" But if any person Assured, desires leave to goe to any other Part of the World than Great Brittain such leave is not to be given, but by the Court of Directors, and for such Additional Premium as they shall think fitt.

" When you have agreed to an assurance on a Life you may take the Premium & 5/- for the Policy & give a Receipt for the same in the following Words—

" Recᵈ. this＿＿＿＿＿Day of ＿＿＿＿＿of A.B. the sum of＿＿＿＿＿being the premium for＿＿＿＿ pounds, assured on the Life of C.D. of ＿＿＿＿aged＿＿Years, for one Year commencing this day, for which I promise to deliver him a Policy under the Seal of the Corporation of the London Assurance of houses and goods from Fire, & for assuring Lives, as soon as the same shall come to Hande.　　　　"

The Mortality Experience of the Washington Life Insurance Company. *

PROBABLY in no branch of actuarial work have American experts more conspicuously shown those qualities of ingenuity and originality which are freely recognized as among the most prominent characteristics of their fellow-countrymen, than in that which may be summed up, generically, in the title, "Mortality Experience." And it is remarkable how many instances of their enterprise in this respect have, within the last few years, been *en évidence,* whereby our available information on vital statistics has been enriched. But a few years ago, the valuable work entitled *System and Tables of Life Assurance,* prepared and compiled under the superintendence of Mr. Levi Meech, was published, and the equally interesting, though perhaps less important, *Experience of the Connecticut Mutual Insurance Company* followed shortly after, in 1883. Other publications of a similar character have been circulated since that time, including the *Experience of the Provident Life and Trust Company of Philadelphia,* reviewed by Mr. Sprague (*J.I.A.,* xxvi, 316); and the list has received its latest addition in the instructive volume whose title appears at the head of this article.

It would well repay the labour essential to such a task, to investigate the methods and results appertaining to these various contributions to mortality statistics, if it were only to establish the nature and tendency of actuarial opinion in America upon this important subject. But in the present brief review we propose to do no more than refer, somewhat cursorily, to the contents of the volume issued by the Washington Life Insurance Company.

The work is divided into three sections, containing historical, actuarial, and medical statistics respectively. The first of these sections deals with the internal affairs of the company, and need not be commented upon; nor will the third section, although containing many facts of interest, detain us. It is with the intermediate division, devoted to the actuarial records, that readers of the *Journal* will be more particularly concerned, and within its comparatively narrow limits there will, indeed, be found instructive results and suggestive ideas in abundance.

---

* *The Washington Life Insurance Company: Historical, Actuarial, and Medical Statistics.* Published by the Company. New York, 1889.

It may appear ungracious to refer, in the first instance, to the more notable omissions in this work, but clearness may be gained by so doing.   Primarily, we may observe that nothing is said about deteriorated, or under-average lives.   Whether the company had any such lives upon its books, or not; if it had such lives, whether, for the purposes of instituting the investigation into the company's mortality, they were regarded as being of their real or rated-up ages; are matters upon which we are left in complete ignorance. This defect is the more singular as it also characterized the *Experience of the Connecticut Mutual,* although in Mr. Levi Meech's investigations (see above) the rated-up ages were apparently adopted in all cases.   The second noteworthy feature, in this sense, is the absence of all reference to, and statistics of, discontinuances—an element, obviously, of no mean importance in considering the comparative results of mortality experience.   A similar omission occurred in the *Experience of the Provident Life and Trust Company of Philadelphia,* and was remarked upon by Mr. Sprague.

Passing from these matters, we may observe that the chief questions determined by the actuary at the outset of his investigations were:

      1. Shall the observations be made by policy or calendar years?

      2. Shall the amounts of insurance or the number of lives (or policies) be the basis?

It scarcely need be remarked that the method in vogue in this country in regard to the first of these questions is that of calendar years.   The Washington Company decided, however, to follow the example of the Connecticut Mutual Company by adopting the system of policy years, well known to British actuaries as Galloway's method.   The reasons which led to this course are shortly stated in the following extract:

" The anniversaries of policies in a given year (dates to which the annual premiums paid would carry them) are as convenient for an observation point as December 31st of that year.   The deaths before the several anniversaries can be easily ascertained.   Policies being grouped as to age at issue and years of insurance, the full amount exposed to risk and the actual and probable mortality for each age can be obtained at once, while the persistence of policies, the effect of medical selection, and other questions can be readily considered   *   *   *   *   *   Moreover, the decimal of a year for the first and last calendar years of observation is avoided by the use of policy years."

The problem here discussed is one of growing importance, and it is impossible not to regard the action of American companies in adopting the mode of policy years as significant. The subject having formed the basis of discussion at the Institute less than three years ago (see *J.I.A.*, xxvi, 249), we refrain from further observation thereon.

The expedient of using the amounts assured as the basis of the observations, instead of the lives assured (or, as in the case of the Seventeen Offices' Experience, the policies issued), is less likely to command approval, we imagine, than the method of policy years already referred to. It is nevertheless worthy of remark that a practical unanimity of opinion appears to prevail in America upon this point, the Thirty American Offices' Experience (Mr. Levi Meech), Connecticut Mutual Experience, and the Washington Experience being all based on amounts assured. The paragraph in the present volume which grapples with this question runs as follows:

" What is the financial risk?   What is the actual loss? being the practical questions, the *amounts* of policies are used as the basis for observation.   These give results most nearly in accord with facts and probability.

" Were policies issued for uniform amounts, lives or policies might be the better data, but not only is there a difference in the amounts of policies issued at the same ages of entry, there is also a disparity in the amounts of policies issued at the several ages, larger amounts of insurance being taken on single lives at the older than at the younger ones.   The premium receipts, disbursements, assets, reserve, and surplus are in dollars and cents.   On this basis neither *lives* nor *policies*, assumed to be of uniform amounts, will exhibit the true experience of the past which is to be a guide for the future."

With all deference to the opinions here expounded, we consider that too restricted a view is taken of the subject.   It will be readily seen that tables based upon amounts assured can only partially fulfil the needs for which tables of experience are framed. If, for the sake of argument, we concede that such tables are the best suited for determining the probable losses, and the reserves that should be held against current insurances, there yet remain such functions as the probabilities of life survivorship, and the expectation of life, which they cannot claim to supply.   And the paramount advantage of the system of constructing tables upon lives assured is that they may be reasonably applied to all problems, whether of pure mortality or financial estimate.   Were the American plan to become general, we should require *two distinct*

sets of tables throughout—one for the multifarious questions of life contingencies, and the other for the estimation of expected losses, reserves, &c.   It becomes, therefore, a matter for consideration whether any theoretical superiority which the advocates of tables based on amounts may lay claim to, would outweigh the serious inconvenience of having to carry through the whole of our experience investigations on two separate bases.

This may be taken as a convenient starting point to debate the question whether, for financial calculations, the amounts assured do, in fact, afford the most suitable foundation for assurance tables.   There is one thing they reflect a clear light upon, beyond reasonable doubt, namely, the joint effect of the company's selection by medical examination, and the policyholder's selection of the amount assured, upon the resulting claim experience.   But this, let it be remembered, relates entirely to the past; and if the investigations are designed to be utilized as a measure of future results, it must be assumed that the character of the business, and the relative distribution of sums assured, will undergo no material change.   Subject to this condition, can it alone be held that tables based on policy amounts are a sound guide to future experience?

In this vital *sine quâ non,* our readers will not fail to perceive a vulnerable point in the American method.   That the death rate varies considerably in different life assurance companies, is true enough ; but the limits of such fluctuations are comparatively narrow.   In the class of business transacted, and the incidence of the policy amounts at the various ages at entry, there exists far wider scope, in our opinion, for disparity.   In employing a mortality table for valuation purposes, we take the risk of the future deaths deviating from the expected ; but when we adopt a claim table for the like purposes, we virtually superimpose upon that risk the greater chance of a deviation in what we may call the business element.   We can well imagine that two companies, exhibiting similar rates of mortality, would show very different rates of claim, and from this point of view an objection to the use of claim tables would be that they would tend to lessen the probability of conformity with any standard experience.   A further consideration that should not be overlooked is that, as regards mortality tables, we possess a general standard in the national experience deduced from census returns, by which the vital statistics of life assurance companies can be tested.   No such criterion exists in respect to claim tables.

The mathematical aspect of the problem does not appear to yield very important reasoning for or against the American practical conclusion. Mr. G. F. Hardy has shown (*J.I.A.*, xxiii, 3) that when policies are observed instead of lives, the probable deviation or error is increased, and *a fortiori* would this be the case if policy amounts were observed instead of lives. But this only seems to us an argument against amounts being taken, if it is conceded that the method of lives (to use a convenient abbreviation) is, strictly speaking, the sounder and more accurate. Whereas, theoretically, we should be inclined to admit that the method of amounts is perfectly correct, provided the condition we have before mentioned were certain of realization.

To return to the work under review, it is remarked that, " where observations have been made, the percentage of mortality is greater on amounts than on lives or policies." To illustrate this, the following interesting figures are given:

				PERCENTAGE OF MORTALITY	
Name of Table				By Lives	By Amounts
Thirty American Offices	.	.	.	1·03	1·10
Connecticut Mutual	.	.	.	1·12	1·16
Mutual (New York)	.	.	.	1·18	1·24
Mutual Benefit	.	.	.	1·12	1·29
Provident Life and Trust	.	.	.	·78	·81
John Hancock	.	.	.	·92	1·10
				By Policies	
Massachusetts Report, Average of all					
Companies	.	.	.	1·29	1·38

The reason of this curious result is left to surmise.

The method of deducing the final tables of experience from the rough observations is explained with tolerable fulness, the most noticeable point in this respect being that the exact number of years of risk observed was strictly calculated, policies terminating (except by death) being taken as exposed for the fraction of the final year during which they had remained in force. The tables are ten in number, the first five containing the exposures and claims for each age at entry and each year of duration, and in groups of years, respectively, together with the "probable mortality", according to various standard tables ; the succeeding four, the final mortality table, premiums and annuity-values; while the remaining table gives some interesting particulars of the varying rates of mortality among whole-life and endowment policies.

The first-mentioned tables afford every facility for analyzing the effect of selection upon the rate of mortality (or, more properly, the rate of loss by claims). The verbal explanations of this part

of the work are not remarkable for perspicuity, as witness the
following paragraph :

" Table IV exhibits the experience by years of membership,
amounts exposed to risk, amounts of death, probable mortality by
American and actuaries' mortality, ratio of actual to probable, and
ratio of deaths to amount exposed.   *   *   *   *   *   *   The column
' Ratio of Deaths to Amount Exposed' illustrates that the ratio may
advance, and the relative amount of mortality increase with increase
of age, while the actual mortality is less than the probable mortality
by the tables on which premiums and reserves are based."

The casual reader would fail to comprehend that the rather
barbarous phrase "amounts of death" implied the "amounts of
claims by death", and that the phrase "ratio of deaths to amounts
exposed" must be interpreted as meaning the ratio of claims to
amounts exposed.   The term mortality is also mis-applied; and
we would suggest that the words "rate of claim" or "rate of loss"
should be used in its stead in any future treatise, where the context
shows the function in question to be derived from amounts in lieu
of lives.   In Table IV, where the observations are massed together
for all ages at entry, and shown for each year of duration separately,
we notice that a comparison is made of actual and probable
"mortality" during the first five years of insurance, by the $H^{M(5)}$
table, among others.   This is such an obviously false test, under
the circumstances, that it is a pity it was used.   To compare like
with unlike can never serve any practical end.   From Table IV,
the benefit of selection would appear to exhaust itself after *three*
complete years from entry, but the "mortality rate", after the
third year of insurance, proceeds very irregularly.

The closing portion of the actuarial section deals, not very
searchingly it must be owned, with the distinctive rates of
mortality (or loss) among (1) life policies as compared with
endowment policies; and (2) reversionary bonuses as compared
with policy amounts (bonuses included).   In the former case, the
figures are scarcely worth reproducing, as all ages are grouped
together, and "the life policies are more numerous and more
advanced as to age than the endowment policies."   The author
remarks on this point: "The results are not, therefore, reliable
" for actual comparison in every respect, but suggest the
" advisability of closer investigation."   Manifestly, the impression
left upon his mind is that the mortality among endowment policies
is far more favourable than that among whole-life policies, an
opinion which we believe to be freely entertained in this country,

and to be well-founded. The actuary who first produces irrefragable evidence on this point will perform a useful act, by supplanting well-informed opinion by actual proof.

The figures treating of the mortality among reversionary additions and total amounts assured, respectively, are more precise. The following extract aptly introduces the subject:

" As the reversionary dividend additions on the younger ages, the current cash dividend is most frequently used each year, thus cancelling the reversion amount. The average amount of reversion per $1,000 of insurance increases with the advance of age. There may be two explanations of this: one, that as the policyholder progresses in business, being better able, he pays the full premium and lets the dividend remain at his credit; the other, that there comes a time when the policyholder fears that he cannot pass a medical examination. Anxious to have more insurance, he purposely takes the option of reversionary dividends to augment the amount of his policy. In fact, as a risk, he is depreciating while his insurance is appreciating."

These views confirm the close personal observation of our veteran actuary, Mr. J. A. Higham,* and are fully borne out by the painstaking investigation made by Mr. G. F. Hardy into the "Mortality observed amongst the various classes of Bonus Policies in the British Empire Mutual Assurance Company" (*J.I.A.*, xxiii, 1). But the divergence in the rates is much more remarkable in the American figures than in those given by Mr. Hardy.

The courtesy of "several life insurance companies which adopt the system of reversionary additions", gave the actuary of the Washington Company (Mr. Izrael C. Pierson) access to a wider field of observation than could have been supplied by his own company, and the figures he has thus brought together are subjoined:

				RATIO OF AMOUNTS AT RISK TO CLAIMS	
				Reversionary Additions	Policies and Additions
In all the Companies .	.	.	.	2·900	1·350
The Lowest Ratio .	.	.	.	2·090	1·156
The Highest Ratio .	.	.	.	3·128	1·562
Washington Company	.	.	.	2·052	1·319

* " I do not know if anyone has framed a table of mortality from bonus-experience alone, but my impression is that selection is exercised against the office to a greater extent with regard to bonus than with regard to original sums assured. A little evidence of this is afforded by the frequency with which, on maturity of a claim, all additions are found to have been surrendered, except the last." (Extract from letter of Mr. J. A. Higham, published in the *Post Magazine* of 15 Nov. 1884.)

Here we may take leave of this interesting work. The public spirit of the directors, and the thoroughness of the actuary of the company, to which we owe its production, are alike worthy of commendation and imitation. If our criticism may seem somewhat microscopical on certain points, the importance of the subject, and the great value we attach to this latest contribution to insurance statistics, will serve as our excuse. After a recent utterance of the President of our Institute, it is unlikely that the third investigation into the mortality experience of British life offices will be long deferred. Hence it is desirable that every available opportunity should be taken, both to discuss important matters of principle bearing on the general question and to bring to the notice of readers of the *Journal* any new publications treating of such subjects; and it is in this belief that we have submitted the foregoing remarks for the consideration of our readers.

<div align="right">G. H. RYAN.</div>

*The Method of Quarter Squares.**

The method of quarter squares consists in the use of the formula

$$ab = \tfrac{1}{4}(a+b)^2 - \tfrac{1}{4}(a-b)^2$$

to effect the multiplication of two numbers, a and b. If we are provided with a table giving the values of $\tfrac{1}{4}n^2$ up to a given value of n, we may obtain, by the aid of this formula, without performing any multiplication, the product of any two numbers whose sum does not exceed the limit of the table.

The method is specially interesting on account of the great simplicity of the formula, by means of which a table of double entry may be replaced by one of single entry. How great a transformation is effected by such a change is evident, if we consider that the largest existing multiplication table of double entry reaches only to $1,000 \times 1,000$ and forms a closely-printed folio of 900 pages, but that a table of quarter squares of the same extent (*i.e.*, of $\tfrac{1}{4}n^2$ up to $n = 2,000$) need only occupy four octavo pages. The disparity becomes even more conspicuous as the limit of the table is extended, for a table of double entry extending to $10,000 \times 10,000$ would require nearly 100 folio volumes; and one extending to $100,000 \times 100,000$

would require nearly 10,000 volumes; whereas, the corresponding quarter square tables need only occupy 40 and 400 octavo pages respectively.

The use of a table of squares in effecting multiplications was recognized as far back as 1690, when Ludolff published his large table of squares, extending to 100,000. In the introduction to the table, Ludolff explained how it could be employed in multiplications. In order to multiply a and b, the table is to be entered with $a+b$ and $a-b$ as arguments, and the difference of the corresponding squares divided by 4. If a and b are both even, or both uneven, their sum and difference will both be even numbers, so that $\frac{1}{2}(a+b)$ and $\frac{1}{2}(a-b)$ will be integers. In either of these two cases we may, therefore, enter the table with the semi-sum and semi-difference of the numbers as arguments, the product being the simple difference of the corresponding squares.

It was not, however, till 1817 that a table of *quarter* squares (*i.e.*, of $\frac{1}{4}n^2$ for argument n) was published, for the purpose of facilitating multiplications. If n be uneven, $\frac{1}{4}n^2$ consists of an integer and the fraction $\frac{1}{4}$. This fraction $\frac{1}{4}$ may be ignored in the use of the table, for if either $a+b$ or $a-b$ is uneven, the other is so too; the fraction $\frac{1}{4}$ therefore occurs in both squares, and disappears from their difference. It may, therefore, be omitted from the table.

The table of 1817, which contained the first practical application of the method, was published by Antoine Voisin, at Paris, under the title: *Tables des Multiplications; ou, Logarithmes des Nombres entiers depuis* 1 *jusqu'à* 20,000. It is curious that Voisin should have called a quarter square a logarithm—he called a the root, and $\frac{1}{4}a^2$ its logarithm. His table extended to 20,000, and was thus available for multiplications up to $10,000 \times 10,000$. On the title-page Voisin described it as effecting multiplications up to 20,000 by 20,000. This statement is justified by the formula

$$ab = 2\{\tfrac{1}{4}a^2 + \tfrac{1}{4}b^2 - \tfrac{1}{4}(a-b)^2\},$$

by which the product was to be obtained if the sum of the numbers exceeded 20,000, the method of quarter squares being then no longer available. It is to be observed, however, that this formula requires three entries besides the final duplication.

Almost simultaneously (1817) a similar table, of the same extent, was published independently by A. P. Bürger, at Carlsruhe. The method was rediscovered by J. J. Centnerschwer, who published a table of the same extent in 1825, at Berlin. In 1832 J. M. Merpaut published, at Vannes, a table of quarter squares extending to 40,000. In 1852, Kulik (well-known for his large table of squares and cubes to 100,000), who had again rediscovered the method, published a table extending to 30,000. In 1856, Mr. S. L. Laundy published, at London, the largest table of quarter squares which had appeared previous to the publication of the present table. Laundy's Table extends to 100,000. It was intended that the multiplications should be effected by means of quarter squares if the sum of the numbers did not exceed 100,000, but other five-figure numbers were to be multiplied by means of Voisin's three-entry formula referred to above.

It is this change of method that has detracted so greatly from the value of Laundy's fine table. It is evident that the table should have been carried to double its actual extent, *i.e.*, to 200,000, so that any two five-figure numbers could be multiplied together by means of the two-entry formula. The late General Shortrede constructed such a table, but it was never printed. In the work under notice, Mr. Blater carries the table as far as 200,000 ; so that, more than 60 years after the publication of the first table effecting the multiplication of two four-figure numbers, the extension to five figures has at last been completed.

The method of quarter squares has had no opportunity of a fair trial in the absence of a table extending to 200,000. Considering the many purposes to which Crelle's Tables (which give the product of any two three-figure numbers by a single entry) are continually applied, it is, perhaps, surprising that no general use should ever have been made of a table which, in a very small compass gives, by only two entries, the product of two four-figure numbers. Still, it is clear that the full power of the method is not felt till we are provided with such a table giving the product of two five-figure numbers. As already stated, the fact that the limit of Laundy's Table was only 100,000, has deprived it of most of its value, for it is obvious that, unless all five-figure numbers can be treated by a uniform method, the table could not be conveniently employed in practice.

Mr. Blater's work consists of the principal table (giving quarter squares up to 200,000), which occupies 200 pages; a small table of four pages, called the index, to facilitate the use of the table in the extraction of square roots ; and an introduction, &c., of 14 pages.

The arrangement of the table (in which the author has followed the plan adopted by Kulik in his table of 1852, already referred to) is somewhat peculiar. The table is first entered (*i.e.*, the required page of the table is found) by means of the *last* three figures of the number: the table is then entered on this page (or, more correctly, double page), by means of the preceding figures. For example, the quarter square corresponding to 126,993 is found by turning to the double page headed 990. In one of the four columns headed 993 we enter the table at the line 126 : from this line we obtain, in the first column, the first four figures of the result, 4,031; in the column under 993, the next three, 805 ; from the bottom of the column we take the last three figures, 512. The result is therefore given in three parts, A, B, C ; A being common to 10 numbers (in the same line) beginning with 126, C being common to 50 numbers (in the same column) ending with 993, and B being special to the combination 126,993.

The table is beautifully printed in large antique figures on thick and excellent paper, and is a handsome piece of typography. The author mentions that it was entirely set up by a single compositor, at the printing office of Mr. Falk, at Mayence, and that it occupied his whole time for 11 months. Besides being admirably printed, the table is no doubt very correct, as a triple calculation was made, and no pains seem to have been spared by Mr. Blater for insuring accuracy.

The book is dedicated to Mr. Anthony Steinhauser, of Vienna,

who has contributed a short historical preface. Mr. Steinhauser, who is himself the author of several logarithmic tables, encouraged Mr. Blater in his work, and rendered him great assistance throughout. The actual calculation occupied 18 months.

With respect to the general employment of Mr. Blater's Table for the performance of multiplications, it is to be feared that its utility has been jeopardized by the size of page adopted. Anyone who has had occasion to make constant use of tables knows the enormous advantage of the octavo form over the quarto. The book is placed to the left of the computer, and the effort of carrying by the eye a series of figures from the left-hand page of a quarto table to the paper—a distance of 18 inches to 2 feet—is but ill compensated for by larger figures or fewer pages. Handsome as the book is to look at, it seems to us that the table would have had much more chance of bringing the method into general use if it had been of octavo form. It is greatly to be regretted that it was not printed on 400 octavo instead of 200 quarto pages, which would have been quite possible with the existing arrangement of the table. If this had been done, and if the type had been somewhat smaller, a neat and handy volume might have been produced.

The mode of entering the table is very insufficiently explained in the introduction. This is unfortunate, as the mode of entry (by the last figures) is so unusual in tables that it should have been explicitly mentioned. Also the translation into English is so very unsatisfactory as to be obscure in places. These, however, are minor blemishes which would have but slight influence on the general utility of the table, if only the form were convenient.

The question of how far the method of quarter squares is likely to come into use is of some interest. Hitherto the method has been very little known, and, so far as we know, it has never been used in practice on any extended scale. The mere fact that it has been so constantly discovered anew is sufficient evidence of the slight attention that it has received. Still, there ought to be room for a table that gives, to the last figure, the products of any two five-figure numbers with only two entries. A seven-figure table of logarithms is inadequate for this purpose, for, besides requiring three entries, it only gives the first seven figures of the results. On the other hand, it may be said that in ordinary calculations seven figures are as many as are required, and that logarithms possess the advantage of being equally convenient for divisions and multiplications. It must be admitted that a five-figure quarter square table is appropriate to only a limited class of calculations; it applies only to multiplications, and the number of figures in each of the two numbers must not be greater than five. These conditions are of a somewhat special kind. In recent years, when heavy multiplications have been required, it has become the custom to make use of Thomas de Colmar's arithmometer; and probably, at the present time, nearly all systematic work of this character is carried out either by Crelle's Tables or by the arithmometer.

Passing now to the general question of multiplication by means of a table of single entry, we have the two methods of quarter squares and logarithms, each possessing its special advantages. There is also

an older method which passed out of notice with the invention of logarithms. This method was called "prosthaphæresis", and depended on the formula

$$\sin a \sin b = \tfrac{1}{2}[\sin\{90° - (a-b)\} - \sin\{90° - (a+b)\}].$$

A table of natural sines could therefore be used as a multiplication table, four entries being required. This method is due to Wittich, of Breslau, who was assistant for a short time to Tycho Brahé, and it was used by them in their calculations in 1582. It is referred to by Raymarus Ursus, Clavius, and Longomontanus; and it seems to have been used for performing multiplications, not only when the numbers occurred as sines, but also in the case of ordinary numbers.

The method of quarter squares depends upon so simple a formula, that it is strange that the first table should not have appeared until 1817. There seems no reason why it should not have been employed before the invention of logarithms, when it would have been a most valuable aid to calculation. The geometrical theorem, which is equivalent to the algebraical identity $(a+b)^2 - (a-b)^2 = 4ab$, on which the method depends, forms Prop. viii, of the second book of Euclid; and one would think that the application of the geometrical or algebraical theorem to arithmetic might have been noticed at any time. The actual history of mathematical tables is, however, entirely different from what we might expect it to have been, owing to the wonderfully early invention of logarithms; and it was, in fact, only just about that time that the importance of tables as an aid to general calculation was beginning to be felt. The date of Herwart ab Hohenburg's great double entry multiplication table, extending to $1,000 \times 1,000$ (the same limit as Crelle's Table, and which has never been exceeded) is only four years earlier (1610) than that of Napier's *Canon Mirificus* (1614).

It is interesting to notice that the method of quarter squares is more closely connected mathematically with the method of prosthaphæresis than with that of logarithms; in fact, from the formula

$$\sin a \sin b = \tfrac{1}{2}\{\cos(a-b) - \cos(a+b)\},$$

we readily deduce

$$ab = \tfrac{1}{4}\{(a+b)^2 - (a-b)^2\},$$

by expanding the sines and cosines in ascending powers of their arguments and equating the terms of two dimensions.

The method of quarter squares enables us to multiply together two numbers of n figures each if we have a table extending to 2×10^n. If the latter only extends to 10^n three entries are required, and the final result has to be doubled whenever the sum of the numbers exceeds 10^n (as in Laundy's Table). If we consider the question of the multiplication of two numbers of n figures each by means of a table extending only to 10^n, the same process being employed in all cases, it appears that three entries are necessary, and that it would be better to tabulate half squares, using the formula

$$ab = \tfrac{1}{2}a^2 + \tfrac{1}{2}b^2 - \tfrac{1}{2}(a-b)^2,$$

In tabulating the half squares the fraction $\frac{1}{2}$ would be thrown off, so that if a and b were both uneven, unity would have to be added to the result.

It would, however, we think, if the table is not to go beyond 10^n, be more convenient to employ a table of triangular numbers. The nth triangular number is $\frac{1}{2}n(n \times 1)$, and if we are provided with a table extending to 10^n we may multiply any two numbers not exceeding 10^n by means of the formula

$$ab = \tfrac{1}{2}(a-1)a + \tfrac{1}{2}b(b+1) - \tfrac{1}{2}(a-b-1)(a-b);$$

or, as we may write it

$$ab = \mathrm{T}(a-1) + \mathrm{T}(b) - \mathrm{T}(a-1-b),$$

$\mathrm{T}(n)$ denoting the nth triangular number.*

Thus, to multiply two numbers we subtract unity from the larger number, and enter the table with the larger number so diminished, with the smaller number, and with the difference of these two numbers. For example, to multiply 5,289 and 2,156, we add the tabular results corresponding to 5,288 and 2,156, and subtract from this sum the tabular result corresponding to 3.132.

The mode of construction of a table of triangular numbers is almost the simplest possible, the numbers being formed by adding to zero the natural numbers 1, 2, 3. . . . . *e.g.*,

$$0+1=1, 1+2=3, 3+3=6, 6+4=10, 10+5=15,$$

and so on. It may be noticed also that any two consecutive triangular numbers are the most nearly equal parts into which a square of points can be divided by a line parallel to the diagonal. For example, in the square of 16 points, the two most nearly equal triangular parts are, $1+2+3=6$, and $1+2+3+4=10$. This is shown in the following diagram:

Whether the square contains an even or an uneven number of points, the diagonal, which is in the middle, has to be given to one of the two parts, which therefore necessarily differ by the number of points it contains. In the square n^2, the two consecutive triangular numbers which form it are $\frac{1}{2}n(n-1)$ and $\frac{1}{2}n(n+1)$, differing, as they should, by n, the number of points in the diagonal. Viewing the

---

* It is interesting to compare the two formulæ which involve half squares and triangular numbers respectively. In the former case we tabulate a discontinuous function, and in the use of the formula a unit has sometimes to be arbitrarily added. In the latter case we tabulate a continuous function, and the formula always holds good (the larger of the arguments being always reduced by unity). One formula depends on squares, n^2; the other on factorials of the second order, $n(n-1)$.

same matter from a slightly different point of view, we see that any two consecutive triangular numbers always make a square, *e.g.*,

$$1+3=4, \; 3+6=9, \; 6+10=16, \; \&c.$$

It is interesting to exhibit, by means of a diagram, the manner in which the rectangle representing the product ab is derived from the three triangular numbers corresponding to $a-1, \; b, \; a-1-b$. As an example, the mode of formation of the product 8×4 is shown below, the triangular number corresponding to 7 being represented by dots, and the triangular number corresponding to 4 by stars:

```
  .

  .   .

  .   .   .
 _____

  .   .   .   .   ☆   ☆   ☆   ☆

  .   .   .   .   .   ☆   ☆   ☆

  .   .   .   .   .   .   ☆   ☆

  .   .   .   .   .   .   .   ☆
```
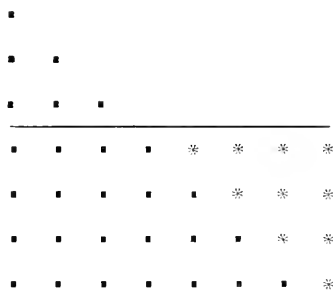
The dots above the line form the triangular number corresponding to $7-4=3$.*

It is not suggested that the method just described by means of triangular numbers is comparable to that of quarter squares. It is certainly better to double the extent of the table and have but two entries. Still, it is interesting to note how readily a table of triangular

---

* We might, of course, also perform the multiplication thus:

```
  .

  .   .

  .   .   .

  .   .   .   .
 _____

  .   .   .   .   .   ☆   ☆   ☆

  .   .   .   .   .   .   ☆   ☆

  .   .   .   .   .   .   .   ☆

  .   .   .   .   .   .   .   .
```
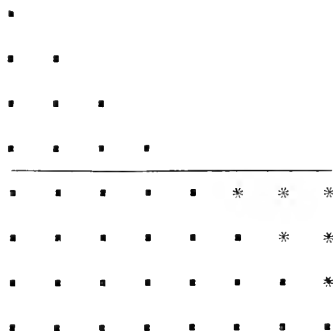
corresponding to the formula

$$ab = \mathrm{T}(a) + \mathrm{T}(b-1) - \mathrm{T}(a-b).$$

But if unity is subtracted from the smaller, instead of from the larger, number, slightly higher numbers are involved in the process.

numbers extending only to 10^n is available for the performance of multiplications of two n-figure numbers. So far as we know, only one extended table of triangular numbers has ever been published. This table, which gives the value of $\frac{1}{2}n(n+1)$ from $n=1$ to $n=20,000$, was published at the Hague, by E. de Joncourt, in 1762, under the title *De Natura et Præclaro Usu Simplicissimæ Speciei Numerorum Trigonalium.* The book is a small and handsomely-printed volume of 267 pages, 224 of which are occupied by the table.

In tabulating quarter squares, the fraction $\frac{1}{4}$ which occurs when the square is uneven is omitted. If we denote by qsq n the tabulated quarter square of n, we have, therefore—

$$\text{qsq}(2n) \quad = n^2,$$
$$\text{qsq}(2n+1) = n^2+n.$$

A table of quarter squares may be formed by adding to zero the numbers 1, 1, 2, 2, 3, 3, ... *e.g.* $0+1=1$, $1+1=2$, $2+2=4$, $4+2=6$, $6+3=9$, $9+3=12$, and so on. Its construction, therefore, is very similar to that of a table of triangular numbers, the only difference being that the added numbers 1, 2, 3, ... are each twice repeated. We may also regard the tabulated quarter squares as defined by this rule: The quarter square of n is equal, if n be even, to the sum of all the uneven numbers less than n, and, if n be uneven, to the sum of all the even numbers less than n. For evidently the series $1+3+5+ \ldots +(2n-1)=n^2$, and the series $2+4+6+ \ldots +2n=n^2+n$.

By means of this definition of a quarter square we may exhibit the method of quarter squares diagrammatically as follows:

Taking as examples the products 8×3 and 7×4, we have

$$\text{qsq}11-\text{qsq}5=8 \times 3,$$

which may be represented by—

and $\text{qsq}11-\text{qsq}3=7 \times 4,$

which may be represented by—

The number of points in the extreme left-hand column of the difference of the quarter squares is always equal to the smaller of the numbers to be multiplied. If this number is uneven, there will be

one middle line containing a number of points equal to the greater of
the two numbers; the points in excess of this number are to be
transferred from the line below the middle one to the line next above
it, the excess from two lines below is to be transferred to make up
the deficiency two lines above, and so on. If the smaller number is
even, as in the second diagram, there are two middle lines differing
from each other by two points; one point from the lower of these
lines is to be transferred to the upper, three points from the line below
the lower middle line to the line above the upper middle line, and
so on.

It will be noticed that the tabulated quarter squares are, as it
were, a species of triangular number in which the succeeding lines of
points differ by two, instead of by one, as in ordinary triangular
numbers (*i.e.*, viewing the matter arithmetically, the quarter squares
are derived alternately from the series $1+3+5+ \dots$ and $2+4+6+ \dots$,
$\dots$, and the triangular numbers from the series $1+2+3+ \dots$).
It is the fact of the lines differing by two which enables us in all cases
to adjust the points in the difference of two quarter squares so as to
form a rectangle in the manner indicated above.

<div style="text-align:right">J. W. L. GLAISHER.</div>

THE METHOD OF QUARTER SQUARES.

May I point out, by way of note to Mr. Glaisher's article
(*Nature*, 10 October 1889) on the method of quarter squares, that
the method is indicated in the second edition of Sir John Leslie's
Philosophy of Arithmetic (Edinburgh, 1820; see pp. 246–57).
Leslie gives what he calls a " specimen " table, extending as far as
2,000, whereby any two numbers containing not more than three
digits each can be multiplied; and he also points out the application
of the table for any two numbers less than 2,000 by using the
formula

$$ab = 2\left\{ \frac{a^2}{4} + \frac{b^2}{4} - \frac{(a-b)^2}{4} \right\}.$$

Apparently, Leslie was an independent discoverer of the method; at
least, this seems to be implied in the remarks which follow his table
in the work cited above: "This application of a table of quarter
" squares, as it is derived from the simplest principles, might have
" readily occurred to a mathematician; yet I have nowhere seen it
" brought into practical use till, last summer, I met with, at Paris, a
" small book by Antoine Voisin, printed in 1817. It contains a table
" of quarter squares for the multiplication of whole numbers from
" 1 to 20,000, with an explanation on the mode of employing them."

University College,   G. CAREY FOSTER.
        London, 13 *October.*

Friendly Society Finance.

Some five years and more ago we published an article under this heading [reprinted *J.I.A.*, xxiv, 301] dealing with the valuation returns made to the Registry of Friendly Societies by registered societies in connection with the five years ending 31 December 1880. We have now before us the last part of the Blue-Book (issued in parts) containing the reports of the Chief Registrar (*Reports of the Chief Registrar of Friendly Societies for* 1886. Part II, F), not only with regard to the valuation returns made relating to the subsequent five years ending 31 December 1885, but also the particulars of the audited annual returns of receipts and expenditure, &c., received from societies as required by the Act relating to the same five years. This last part contains elaborate summaries, very carefully compiled, of the results published in the preceding parts, and, in addition, a " Memorandum by the Actuary to the Central Office on the Summary " of Annual Returns and of Valuations received relating to the five " years 1881-1885." In this memorandum the actuary, Mr. William Sutton, has condensed into a small compass the broad results disclosed by the mass of figures which set out the details with regard to particular societies. It is with regard to the broad results set out in his memorandum that we propose to deal.

Mr. Sutton first calls attention to the fact that the Blue-Book particulars only relate to independent societies, and in no way to those societies which are known as affiliated societies and their registered branches—in other words, the figures do not relate to the Manchester Unity of Oddfellows, Ancient Order of Foresters, and the other orders, whose lodges, courts, &c., have gone through the forms required under the Act, and become formally registered as branches of such orders. This, it must be remembered, is a very important qualification, as it must necessarily very largely reduce the number of friendly societies (so-called) to which the Blue-Book relates.

The next point brought out in the memorandum is the enormous number of societies which fail to comply with almost the first duty laid down by the Act. Mr. Sutton tells us that "out of 12,848 " societies, no less a number than 5,733, or 44·6 per-cent, made no " annual returns at all in the five years ending 31 December 1885"; and we find from the other particulars given that only 24·7 per-cent made returns for all the five years. Even after allowing for societies registered during the five years, these figures are extraordinary, and clearly show that the Act as at present administered is practically in this respect a dead letter to a large extent. We presume that annual return forms are sent out by the Registry Office to all societies on the register, so that it is clear from these figures that societies are not, at all events at present, profoundly impressed with the necessity of fulfilling the duties imposed upon them by the Act. We have not the materials before us to pronounce an opinion upon whom the blame rests, but it is clearly the duty of the Registry Office to insist upon

the due execution of the duties imposed by the Act, and if that office
is prevented by financial reasons from doing its duty, then it is unfair
for Members of Parliament or others interested in the subject to
throw the blame upon the Registry of Friendly Societies. It is
manifest that if the Act is to be carried out, and societies will not of
their own accord comply with it, they should be made to do so, or the
Act should be amended. To make societies comply must from the
nature of the case involve expenditure, and it is for Parliament in its
discretion to vote this money or not. There is, however, another
view of the matter, which has not been hitherto brought out.
Registered societies have well-defined privileges, and in many cases
are practically subsidized by the State. Why should not the State
insist, for instance, that before income tax is returned the Revenue
Department should be authorized to require the production of an
acknowledgment from the Registry Office that its last due annual
return has been sent in? And similarly with the other privileges,
such as exemption from stamp duty, protection of funds in hands of
officers, unlimited deposits with Post Office Savings Banks, &c.? It
would be an interesting question for some member of the House of
Commons to ask whether every society receiving at the present time
a privileged rate of interest from the National Debt Office, involving
a sensible charge upon the State, had in every respect, as far as the
Registry Office was concerned, duly complied with the Act.

    To return, however, to Mr. Sutton's memorandum. From it we
gather that the average annual contribution of members of ordinary
friendly societies, judged by all of the annual returns sent in, is about
15s., and that out of the total receipts some 75 per-cent is spent in
benefit to members, under 10 per-cent in management expenses, and
the remainder is put by for the accumulating liabilities of the future.
Mr. Sutton points out how difficult it is, from these consolidated
figures, relating, as they do, to every class of society,—ordinary sick
and burial societies, burial societies only, dividing societies, &c.,—to
draw any trustworthy conclusions of a general character; and he
further points out that many societies of long standing (some of
them very large) are still exempt from the requirement to have a
separate account of management expenses, receipts, and disburse-
ments. This last has apparently led Mr. Sutton to employ a different
method of tabulation from that he would have desired. He says,
" In consequence of many societies (among them some of the largest)
" being still exempted from keeping a separate account of receipts as
" well as expenditure in respect of management, some particulars of
" great interest cannot be given."

    In another table are given particulars, county by county, deduced
from the returns sent in by societies duly complying with the Act for
all the five years.

    We have referred hitherto to the average annual contributions,
and how the total receipts are disposed of, but in other matters quite
as interesting and suggestive information is furnished. Thus it
appears that for societies other than collecting societies the approxi-
mate annual percentage dying is 1·53, leaving 5·01, and entering
7·69.

    Turning now to collecting societies (the penny-a-week collected

death societies coming under section 30 of the Act, as to which a
Special Committee of the House of Commons is now sitting, with
Sir Herbert Maxwell as chairman), it would appear that the approxi-
mate annual percentage dying is 2·39, leaving 18·65, and entering
26·53, and the average annual contribution per member some 7*s.*, and
that of the total receipts no less than 43·29 is spent in management,
47·00 per-cent is benefit, and the remaining 9·71 per-cent saved for
the liabilities of the future.

These figures are remarkable, and although the percentage spent
in management may be considered to an extent accounted for by
the nature of the business transacted, the high rate of leaving is a
matter for inquiry and consideration. Possibly the difference between
the rate of mortality, 2·39 per-cent and 1·53 for ordinary societies, is
practically accounted for by the fact that in the former, children form
a very large proportion of the whole number assured.

The remainder of Mr. Sutton's memorandum relates to the
valuation returns received from societies made as at some date or
other in the five years ending with 1885. A classified summary,
similar to that given five years ago, has been compiled, and
Mr. Sutton has given some general criticisms thereon, which should
be read with great interest by all who study the important question
of friendly society finance. We have not space to refer in detail to
the various points raised, and must content ourselves with making
one quotation from this portion of the memorandum. Speaking of
the comparison made by him between the results of this and the
previous five years, Mr. Sutton says:

"This apparent improvement is, it is only too probable, partly to be ascribed
to the non-inclusion in the second five years of many courts and lodges that had
become registered as branches of their respective orders, rather than to any
pronounced improvement in the financial position of the societies still remaining.
It is true that in a few cases praiseworthy endeavours appear to have been made
to put societies on a sound financial basis; but on the whole it is to be feared
that the members of societies have not thoroughly grasped the situation, and
when they have done so the older, and presumably more influential, members
have been content to let things alone, and been successful in inducing the
younger members to take the same view, the latter little thinking how bitterly
they will have occasion to repent hereafter of their present disregard of the
results brought out by the valuations. In connection with this all-important
question, it may be mentioned that it does not appear by any means certain that
the results of valuation become clearly made known to every member of a society,
and until this is done it is perhaps improbable that any general tendency to
improvement in their financial condition will take place."

It appears that, as regards England and Wales, the societies
making valuation returns show (excluding collecting societies) a net
deficiency of 48·7 per-cent in the amount of funds, which could be
removed by a reduction in benefits of 14·0 per-cent, or an increase of
contributions of 23·5 per-cent.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1887.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	151,275,956	5,523,811	156,799,767
Adjustment of Briton Medical and General balance (−£1,653), and for an additional return (+£1,443)	−1,653	+1,443	−210
	151,274,303	5,525,254	156,799,557
Premiums	13,586,325	4,008,912	17,595,237
Consideration for Annuities .	812,660	375	813,035
Interest and Dividends (less Tax)	6,170,171	196,429	6,366,600
Increase in value of Investments .	155,958	95	156,053
Fines, Fees, &c.	7,573	290	7,863
Capital Paid-up	23,097	127,488	150,585
Customs Timber Measuring, &c. .	2,231	...	2,231
Donations (Itinerant Methodists) .	3,178	...	3,178
Transfers from other Accounts .	40,089	...	40,089
Dividends from the "Emperor" to the "Whittington" . .	1,972	...	1,972
Miscellaneous	1,215	616	1,831
	172,078,772	9,859,459	181,938,231

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	11,710,695	1,588,174	13.298,869
Cash Bonuses and Reduction of Premiums	949,963	10	949,973
Annuities	704,989	27	705,016
Surrenders	870,963	8,393	879,356
Commission	671,975	1,102,107	1,774,082
Expenses of Management . .	1,376,413	668,659	2,045,072
Bad Debts	6,289	85	6,374
Decrease in value of Investments .	47,060	196	47,256
Interest on Capital and Dividends, and Bonuses to Shareholders .	492,327	253,560	745,887
Transfers to other Accounts . .	7,582	35,778	43,360
For Canadian Policyholders and sums written off (Briton Medical and General)	26,457	...	26,457
Miscellaneous	5,857	...	5,857
Balance* at the end of the Year .	155,208,202	6,202,470	161,410,672
	172,078,772	9,859,459	181,938,231

* This Balance includes the whole of the Life and Annuity Funds (£155,834,116), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1887).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . .	11,457,906	294,921	11,752,827
Life and Annuity Funds . .	149,915,277	5,918,839	155,834,116
Fire Funds of Companies transacting Life Business . .	9,370,461	...	9,370,461
Marine Funds of Companies transacting Life Business . . .	561,886	...	561,886
Reserve Funds	3,486,885	...	3,486,885
Other Funds	564,148	888	565,036
Profit and Loss Balances . .	2,298,615	...	2,298,615
Depreciation and Investment Balances	478,576	...	478,576
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims . . .	3,687,085	30,627	3,717,712
Outstanding Accounts . . .	396,206	951	397,157
Temporary Loans	70,200	2,287	72,487
Sundries	80,719	...	80,719
	183,470,764	6,248,513	189,719,277

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	77,666,183	148,715	77,814,898
Loans on Policies	8,629,088	7,952	8,637,040
,, Rates (and Rent-charges)	21,950,821	1,699,169	23,649,990
British Government Securities .	5,341,197	692,568	6,033,765
Indian and Colonial Government Securities	11,788,988	27,174	11,816,162
Foreign Government Securities .	3,676,810	...	3,676,810
Debentures	14,862,221	910,764	15,772,985
Shares and Stocks . . .	12,040,539	12,700	12,053,239
Companies' own Shares . .	565,455	...	565,455
Land and House Property, and Ground Rents	11,206,546	2,171,941	13,378,487
Life Interests and Reversions .	3,258,625	19,262	3,277,887
Loans on Personal Security . .	1,237,080	13,547	1,250,627
Agents' Balances and Outstanding Premiums	3,785,212	238,066	4,023,278
Outstanding Interest . . .	1,732,370	58,249	1,790,619
Cash, Bills, Stamps, &c. . .	5,416,025	221,284	5,637,309
Customs Timber Measuring Balances, &c.	1,683	...	1,683
Book-Room Grant (Itinerant Methodists)	75,000	...	75,000
Deficiencies, Preliminary Expenses, &c.	236,921	27,122	264,043
	183,470,764	6,248,513	189,719,277

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's*
Summary (1887), *when compared with the corresponding Items
for the previous Year.*

	Ordinary Companies	Industrial Companies
INCOME	£	£
Premiums	+ 552.380	+ 262,671
Consideration for Annuities	+ 108,667	+ 375
Interest and Dividends (less Tax)	+ 130,466	+ 26,770
Net Increase in Value of Investments	+ 77,172	+ 777
OUTGO		
Claims	+ 354,258	+ 126,342
Annuities	+ 15,648	+ 12
Surrenders	+ 10,155	− 710
Commission	+ 35,096	+ 115,820
Expenses of Management	+ 83,160	+ 93,159
LIABILITIES		
Paid-up Capital (including sundry Shareholders' Balances)	+ 46,106	+ 128,931
Life and Annuity Funds	+3,818,586	+ 551,250
ASSETS		
Mortgages (including Loans on Rates* and Rent-charges)	+ 341,134	+ 47,614
Life Interests and Reversions	− 103,660	− 4,435
Loans on Policies	+ 248,540	+ 1,264
British Government Securities	+ 188,083	+ 31,398
Indian and Colonial Government Securities*	+ 555,573	+ 14,248
Foreign Government Securities*	− 304,602	...
Debentures*	+1,721,035	+ 180,689
Shares and Stocks*	+ 375,647	+ 8,700
Companies' own Shares	− 8,608	...
Land and House Property and Ground Rents	+ 790,120	+ 230,378
Loans on Personal Security	+ 23,072	− 1,356

* One of the Companies states that £318,415, described in the previous year as Indian and Colonial Government Securities, and £57,674, described as Foreign Government Securities, should have been described, respectively, as Colonial and Foreign Municipal Securities. Another Company explains that £373,596, described as Railway and other Debentures and Stocks, consisted of Preference and Guaranteed Capital Stocks. These corrections have been allowed for in making the above comparison.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 108, of which 95 have been classed as Ordinary, 12 as Industrial, and 1 appears in both Classes, this Company's Return showing its Ordinary and Industrial business separately.

During the year one name has been removed from the official List of Companies, namely, that of the Briton (Limited), the business of which has been transferred. One new name has been added, namely, that of the London Amicable Assurance Society (Limited). And one has been restored, namely, that of the Leicester Industrial Assurance and Building Company (Limited).

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies.*
ORDINARY BUSINESS.

| | With Profits | | Without Profits | | Total | | Re-assurances | Net Amounts |
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.		£		£		£	£	£
Whole Term of Life	665,914	338,628,167	79,921	55,015,539	745,835	393,643,706	17,977,188	375,666,518
Limited number of Premiums .	24,854	14,096,205	4,256	1,979,781	29,110	16,075,986	556,362	15,519,624
	690,768	352,724,372	84,177	56,995,320	774,945	409,719,692	18,533,550	391,186,142
Endowments .	2,808	512,068	5,318	919,580	8,126	1,431,648	4,035	1,427,613
Endowment Assurances .	116,831	24,853,964	24,500	6,904,291	141,331	31,758,255	389,733	31,368,522
Joint Lives .	7,827	2,112,657	2,049	964,985	9,876	3,077,642	319,629	2,758,013
Last Survivor .	1,259	1,005,546	1,200	1,124,672	2,459	2,130,218	144,291	1,985,927
Contingent .	57	66,985	2,716	4,646,902	2,773	4,713,887	1,085,159	3,628,728
Issue .	5	6,150	640	2,723,519	645	2,729,669	720,499	2,009,170
Miscellaneous .	282	116,778	3,612	3,776,745	3,894	3,893,523	747,126	3,146,397
	819,837	381,398,520	124,212	78,056,014	944,049	459,454,534	21,944,022	437,510,512
ANNUITIES.						Per Annum		
Immediate .	...	...	...	...	15,620	691,547	3,603	687,944
Deferred .	...	...	...	...	4,538	139,465	1,210	138,255
	...	...	...	...	20,158	831,012	4,813	826,199

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

| | With Profits | | Without Profits | | Total | | Re-assurances | Net Amounts |
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.						£		£
Whole Term of Life	...	...	...	...	9,012,982	81,000,216	...	81,000,216
Limited number of Premiums .	...	...	...	...	3	7	...	7
	...	...	...	...	9,012,985	81,000,223	...	81,000,223
Endowments .	...	...	...	...	25,864	348,006	...	348,006
Endowment Assurance .	...	...	...	...	70,168	1,090,574	...	1,090,574
Joint Lives .	...	...	...	...	99,654	1,452,817	...	1,452,817
	...	...	...	...	9,208,671	83,891,620	...	83,891,620
ANNUITIES.						Per Annum		
Immediate .	...	..	...	...	1	15	...	15

The above figures are based on Returns deposited for the most part during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of five Companies, namely, the Co-operative, Industrial of Great Britain, Northern, Provincial, and Royal, the amount of business at a more recent date has been included.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1888.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	147,933,581	4,524,194	152,457,775
Adjustment for Returns from two additional Companies (£182, and a Deficiency Balance of £1,703)	...	−1,521	−1,521
	147,933,581	4,522,673	152,456,254
Premiums	13,033,945	3,746,241	16,780,186
Consideration for Annuities . .	703,993	...	703,993
Interest and Dividends (less Tax)	6,039,705	169,659	6,209,364
Increase in value of Investments .	89,572	...	89,572
Fines, Fees, &c.	7,243	227	7,470
Capital Paid-up	266	28,064	28,330
Customs Timber Measuring, &c. .	2,111	...	2,111
Donations (Itinerant Methodists) .	3,116	...	3,116
Transfers from other Accounts .	20	100,000	100,020
Dividends from the "Emperor" to the "Whittington" . . .	3,944	...	3,944
Miscellaneous	2,707	...	2,707
	167,820,203	8,566,864	176,387,067

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	11,356,437	1,461,832	12,818,269
Cash Bonuses and Reduction of Premiums	1,068,334	53	1,068,387
Annuities	689,341	15	689,356
Surrenders	860,808	9,103	869,911
Commission	636,879	986,287	1,623,166
Expenses of Management . .	1,293,253	575,500	1,868,753
Bad Debts	1,214	499	1,713
Decrease in value of Investments .	57,846	878	58,724
Interest on Capital and Dividends and Bonuses to Shareholders .	562,512	8,219	570,731
Transfers to other Accounts . .	3,501	651	4,152
Miscellaneous	14,122	16	14,138
Balance* at the end of the Year .	151,275,956	5,523,811	156,799,767
	167,820,203	8,566,864	176,387,067

* This Balance includes the whole of the Life and Annuity Funds (£151,464,280), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1888).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . .	11,411,800	165,990	11,577,790
Life and Annuity Funds . .	146,096,691	5,367,589	151,464,280
Fire Funds of Companies transacting Life Business . . .	8,915,648	...	8,915,648
Marine Funds of Companies transacting Life Business . . .	607,169	...	607,169
Reserve Funds	3,450,659	...	3,450,659
Other Funds	519,308	1,001	520,309
Profit and Loss Balances . .	2,221,158	...	2,221,158
Depreciation and Investment Balances	331,102	...	331,102
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims . . .	3,469,991	28,820	3,498,811
Outstanding Accounts . . .	497,075	738	497,813
Temporary Loans	62,470	1,993	64,463
Sundries	11,805	...	11,805
	178,697,676	5,566,131	184,263,807

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	77,169,666	144,268	77,313,934
Loans on Policies	8,380,548	6,688	8,387,236
,, Rates (and Rent-charges)	21,700,115	1,656,002	23,356,117
British Government Securities .	5,153,114	661,170	5,814,284
Indian and Colonial Government Securities	11,581,830	12,926	11,594,756
Foreign Government Securities .	4,039,086	..	4,039,086
Debentures	13,514,782	730,075	14,244,857
Shares and Stocks . . .	11,291,296	4,000	11,295,296
Companies' own Shares . .	574,063	...	574,063
Land and House Property and Ground Rents	10,416,426	1,941,563	12,357,989
Life Interests and Reversions .	3,362,285	23,697	3,385,982
Loans on Personal Security . .	1,214,008	14,903	1,228,911
Agents' Balances and Outstanding Premiums	3,586,232	211,243	3,797,475
Outstanding Interest . . .	1,656,956	53,111	1,710,067
Cash, Bills, Stamps, &c. . .	4,799,237	88,847	4,888,084
Customs Timber Measuring Balances, &c.	1,961	...	1,961
Book-Room Grant (Itinerant Methodists)	75,000	...	75,000
Outstanding Accounts . . .	8,344	...	8,344
Deficiencies, Preliminary Expenses, &c.	172,727	17,638	190,365
	178,697,676	5,566,131	184,263,807

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's Summary* (1888), *when compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
	£	£
INCOME.		
Premiums	+ 187,020	+ 196,206
Consideration for Annuities . . .	+ 102,806	...
Interest and Dividends (less Tax) . .	− 7,707	+ 30,267
Net Increase in Value of Investments .	− 94,564	− 395
OUTGO.		
Claims	+ 7,265	+ 95,295
Annuities	+ 31,010	...
Surrenders	+ 80,233	+ 1,304
Commission	+ 29,815	+ 51,410
Expenses of Management	+ 34,469	÷ 6,928
LIABILITIES.		
Paid-up Capital (including sundry Shareholders' Balances)	+ 4,491	+ 32,750
Life and Annuity Funds	+ 3,344,984	+ 974,906
ASSETS.		
Mortgages (including Loans on Rates* and Rent-charges)	+ 1,418,985	+ 160,825
Life Interests and Reversions . .	+ 89,101	− 27,196
Loans on Policies	+ 185,038	+ 1,607
British Government Securities . .	+ 134,678	+ 62,245
Indian and Colonial Government Securities*	+ 596,049	+ 12,926
Foreign Government Securities . .	− 28,854	...
Debentures	+ 764,206	+ 320,609
Shares and Stocks	− 95,906	+ 4,000
Companies' own Shares . . .	− 16,502	...
Land and House Property and Ground Rents	+ 638,902	+ 337,197
Loans on Personal Security . . .	− 76,126	− 1,324

* One of the Companies has explained that in its Returns for the previous year a sum of £324,064. 15s., then included as Indian and Colonial Government Securities, should have been described as Indian and Colonial Municipal and other Bonds.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary (including the Monthly) is 107, of which 95 have been classed as Ordinary, 11 as Industrial, and 1 appears in both Classes, this Company's Return showing its Ordinary and Industrial business separately.

During the year, in addition to restoring the name of the Industrial of Great Britain (Limited), the names of two Ordinary Companies and one Industrial have been added to the official List of Companies, namely, Co-operative (Limited), Scottish Economic (Limited), United Brothers.

Summary of the Assurances in Force, *as shown by the last Valuation Returns of the Companies.*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amounts
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.		£		£		£	£	£
Whole Term of Life	654,692	331,625,527	78,017	54,371,974	732,709	385,997,501	17,981,330	368,016,171
Limited number of Premiums . .	20,676	11,552,369	3,361	1,612,962	24,037	13,165,331	428,558	12,736,773
	675,368	343,177,896	81,378	55,984,936	756,746	399,162,832	18,409,888	380,752,944
Endowments . .	2,575	419,850	5,054	881,386	7,629	1,301,236	1,000	1,300,236
Endowment Assurances . . .	98,176	19,887,707	22,825	6,503,819	121,001	26,391,526	410,783	25,980,743
Joint Lives . .	7,633	1,987,906	2,157	919,902	9,790	2,907,808	267,832	2,639,976
Last Survivor . .	1,433	1,119,036	1,203	1,077,854	2,636	2,196,890	114,336	2,082,554
Contingent . .	67	86,787	2,588	4,272,178	2,655	4,358,965	1,021,031	3,337,934
Issue . . .	5	6,150	575	2,568,202	580	2,574,352	738,579	1,835,773
Miscellaneous . .	261	111,701	3,770	3,647,693	4,031	3,759,394	627,786	3,131,608
	785,518	366,797,033	119,550	75,855,970	905,068	442,653,003	21,591,235	421,061,768
ANNUITIES.						Per Annum		
Immediate . .	...	...	...	...	15,013	661,593	3,153	658,440
Deferred . .	...	...	...	...	4,824	140,381	680	139,701
	...	...	...	...	19,837	801,974	3,833	798,141

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amounts
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.						£		£
Whole Term of Life	...	...	...	...	8,951,207	80,552,109	...	80,552,109
Limited number of Premiums . .	...	...	...	...	3	7	...	7
	...	...	...	...	8,951,210	80,552,116	...	80,552,116
Endowments . .	...	...	...	...	25,192	344,736	...	344,736
Endowment Assurance . . .	...	...	...	...	70,116	1,089,610	...	1,089,610
Joint Lives . .	...	...	...	...	99,326	1,448,025	...	1,448,025
	...	...	...	...	9,145,844	83,434,487	...	83,434,487
ANNUITIES.						Per Annum		
Immediate . .	...	...	...	...	1	15	...	15

The above figures are based on Valuation Returns deposited for the most part during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. The Board of Trade are in communication with the Companies.

A Method of using Mr. James Chisholm's Tables of the Values of Policies of all Durations, according to any Rate of Interest and Mortality.   By D. J. McG. McKENZIE, *Wellington, N.Z.*

ON account of the very extensive range of application of these tables, and the improbability of complete tables of policy-values ever being constructed for the various statuses to which they are applicable, the following method of obtaining exact values from them may be of interest and prove useful on occasions. I say exact, because the policy-values obtained will either be identical with those obtained by established methods, or will differ from them by only a very few units in the third place of decimals. It consists of an easy interpolation to suit the initial annuity-values by the aid of a small tablet of multipliers, the proportional parts inserted in the volume being used for the terminal annuity-values. It will be noted that complementary proportional parts are used, so as to make them additive throughout.

The subjoined examples exhibit the workings in full.

Table of Multipliers.

Δ'	Multi-pliers	Δ'	Multi-pliers	Δ'	Multi-pliers	Δ'	Multi-pliers	Δ'	Multi-pliers	Δ'	Multi-pliers	Δ'	Multi-pliers
1	·02	8	·16	15	·30	22	·44	29	·58	36	·72	43	·86
2	·04	9	·18	16	·32	23	·46	30	·60	37	·74	44	·88
3	·06	10	·20	17	·34	24	·48	31	·62	38	·76	45	·90
4	·08	11	·22	18	·36	25	·50	32	·64	39	·78	46	·92
5	·10	12	·24	19	·38	26	·52	33	·66	40	·80	47	·94
6	·12	13	·26	20	·40	27	·54	34	·68	41	·82	48	·96
7	·14	14	·28	21	·42	28	·56	35	·70	42	·84	49	·98

EXAMPLES.

WHOLE-LIFE POLICY-VALUES.

(1) Required $100_{15}V_{44}$ H^M $3\frac{1}{2}$ per-cent; $1 + a_{44} = 16\cdot001$, $1 + a_{59} = 11\cdot180$.

$\left.\begin{array}{l} 16\cdot050 \\ 11\cdot200 \end{array}\right\} = 30\cdot218.$ $\left.\begin{array}{l} 16\cdot000 \\ 11\cdot200 \end{array}\right\} = 30\cdot000,$ $\Delta = 218,\ 16\cdot001 - 16\cdot000 = \Delta' = 1,$

$$\begin{array}{r} \times\ ·02 \\ \hline ·004(36 \end{array}$$ Multiplier = ·02

$30\cdot000$

$\dfrac{·125}{} = $ Proportional part for ·020

$30\cdot129\ = 100_{15}V_{44}$

$30\cdot128\ =$,,   by Hardy's Tables

(2) Required 100_5V_{20} H^M 4   per-cent; $1+a_{20}=19{\cdot}644$, $1+a_{25}=18{\cdot}961$.

$\left.\begin{array}{l}19{\cdot}650\\19{\cdot}000\end{array}\right\}=3{\cdot}308,$ $\left.\begin{array}{l}19{\cdot}600\\19{\cdot}000\end{array}\right\}=3{\cdot}061,$ $\Delta=247,$ $19{\cdot}644-19{\cdot}610=\Delta'=44,$

$$\begin{array}{r}\times\ {\cdot}88\\\hline{\cdot}217(4\end{array}\qquad\text{Multiplier}={\cdot}88$$

$3{\cdot}061$

${\cdot}153$ = Proportional part of ${\cdot}03$

$\underline{45(8}$ =   ,,   ,, ${\cdot}009$

$3{\cdot}477$ $=100_5V_{20}$

$\underline{\underline{3{\cdot}477}}$ =   ,,   by Hardy's Tables

Endowment Assurance Policy-Values (H^M 4 per-cent).

(3) Required

$$100_5V_{20.\,\overline{30|}},\ \ 1+a_{20.\,\overline{29|}}=16{\cdot}490,\ \ 1+a_{25.\,\overline{24|}}=14{\cdot}993.$$

$\left.\begin{array}{l}16{\cdot}500\\15{\cdot}000\end{array}\right\}=9{\cdot}091,$ $\left.\begin{array}{l}16{\cdot}450\\15{\cdot}000\end{array}\right\}=8{\cdot}815,$ $\Delta=276,$ $16{\cdot}490-16{\cdot}450=\Delta'=40,$

$$\begin{array}{r}\times\ {\cdot}80\\\hline{\cdot}220(80\end{array}\qquad\text{Multiplier}={\cdot}80$$

$8{\cdot}815$

$\underline{42(4}$ = Proportional part of ${\cdot}007$

$9{\cdot}078$ $=100_5V_{20.\,\overline{30|}}$

$\underline{\underline{9{\cdot}078}}$ =   ,,   by Carment's Tables

(4) Required

$$100_{15}V_{36.\,\overline{24|}},\ \ 1+a_{36.\,\overline{23|}}=14{\cdot}185,\ \ 1+a_{51.\,\overline{8|}}=7{\cdot}204.$$

$\left.\begin{array}{l}14{\cdot}200\\7{\cdot}300\end{array}\right\}=48{\cdot}592,$ $\left.\begin{array}{l}14{\cdot}150\\7{\cdot}300\end{array}\right\}=48{\cdot}410,$ $\Delta=182,$ $14{\cdot}185-14{\cdot}150=\Delta'=35,$

$$\begin{array}{r}\times\ {\cdot}70\\\hline{\cdot}127(40\end{array}\qquad\text{Multiplier}={\cdot}70$$

$48{\cdot}410$

634 = Proportional part of ${\cdot}09$

$\underline{42(3}$ =   ,,   ,, ${\cdot}006$

$49{\cdot}214$ $=100_{15}V_{36.\,\overline{24|}}$

$\underline{\underline{49{\cdot}212}}$ =   ,,   by Carment's Tables

Last Survivor Policy-Values (see Introduction to Mr. Chisholm's Book, pp. 6, 7).

(5) Required the value, after 15 years' duration, of a last survivor policy for 100, taken out on two lives, ages 35 and 25, H^M $3\frac{1}{2}$ per-cent.

$$1+a_{\overline{35.25}}=22{\cdot}569,\ \ 1+a_{\overline{50.40}}=19{\cdot}006,\ \ 1+a_{40}=17{\cdot}103,$$

(a) Both lives surviving at the end of 15 years.

$$\left.\begin{array}{c}22{\cdot}600 \\ 19{\cdot}100\end{array}\right\} = 15{\cdot}487, \quad \left.\begin{array}{c}22{\cdot}550 \\ 19{\cdot}100\end{array}\right\} = 15{\cdot}299, \quad \Delta = 188, \quad 22{\cdot}569 - 22{\cdot}550 = \Delta' = 19,$$

$$\begin{array}{r}\times \cdot38 \\ \hline \cdot071(44 \\ 15{\cdot}299\end{array}$$

Multiplier $= \cdot38$

$\cdot399$ = Proportional part of $\cdot09$

$17(7$ =   ,,   ,, $\cdot004$

$\overline{15{\cdot}787}$ = The exact value

(b) The elder life dead at the end of 15 years.

$$\left.\begin{array}{c}22{\cdot}600 \\ 17{\cdot}200\end{array}\right\} = 23{\cdot}894, \quad \left.\begin{array}{c}22{\cdot}550 \\ 17{\cdot}200\end{array}\right\} = 23{\cdot}725, \quad \Delta = 169, \quad \Delta' = 19, \quad \text{Multiplier} = \cdot38$$

$$\begin{array}{r}\times \cdot38 \\ \hline \cdot064(22 \\ 23{\cdot}725\end{array}$$

$\cdot399$ = Proportional part of $\cdot09$

$31(0$ =   ,,   ,, $\cdot007$

$\overline{24{\cdot}219}$ = The exact value

Errata in Erskine Scott's Logarithms.

Mr. RALPH P. HARDY sends the following list of errata, which he has found in Erskine Scott's *Table of Logarithms*, during a very extensive use of the same:

Page 24—For 32824, read 32834.

,,   25— ,, 34165 ,, 34175.

,,   40— ,,   462   ,,   463   (the following 0's require

,,   57— ,, 57467 ,, 57497.   thickening).

..   70— ,,   655   ,,   645.

,,   71— ,, 64969 ,, 64959.

,,   72— ,,   757   ,,   657.

,,   148— ,,   928   ,,   920.

,,   290— ,,   241   ,,   341.

,,   305— ,,   306   ,,   406.

,,   325— ,,   312   ,,   512.

CORRESPONDENCE.

ASSURANCES WITH RETURN OF PREMIUMS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—On p. 163 of the October No. of the *Journal*, Mr. King gives a corrected solution of the problem discussed on p. 293 of the *Text-Book*, Part II, namely:—" To find the annual premium for a " whole-life assurance, under the condition that all the premiums " paid are to be returned, with compound interest at rate j, along " with the sum assured, the premiums to be calculated at rate i; " when of necessity i is $>j$." By slightly modifying the solution, the result may, I think, be made rather more suitable for numerical calculation. We have, denoting the annual premium by π,

$$\left.\begin{array}{l}\text{Value of return}\\ \text{in respect of}\\ \text{1st premium}\end{array}\right\} = \frac{\pi}{\mathrm{D}_x}\{(1+j)\mathrm{C}_x \quad +(1+j)^2\mathrm{C}_{x+1}+(1+j)^3\mathrm{C}_{x+2}+\&\mathrm{c}.\}$$

$$\left.\begin{array}{l}\text{Value of return}\\ \text{in respect of}\\ \text{2nd premium}\end{array}\right\} = \frac{\pi}{\mathrm{D}_x}\{(1+j)\mathrm{C}_{x+1}+(1+j)^2\mathrm{C}_{x+2}+(1+j)^3\mathrm{C}_{x+3}+\&\mathrm{c}.\}$$

&c. &c.

The total value of the return in respect of the premiums will therefore be (by summing perpendicularly)

$$\frac{\pi}{\mathrm{D}_x}\{(1+j)\mathrm{M}_x+(1+j)^2\mathrm{M}_{x+1}+\&\mathrm{c}.\}.$$

We see that the terms in this expansion respectively represent the value of the return in respect of the *last* premium paid, the last premium but one, and so on; and, in fact, the formula might have been at once deduced in this manner.

The formation of the selected terms necessary for summation by Lubbock's formula, if we use Mr. King's expression, involves the calculation of $\log \mathrm{M}'_x$ at a special rate of interest, and even if we use the modified formula $\dfrac{\mathrm{C}'_x+\mathrm{C}'_{x+1}(1+a_{\overline{1}|})+\&\mathrm{c}.}{\mathrm{D}'_x}$, we have to form $\log \mathrm{C}_x$ at a special rate of interest. By the formula given above, however, the calculation takes the form $\log \mathrm{M}_{x+n}+\log(1+j)^{n+1}$, and as both these logs are usually tabulated the work will be small.

I am, Sir,
Your obedient servant,
G. J. LIDSTONE.

4 & 5 King William Street, E.C.,
    17 *October* 1889.

ON THE VARIOUS CLASSES OF ASSURANCE BUSINESS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In his speech at the annual meeting in June last, the President foreshadowed a new combined Mortality Experience of Offices, to be taken out under the superintendence of the Institute.

Since the publication of the Twenty Offices' Experience, many suggestions have been offered, from time to time, as to the methods desirable to be adopted, both in the collection of data and in their subsequent arrangement, in a future combined experience. I should like to be permitted to contribute the following analysis and remarks, in the hope that they may be of some service to the Council, when the subject of the proposed new experience is before them.

Mr. George King, in his paper "On the Mortality amongst Assured Lives, and the requisite Reserves of Life Offices" (*J.I.A.*, xix, 382), writes as follows:—"We have principally in view to measure " the effects of the 'selection against a company' after its policies " have been issued; but the force, and its consequent action upon " the mortality, must vary according to the description of policy. " If, therefore, there were any large admixture of other kinds with " whole-life policies, and especially of such kinds as might follow a " very different law of discontinuance, it would be useless to take the " trouble which a laborious analysis involves," the outcome of the analysis referred to being Mr. King's well-known "Analyzed Mortality Tables", see pp. 398-404 of the same volume.

Mr. King, however, shows by his Table A—given on page 383— that, since no less than 95·35 per-cent of the total business of the 19 offices whose returns he analyzed was for the whole of life, the usual assumption, namely, that all the policies included in the Twenty Offices' Experience were whole-life policies, did not involve any material error.

The same point is also brought out in Mr. Sutton's paper "On the Determination of an Average Life Office" (*J.I.A.*, xx, 193).

On looking at the column in Mr. King's Table A, headed " Percentage to Total Business", I was struck with the insignificant percentage of endowment assurances, and thought that—in view of the rapid strides in popular favour which that class of assurances has been making in recent years—it would probably be found to constitute a considerably larger percentage of the total business of the same offices at the present time, so that we might be within measurable distance of the "large admixture of other kinds with whole-life policies" to the possibility of which Mr. King refers.

I am aware that, further on in the same paper (*J.I.A.*, xix, 384), Mr. King says, "Many of the special whole-life cases, and the joint- " life and endowment assurances, *probably* follow a very similar law " of discontinuance" (to that of the ordinary uniform-premium whole- life assurances); but, judging by the kind manner in which he has expressed himself with regard to the present suggestion, I think it not unlikely that he would now approve of an investigation into the *actual* law of discontinuance in endowment-assurance policies being undertaken.

The appended tabular statement sets side by side the figures for "the eight broad classes" of policies into which Mr. King divided the

Analysis of the Business of Offices which contributed Data to the Institute Experience :

(1) Mr. King's Analysis of the business of 19 of the 20 Offices, 1876 (*J.I.A.*, xix, 383, Table A).
(2) Analysis of the business of 17 of the 20 Offices, 1889.

Description of Assurances	Amounts Assured		Amounts Reassured		Percentage of Reassurances		Amounts Assured, less Reassurances		Percentage to Total Business	
	1876	1889	1876	1889	1876	1889	1876	1889	1876	1889
	£	£	£	£			£	£		
Whole-Life, Uniform Premiums	103,039,506	133,677,013	5,661,934	7,777,603	5·49	5·82	97,377,572	125,899,410	90·33	84·22
Do. other Descriptions	5,808,330	11,383,836	397,612	306,805	6·85	3·48	5,410,688	10,987,031	5·02	7·35
Joint Lives	862,897	980,875	180,600	244,550	20·93	24·93	682,297	736,325	·63	·49
Endowment Assurances	1,683,415	8,312,145	21,250	83,500	1·26	1·00	1,662,165	8,258,615	1·54	5·53
Short Term	1,358,998	1,733,151	245,997	408,651	18·10	23·58	1,113,001	1,324,500	1·03	·89
Contingent Survivorship	895,679	2,105,498	138,680	637,213	15·48	30·26	756,999	1,468,285	·70	·98
Last Survivor	779,023	681,504	77,850	118,250	9·99	17·35	701,173	563,254	·65	·38
Endowments	108,175	240,665	Nil	Nil	...	...	108,175	240,665	·10	·16
	114,536,023	159,144,687	6,723,953	9,666,572	5·87	6·07	107,812,070	149,478,115	100·00	100·00

NOTE.—From among the 20 offices, there had been one amalgamation prior to 1876, and two others have taken place since that date ; as one of these latter amalgamations, however, was a union of two companies, both of which were included in the 20 offices, my figures in effect comprise the returns of 18 of the offices.

business of the offices in his Table A, in the year 1876, with those I have extracted from the latest valuation returns of the offices.

In most instances no difficulty was found in obtaining the required information from the Board of Trade schedules; but as one of the offices did not separate "short term" and "contingent" assurances and reassurances in its return, some method of approximation was necessary, in order to obtain the separate figures and complete the table. Mr. King very kindly came to my rescue in this difficulty, and, acting on his suggestion, I have divided the assurances in question, for that office, in the ratio subsisting between the aggregate "short term" and "contingent" assurances of the remaining offices; and similarly with the reassurances.

There are four features of the comparison to which I should like to call attention.

(1) It may be of interest to note (as exhibiting the relative popularity of the various classes of assurance) that, since 1876, the total net amounts assured under

Whole-life Policies (uniform premiums) have increased by	£28,521,838	or	29·29 %
Whole-life Policies (other descriptions) have increased by	5,576,343	,,	103·06 ,,
Joint Lives have increased by	54,028	,,	7·92 ,,
Endowment Assurances have increased by	6,596,480	,,	396·86 ,,
Short Term ,, ,, ,,	211,499	,,	19·00 ,,
Contingent Survivorship ,, ,,	711,286	,,	93·96 ,,
Endowments ,, ,,	132,490	,,	122·47 ,,

while the only instance of an actual decrease in the amount of existing business is that of last-survivor assurances, which have decreased by £137,919, or 19·67 per-cent.

(2) The chief feature of the table will at once be seen to be the rapid rise in importance of endowment assurances. In less than 14 years they have risen from 1·54 per-cent of the total business of the offices analyzed to 5·53 per-cent; and the actual increase in the net amounts assured has been £6,596,480, or 396·86 per-cent of the figures of 1876; in other words, the total net amount of assurances in this class, in the later returns, is nearly five times what it was in the earlier.

But since, for the purposes of a mortality experience, the number of lives in the respective classes, rather than the amount of assurances, is the element to be considered, even these figures under-estimate the relative importance of the endowment-assurance class, since it is, I believe, the experience of all offices granting such policies that, on an average, the sum assured on a life is much smaller than in the case of whole-life assurances ; so that the addition of six and a-half millions to the sums assured involves, no doubt, the addition of a much larger body of lives to the endowment-assurance class than would be represented by the addition of an equal aggregate amount of sums assured to the whole-life class.

Again, on striking an average of the dates of the last valuations of the 17 offices whose returns I have analyzed, I find the mean date of valuation to be (say) May 1886; so that, if the new experience were to include the data up to the end of the present year, there

would be, taking one office with another, the results of three and-a-half years to be added to those comprised in the present statement. I suppose there can be no doubt but that this would still further enhance the relative importance of the endowment-assurance class.

(3) It will be noticed that the percentage of sums reassured in the endowment assurance class was very small, namely, 1·26 in 1876 and 1·00 in 1889.

This is only what might have been expected from the nature of the case, because the average sum assured on each life in this class is, as we have already seen, comparatively small; and the connection between the average amount of the assurances in any class and the percentage of reassurances in the same class is one of the points to which Mr. King draws special attention in discussing the figures of his Table A.

(4) The considerable increase in the net amount of whole-life assurances paid for otherwise than by uniform premiums, and in the percentage which that class of assurances bears to the total business, is almost entirely due to the very large volume of "limited premium" business transacted by one of the Scottish offices.

As the five minor classes of assurance amount, in the aggregate, to but a very small percentage of the total business (3·11 per-cent in 1876 and 2·90 per-cent in 1889), they do not seem to call for any special notice.

<div style="text-align:center">

I am, Sir,

Your obedient servant,

A. E. MOLYNEUX.

</div>

National Provident Institution,
 48 Gracechurch Street, E.C.,
  29 November 1889.

THE INSTITUTE OF ACTUARIES.

EXAMINATION OF THE INSTITUTE, 26 OCTOBER 1889.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

Examiners—J. McGOWAN, Esq., B.A.; ERNEST WOODS, Esq.; T. E. YOUNG, Esq., B.A.

First Paper.

ARITHMETIC AND ALGEBRA.

1. What fraction of $\dfrac{\frac{1}{2} \text{ of } 2\frac{1}{3}}{2\cdot36 + 1\cdot7}$ of £3. 10s. is $\dfrac{4\cdot\dot{0}\dot{3} - 3\cdot\dot{3}}{2\frac{2}{3} \text{ of } 3\frac{3}{4}}$ of £1?

2. Within what limits are the approximations obtained by the use of a table of *proportional parts* in a book of logarithms accurate?

3. A tradesman sells his goods at a price which gives a profit of x per-cent on the selling price, and $x+50$ per-cent on the cost price. If his profit for a single day be £20, find his gross receipts for that day.

4. Which is the larger of the two fractions—

$$\frac{2\frac{1}{5} \times \frac{3}{17} \times 2\frac{2}{3}}{\frac{1}{2}+\frac{2}{19} \times 28\frac{1}{2}} \quad \text{and} \quad \frac{2\frac{1}{5} \times \frac{3}{11} \times 1\frac{2}{3}}{2\frac{3}{8}+\frac{3}{4}+\frac{1}{2} \times \frac{3}{2}} ?$$

5. Solve the following equations—

(*a*) $x^2-3x+2\sqrt{x^2-3x+1}=2$;

(*b*) $x+\dfrac{1}{x}+\dfrac{1}{x+\dfrac{1}{x}}=2\frac{9}{10}.$

6. Define and illustrate the term *Expectation* as used in the Doctrine of Probabilities.

7. Find the general term in the expansion of $(1-x)^{-5}$.

8. Find the G. C. M. of

$$3x-x^2-5x^3-21x^4,$$

and

$$7x^2+16x^3+14x^4-16x^5-33x^6.$$

9. Sum the series $\dfrac{n-1}{n}+\dfrac{n-2}{n}+\dfrac{n-3}{n}$ . . . to n terms.

10. Define and illustrate the modulus of a system of logarithms. What are the advantages of the base 10 ?

BOOK-KEEPING.

11. Describe briefly the various books necessary in book-keeping by double entry, and state which would be dispensed with if the method of single entry were used.

12. Explain what is meant by (1) a Profit and Loss Account, (2) a Balance Sheet, and (3) Debtor and Creditor sides of an Account?

Second Paper.

EUCLID (Books I to IV, and VI, and Definitions of Book V).

1. In any right-angled triangle, the square which is described on the side subtending the right-angle is equal to the squares described on the sides which contain the right angle.

2. Show that the square on the side subtending an acute angle of a triangle is less than the squares on the sides containing the acute angle.

3. Define compound ratio, and give an example.

ARITHMETIC AND ALGEBRA.

4. Explain the contracted method of division of decimals, and divide ·123456 by ·234567 by the contracted method.

5. Given

$$\log_e 2 = \cdot 693147$$
$$\log_e 3 = 1 \cdot 098612$$
$$\log_e 10 = \frac{1}{\cdot 434294},$$

find $\log_{10} 6$, $\log_{10} 8$, $\log_{10} 12$.

6. Find the condition that the roots of the equation $ax^2 + bx + c = 0$ may be (1) both positive, and (2) opposite in sign, but the greater of them negative.

7. A man writes three letters and directs three envelopes into which he puts them at random: find the chance that all the letters will go wrong.

8. A ball, one inch in diameter, is thrown against a wire netting, the apertures of which are squares of three inches a side: find the chance that it goes through without hitting a wire. The thickness of the wire may be neglected.

FINITE DIFFERENCES.

9. Investigate the expressions—

  (a) For $\Delta^n u_x$ in terms of u_x and its successive values.

  (β) For u_{x+n} in terms of u_x and its first n leading differences.

10. Prove that if u_x be a rational integral function of x of the nth degree, then the nth difference is constant.

11. Given

$$\begin{aligned}
\log 350 &= 2 \cdot 54407 \\
351 &= 2 \cdot 54531 \\
352 &= 2 \cdot 54654 \\
354 &= 2 \cdot 54900
\end{aligned}$$

find $\log 353$.

It is optional on the part of the Candidate to answer one or more of the following questions; but due weight will be given to answers sent in.

PLANE CO-ORDINATE GEOMETRY.

12. Show that the equation to a circle, when the axes are rectangular, is always of the form $x^2 + y^2 + \mathrm{A}x + \mathrm{B}y + \mathrm{C} = 0$.

13. What straight lines are represented by the equation $y^2 - 4xy + 3x^2 = 0$?

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

VOL. XXVIII.—PART IV.

APRIL 1890.

CONTENTS OF NO. CLVI.

JOURNAL

INSTITUTE OF ACTUARIES.

On Bonuses on Endowment-Assurance Policies. By Arthur W. Sunderland, M.A., *Actuary of the National Life Assurance Society.*

[Read before the Institute, 23 December 1889.]

In a paper which I read before the Institute of Actuaries in April 1887 (*J.I.A.*, xxvi, 357) I gave some tables which seemed to me likely to be of use, in the case of certain life offices, for the purpose of forming estimates of the amounts of bonuses arising on ordinary whole-term with-profit policies. The offices contemplated were mutual societies, making their periodical valuations every five years by the H$^{\text{M}}$ Table, at 3 per-cent interest. Life offices differ so much in regard to rates of premium charged and rate and incidence of expenses, and also in regard to the rate of interest earned by their funds, that it appeared to me it would be desirable to construct tables in such a form that the bonuses, which would fit the circumstances of any particular office, might, without difficulty, be calculated from them, rather than to calculate bonuses for a hypothetical office. An illustration was given, it is true, but this was subsidiary to the main purpose of the paper, and was furnished with the object of showing how the tables might be applied.

At that time I was not prepared to extend the investigation to

bonuses on endowment-assurance policies, the tables applicable to policies for the whole term of life being somewhat extensive, and having occupied considerable time to compute. Participating endowment assurances are, however, continually becoming a more and more important section of the business of life assurance companies, and the question of the mode in which the surplus funds of life offices should be apportioned between them and the ordinary whole-term policies, is one to which, of late, much attention has been directed.

The cash value of a given bonus, payable with the sum assured, is obviously greater on an endowment-assurance policy than on one for the whole term of life; and it has been matter for very serious thought whether, in the case of offices which divide their surplus funds among all classes of assurances alike—

1. By way of uniform percentage additions to the sums assured for each premium paid in the valuation period; or

2. In proportion to the premium receipts;

endowment assurances, regarding them as a whole, do not get more than their fair share of the surplus.

The question whether or not endowment assurances are unduly favoured cannot, by one general investigation, be solved for all offices alike. Important variations in the rates of premiums for these assurances are to be found on referring to the prospectuses of different companies, and, in addition to this, the rate of interest which a life office makes upon its funds may be an important element, not only in determining the total bonuses, but also the ratio of the bonuses on endowment assurances to those on ordinary whole-term policies. I have, therefore, thought that the preparation of tables similar to those of my former paper, but applicable to endowment assurances, might prove useful at the present time, and that the tables themselves and certain results which seem to follow from them might be of interest to the Institute.

The tables which I have prepared, and which are given in the Appendix, are more limited in extent than those published in 1887. Many actuaries seem to be of the opinion that although a considerable part of the office expense attaching to a policy of assurance falls upon the year of issue, yet, for purposes of apportioning bonuses, it is only reasonable that this initial expenditure should be distributed throughout the duration of the

policy. In deference to this opinion, I have omitted from the calculations any table corresponding to Table C (bonuses from single surplus margin, £1), and have assumed that the expenses attaching to a policy are distributed uniformly over its duration.

On enquiry at a number of offices, I find that the ages at maturing for which endowment assurances are usually effected range between 45 and 65, it being a rare occurrence for an endowment assurance to be effected maturing at so high an age as 70. I have accordingly constructed tables for each of the three ages at maturing at 45, 55, and 65, and for ages at entry, 20, 30, 40, and 50; those cases being omitted in which the difference between the age at entry and the age at maturing is less than 15 years. It will be seen that the greatest difference between the age at entry and the age at maturing is 45 years. The tables of my previous paper were calculated for each of the experience rates of interest, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, and 5 per-cent. The object with which tables at so many rates of interest were calculated was this: that from them could be found by interpolation the bonuses for any intermediate experience rate. It might have been thought that, for all practical purposes, tables at experience rates 4 and $4\frac{1}{2}$ per-cent would have been sufficient, and that from these the bonuses for any intermediate or adjacent rate could have been found with sufficient accuracy by simple interpolation. Simple interpolation, however, would scarcely have given bonuses sufficiently exact in the case of old standing policies. As, however, for endowment assurances with the ages above mentioned the maximum policy age is 45 years, I have thought it sufficient in the present paper to take two experience rates of interest, 4 and $4\frac{1}{2}$ per-cent.

Assuming that the surplus margin is uniform in amount, and denoting it by X, the formula for the total cash surplus on an endowment-assurance policy corresponding to formula 1 (*J.I.A.*, xxvi, 359) is the following, $x+t$ being the age at which the endowment matures : *

$$X(1+i)\frac{(1+i)^5-1}{i}$$
$$+ \pi'_{x\bar{t}}(i-i')\frac{(1+i)^5-1}{i}$$
$$+ (i-i')\left[{}_nV'_{x\bar{t}}(1+i)^4 + {}_{n+1}V'_{x\bar{t}}(1+i)^3 + \ldots + {}_{n+4}V'_{x\bar{t}}\right]$$
$$+ {}_nB_{x\bar{t}}(i-i')\left[A'_{\overline{x+n}.\overline{t-n}}(1+i)^4 + \ldots + A'_{\overline{x+n+4}.\overline{t-n-4}}\right]$$

* For explanation of the symbols employed, see *J.I.A.*, xxvi, 358, 359.

Now $\qquad {}_nV'_{\overline{x}\overline{t}|} = 1 - \dfrac{1 + {}_{|t-n-1|}a'_{x+n}}{1 + {}_{|t-1|}a'_x} ;$

so that the third line becomes

$$(i-i')\left[\frac{(1+i)^5-1}{i} - \frac{1}{1 + {}_{|t-1}a'_x} \left\{ (1+i)^4(1 + {}_{|t-n-1}a'_{x+n}) \right. \right.$$
$$\left. \left. + (1+i)^3(1 + {}_{t-n-2}a'_{x+n+1}) + \ldots \right\} \right]$$

Also, since $\qquad A'_{\overline{x+n}.\overline{t-n}|} = 1 - d'[1 + {}_{t-n-1}a'_{x+n}],$

the fourth line becomes

$${}_nB_{x\overline{t}|}(i-i')\left[\frac{(1+i)^5-1}{i} - d'\{(1+i)^4(1 + {}_{t-n-1}a'_{x+n}) + \ldots\} \right]$$

If we denote the expression

$$(1+i)^4(1 + {}_{t-n-1}a'_{x+n}) + (1+i)^3(1 + {}_{|t-n-2}a'_{x+n+1}) + \ldots$$

by $f(t, x, n)$, the expression for the cash bonus becomes

$$X(1+i)\frac{(1+i)^5-1}{i}$$
$$+ (\pi'_{x\overline{t}|} + 1)(i-i')\frac{(1+i)^5-1}{i} - \frac{(i-i')f(t, x, n)}{1 + {}_{|t-1}a'_x}$$
$$+ {}_nB_{x\overline{t}}(i-i')\left(\frac{(1+i)^5-1}{i} - d'f(t, x, n) \right) \quad . \quad . \quad . \quad . \quad (1)$$

By the aid of this formula have been constructed the appended Tables A and B, which correspond to the similarly designated tables of my former paper. The function $f(t, x, n)$ was first calculated by means of the arithmometer, and then the rest of the process followed without difficulty. It may, perhaps, be well to mention, that to form Table A, X must be put equal to nothing in formula 1, while to form Table B, X is put equal to ·5, and the second line in formula (1) is omitted.

I venture to think that these tables may be made to yield a good deal of information as to the bonuses likely to arise upon endowment-assurance policies. Let us, in the first place, fix our attention upon that part of Table A corresponding to experience rate of interest 4 per-cent. If we compare for the three ages at maturing, 45, 55, and 65, the bonuses upon endowment assurances having the same period to run, for example, if we compare the

three bonus tables for 20 at 45, 30 at 55, and 40 at 65, it will be seen that the bonus tables are in the three cases nearly identical. The same is true, or nearly so, of the bonuses of Table B, and we infer that the bonus tables for endowment-assurance policies having the same period to run are practically the same, so that we may say, with reasonable accuracy, that the tables of bonuses are independent of the age at entry, depending only on the difference between the age at entry and the age at which the policy matures. The same will be observed to be true for the rate of interest $4\frac{1}{2}$ per-cent. In discussing the bonuses upon endowment assurances it will, therefore, be useful to arrange the tables in a somewhat different manner, and this has accordingly been done in Tables a and β. It will be sufficient if I describe one of them. Table a may be described as an average bonus table for endowment assurance policies having 15, 25, 35, and 45 years to run. What may be the most appropriate way of forming such average tables is, perhaps, a matter of opinion. The average age at which endowment assurances mature would probably be found to depend on the term of the endowment assurance; thus, for instance, since very few policies are effected before the age 20, the average age of maturing for endowment assurances having 45 years to run would probably be at least 65 years of age. On the other hand, the average age at maturing for endowment-assurance policies having only 25 years to run might be something like 55. To form the Tables a and β, I have taken the arithmetic mean of the bonuses given in Tables A and B, using all the bonus tables available. Thus, for instance, for endowment assurances having 45 years to run, the Tables of a and β will be identical with those of A and B, age at entry 20, age at maturing 65; for endowment assurances having 35 years to run, the Tables of a and β are formed from Tables A and B by taking the arithmetic mean of the bonus tables for 20 at 55, and 30 at 65; and so on.

The fact that the bonus tables depend only on the term that the endowment assurance has to run is one which might, perhaps, have been anticipated. It seemed to me, however, desirable that it should be actually investigated by calculation. It is important from two points of view. In the first place it very much lessens the number of tables which have to be constructed for the purpose of estimating the bonuses upon endowment-assurance policies; and in the second place it facilitates, I think, the calculation of

the effect upon those bonuses which would arise from the mortality experienced being different from that anticipated. To this second point I shall recur later on.

I now proceed to illustrate the use of the Tables a and β, and desire to draw particular attention to the fact that what follows is only an illustration. There may be wide differences of opinion as to what are reasonable rates of premium to take, and reasonable rates of expense to assume.

For the purpose of estimating the bonuses likely to arise on endowment-assurance policies, I shall, in what follows, make use only of the Tables a and β; that is to say, endowment-assurance policies will be regarded as classified according to the term of the endowment irrespective of the age at entry. From the tables of rates quoted by certain offices, I have deduced the following table of average rates for endowment-assurance policies, and certain other tables which are followed by their explanation.

	Age at Entry	MATURING IN			
		15 Years	25 Years	35 Years	45 Years
(1) Average Office Rates	20	...	4·000	2·900	2·400
	30	6·750	4·075	3·075	...
	40	6·925	4·375	...	...
	50	7·375	...	...	...
(2) Ditto, less 5 per-cent and 5s.	20	...	3·550	2·505	2·030
	30	6·162	3·621	2·671	...
	40	6·329	3·906	...	...
	50	6·756	...	...	...
(3) H^M 3 per-cent Net Premiums	20	...	3·102	2·131	1·682
	30	5·684	3·244	2·342	...
	40	5·873	3·547	...	...
	50	6·348	...	...	...
(4) Difference between (2) and (3) = Annual Surplus Margin	20	...	·448	·374	·348
	30	·478	·377	·329	...
	40	·456	·359	...	...
	50	·408	...	...	...
(5) Adjusted Average Annual Surplus Margin	...	·450	·400	·350	·300

Table No. (1), as already mentioned, shows, for a £100 policy, the average premiums for certain life offices. Table No. (2) shows the premiums which remain after deducting for expenses of management 5 per-cent of the office premiums and 5s. By deducting the H^M 3 per-cent net premiums of Table (3) we get

the annual surplus margins of Table (4). These margins of Table (4) have been used as a guide in forming the adjusted average annual surplus margins of Table (5).

To form a bonus table under the above hypothesis as to the amounts of annual surplus margin, we take Table β (which assumes a surplus margin of ·5), multiply the bonuses for 15-year endowments by ·9, those for 25-year endowments by ·8, those for 35-year endowments by ·7, and those for 45-year endowments by ·6, and add the results to the bonuses of Table *a*. For experience rate of interest 4 per-cent we have the following results:

REVERSIONARY BONUSES.

£100 *Endowment Assurance, maturing in 15 Years.*

Policy Age			New Bonus			Total Bonus
5	.	.	4·17	.	.	4·47
10	.	.	5·97	.	.	10·44
15	.	.	7·41	.	.	17·85

£100 *Endowment Assurance, maturing in 25 Years.*

Policy Age			New Bonus			Total Bonus
5	.	.	4·55	.	.	4·55
10	.	.	5·45	.	.	10·00
15	.	.	6·34	.	.	16·34
20	.	.	7·21	.	.	23·55
25	.	.	8·05	.	.	31·60

£100 *Endowment Assurance, maturing in 35 Years.*

Policy Age			New Bonus			Total Bonus
5	.	.	4·71	.	.	4·71
10	.	.	5·37	.	.	10·08
15	.	.	6·02	.	.	16·10
20	.	.	6·68	.	.	22·78
25	.	.	7·34	.	.	30·12
30	.	.	7·98	.	.	38·10
35	.	.	8·60	.	.	46·70

£100 *Endowment Assurance, maturing in 45 Years.*

Policy Age			New Bonus			Total Bonus
5	.	.	4·72	.	.	4·72
10	.	.	5·25	.	.	9·97
15	.	.	5·80	.	.	15·77
20	.	.	6·36	.	.	22·13
25	.	.	6·91	.	.	29·04
30	.	.	7·48	.	.	36·52
35	.	.	8·05	.	.	44·57
40	.	.	8·61	.	.	53·18
45	.	.	9·13	.	.	62·31

In connection with the foregoing investigation it is important to notice that the valuations are supposed to be net-premium

valuations by *a table of mortality coinciding with the experience.*
Under these suppositions it appears that the bonus tables for
endowment-assurance policies having the same term to run are
not greatly affected by the mortality which prevails among the
lives, for instance, the bonus tables for 20 at 45 do not differ
seriously from those for 40 at 65. If, instead of comparing
different parts of the same mortality table, *e.g.*, the interval 20 to
45 with the interval 40 to 65, we compare intervals of equal
extent in different mortality tables, we should naturally get the
same result, so that, for instance, for endowment assurances
maturing in 25 years the bonus tables would be approximately
the same for the H^M Table as the Carlisle.

The object of what follows is to show how to form an
estimate of the effect upon the bonuses which would result from
a mortality experience more favourable than that on which the
valuations are based, and, for the purpose of giving clearness to
our ideas, I think it will be well to proceed by way of illustration.
Let us compare endowment assurances maturing in 25 years, one
effected at age 30, the other at age 40. The net H^M 3 per-cent
annual premiums are 3·244 and 3·547 per £100 assured. The
difference is ·303, or 6s. 1d. I give below, for intervals of 5
years, on the basis of the H^M mortality and 3 per-cent interest,
the reserve values of the policies and the reserve values of £100
of bonus. The former are taken from Mr. D. Carment's Tables
(see *J.I.A.*, xxii, 372, &c.), the latter from Mr. R. P. Hardy's
Valuation Tables (see pp. 22 and 23).

Endowment Assurances maturing in 25 Years.

Age of Policy	POLICY-VALUES		$A_{\overline{zn}}$	
	30 at Entry	40 at Entry	30 at Entry	40 at Entry
5	13·8	14·1	59·2	61·3
10	29·9	30·2	66·8	68·5
15	49·0	49·0	75·9	77·0
20	71·9	71·4	86·7	87·1

Now, suppose by way of illustration, that for an endowment-
assurance policy effected at age 40, maturing at age 65, the
valuations were made by H^M 3 per-cent, while the mortality
experience coincided with the H^M for the age interval 30–55.

The bonuses may now be divided under three heads: bonuses from surplus interest corresponding to Table A, bonuses from surplus loading corresponding to Table B, and profit from mortality experience.

Under the supposition of these three sources of surplus it will be convenient to define the bonuses corresponding to Table A, and also those corresponding to Table B, as those which, in each case respectively, would arise by the methods of calculation under which those tables are formed were the mortality actually experienced made use of in the valuations. It will then be seen that, with the illustration assumed, the bonuses from favourable mortality experience would be exactly those of the H^M Table for age at entry 30, age at maturing 55, given by an annual surplus margin ·303. In fact, X being the annual surplus margin in the office premium obtained on the supposition of 3·547 being the risk premium, then if the true mortality table were used in valuing, the bonuses would be exactly those of Table A, age at entry 30, age at maturing 55, + the bonuses derived for the same age interval from Table B for the annual surplus margin X + ·303. The premium charged can yield these bonuses, and would yield them exactly if the valuation mortality table coincided with the experience.

The effect of valuing by a table with rates of mortality higher than that experienced will be, speaking generally, to slightly diminish the bonuses in the early years of a policy's existence, when these bonuses are not large, and the slight alteration in the bonuses given at the beginning can alter but slightly the benefits to the policyholders, looked at from the point of view of their dying early. A little will be taken off the bonuses at the beginning and somewhat more put on the bonuses at the end. I think, however, that a little consideration will be sufficient to convince those who reflect on the subject that these differences will not be of significant amount. In the case of the illustration which has been taken, it will be noted, on reference to the table given on page 264, that the two tables of policy-values differ but slightly, the difference in the net premiums for 30 at 55 and 40 at 65 being mainly absorbed in meeting the current risk, and that the two tables of $A_{x\overline{n}|}$ (the divisor used in converting cash bonus into reversion) are almost identical; so that, if for 40 at 65 the experience mortality coincided with the H^M for 30 at 55, a premium 3·547 would for 40 at 65 provide

the sum assured, and, for practical purposes, the same bonuses, whether the valuation were by the H^M Table using the interval 30 to 55 or the interval 40 to 65.

We obtain then, for practical purposes, this result. If 100π is the pure premium for an endowment assurance calculated according to the experience of lives assured under endowment-assurance policies, and $100\pi + \delta$ the pure valuation premium for the same assurance, and X the annual surplus margin according to the valuation table of mortality, so that $X + \delta$ is the annual surplus margin according to the experience mortality, then average tables, such as a and β, being calculated at the experience rate of interest, the bonuses for the particular policy may be found by adding the bonuses of the a table to the bonuses got from the β table by taking $X + \delta$ as the annual surplus margin.

Let us, as an example, find, for valuation rate of interest 3 per-cent and experience rate 4 per-cent, the bonuses which will arise on an endowment-assurance policy of £100, maturing in 25 years, the annual surplus margin, according to the valuation data, being ·4, but the pure premium, by the mortality experience and 3 per-cent interest, being ·3 less than by the valuation data.

We have to use the Tables a and β of the appendix for a policy maturing in 25 years, and experience rate of interest 4 per-cent. The bonuses of the β table must be multiplied by 1·4, and added to those of the a table. The following are the results:

REVERSIONARY BONUSES.

Policy Age			New Bonus			Total Bonus
5	.	.	7·39	.	.	7·39
10	.	.	8·11	.	.	15·50
15	.	.	8·83	.	.	24·33
20	.	.	9·55	.	.	33·88
25	.	.	10·25	.	.	44·13

On comparing these bonuses with those of the corresponding table of page 263, it will be seen that a favourable mortality experience among the lives assured under endowment-assurance policies may very appreciably affect the rate of bonus.

TABLE A.—*Experience* H^M 4 *per-cent—all Loading spent.*

Age	CASH BONUS			REVERSIONARY BONUS		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
	ENDOWMENT ASSURANCES MATURING AT AGE 45					
	Age at Entry 20					
25	·444	...	·444	·76	·76	25
30	1·239	·025	1·264	1·91	2·67	30
35	2·181	·101	2·282	3·03	5·70	35
40	3·296	·246	3·542	4·09	9·79	40
45	4·634	·485	5·119	5·12	14·91	45
	Age at Entry 30					
35	·859	...	·859	1·14	1·14	35
40	2·453	·049	2·502	2·89	4·03	40
45	4·366	·200	4·566	4·57	8·60	45
	ENDOWMENT ASSURANCES MATURING AT AGE 55					
	Age at Entry 20					
25	·282	...	·282	·60	·60	25
30	·763	·016	·779	1·48	2·08	30
35	1·323	·062	1·385	2 34	4·42	35
40	1·973	·149	2·122	3·17	7·59	40
45	2·736	·289	3·025	3·99	11·58	45
50	3·645	·501	4·146	4·78	16·36	50
55	4·741	·811	5·552	5·55	21·91	55
	Age at Entry 30					
35	·454	...	·454	·77	·77	35
40	1·248	·026	1·274	1·91	2·68	40
45	2·179	·102	2·281	3·01	5·69	45
50	3·289	·246	3·535	4·08	9·77	50
55	4·627	·484	5·111	5·11	14·88	55
	Age at Entry 40					
45	·866	...	·866	1·14	1·14	45
50	2·449	·049	2·498	2·88	4·02	50
55	4·358	·200	4·558	4·56	8·58	55

TABLE A.—*Experience* H^M 4 *per-cent—all Loading spent.*

Age	Cash Bonus			Reversionary Bonus		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	

ENDOWMENT ASSURANCES MATURING AT AGE 65

Age at Entry 20

25	·208	...	·208	·52	·52	25
30	·542	·012	·554	1·24	1·76	30
35	·926	·044	·970	1·96	3·72	35
40	1·361	·104	1·465	2·67	6·39	40
45	1·858	·198	2·056	3·36	9·75	45
50	2·432	·338	2·770	4·04	13·79	50
55	3·088	·535	3·623	4·71	18·50	55
60	3·860	·808	4·668	5·36	23·86	60
65	4·803	1·185	5·988	5·99	29·85	65

Age at Entry 30

35	·303	...	·303	·61	·61	35
40	·801	·017	·818	1·49	2·10	40
45	1·369	·065	1·434	2·34	4·44	45
50	2·025	·154	2·179	3·18	7·62	50
55	2·776	·296	3·072	3·99	11·61	55
60	3·658	·508	4·166	4·78	16·39	60
65	4·737	·815	5·552	5·55	21·94	65

Age at Entry 40

45	·477	...	·477	·78	·78	45
50	1·284	·027	1·311	1·91	2·69	50
55	2·207	·104	2·311	3·00	5·69	55
60	3·292	·249	3·541	4·07	9·76	60
65	4·618	·485	5·103	5·10	14·86	65

Age at Entry 50

55	·884	...	·884	1·15	1·15	55
60	2·439	·050	2·489	2·86	4·01	60
65	4·310	·199	4·539	4·54	8·55	65

TABLE A.—*Experience* H^M $4\frac{1}{2}$ *per-cent—all Loading spent.*

Age	Cash Bonus			Reversionary Bonus		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
ENDOWMENT ASSURANCES MATURING AT AGE 45						
Age at Entry 20						
25	·670	...	·670	1·15	1·15	25
30	1·875	·058	1·933	2·92	4·07	30
35	3·301	·233	3·534	4·69	8·76	35
40	4·990	·572	5·562	6·43	15·19	40
45	7·016	1·140	8·156	8·16	23·35	45
Age at Entry 30						
35	1·298	...	1·298	1·72	1·72	35
40	3·712	·112	3·824	4·42	6·14	40
45	6·608	·461	7·069	7·07	13·21	45
ENDOWMENT ASSURANCES MATURING AT AGE 55						
Age at Entry 20						
25	·427	...	·427	·91	·91	25
30	1·154	·037	1·191	2·26	3·17	30
35	2·003	·143	2·146	3·62	6·79	35
40	2·988	·346	3·334	4·99	11·78	40
45	4·143	·678	4·821	6·35	18·13	45
50	5·520	1·188	6·708	7·74	25·87	50
55	7·180	1·943	9·123	9·12	34·99	55
Age at Entry 30						
35	·686	...	·686	1·16	1·16	35
40	1·888	·059	1·947	2·91	4·07	40
45	3·299	·234	3·533	1·66	8·73	45
50	4·980	·572	5·552	6·40	15·13	50
55	7·006	1·136	8·142	8·14	23·27	55
Age at Entry 40						
45	1·307	...	1·307	1·72	1·72	45
50	3·705	·113	3·818	4·40	6·12	50
55	6·597	·460	7·057	7·06	13·18	55

TABLE A.—*Experience* H^M $4\frac{1}{2}$ *per-cent—all Loading spent.*

Age	CASH BONUS			REVERSIONARY BONUS		Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	

ENDOWMENT ASSURANCES MATURING AT AGE 65

Age at Entry 20

Age	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	Age
25	·314	...	·314	·78	·78	25
30	·820	·027	·847	1·90	2·68	30
35	1·402	·102	1·504	3·04	5·72	35
40	2·061	·242	2·303	4·19	9·91	40
45	2·813	·466	3·279	5·35	15·26	45
50	3·683	·802	4·485	6·54	21·80	50
55	4·677	1·283	5·960	7·74	29·54	55
60	5·845	1·957	7·802	8·96	38·50	60
65	7·273	2·899	10·172	10·17	48·67	65

Age at Entry 30

Age	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	Age
35	·458	...	·458	·93	·93	35
40	1·212	·039	1·251	2·28	3·21	40
45	2·072	·151	2·223	3·63	6·84	45
50	3·067	·359	3·426	5·00	11·84	50
55	4·204	·696	4·900	6·36	18·20	55
60	5·540	1·205	6·745	7·75	25·95	60
65	7·174	1·953	9·127	9·13	35·08	65

Age at Entry 40

Age	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	Age
45	·721	...	·721	1·18	1·18	45
50	1·943	·062	2·005	2·92	4·10	50
55	3·341	·241	3·582	4·65	8·75	55
60	4·984	·580	5·564	6·39	15·14	60
65	6·992	1·140	8·132	8·13	23·27	65

Age at Entry 50

Age	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	Age
55	1·336	...	1·336	1·74	1·74	55
60	3·690	·115	3·805	4·37	6·11	60
65	6·569	·460	7·029	7·03	13·14	65

TABLE B.—*Bonuses from Annual Surplus Margin,* 10s.

	Experience H^M 4 per-cent Cash Bonus from Surplus Margin 2·8165			Experience H^M 4½ per cent Cash Bonus from Surplus Margin 2·8584			
Age	Cash Bonus	Reversionary Bonus		Cash Bonus	Reversionary Bonus		Age
		New	Total		New	Total	
Endowment Assurances maturing at Age 45							
Age at Entry 20							
25	2·817	4·84	4·84	2·858	4·91	4·91	25
30	2·977	4·50	9·34	3·105	4·70	9·61	30
35	3·168	4·20	13·54	3·407	4·52	14·13	35
40	3·400	3·93	17·47	3·780	4·37	18·50	40
45	3·682	3·68	21·15	4·246	4·25	22·75	45
Age at Entry 30							
35	2·817	3·73	3·73	2·858	3·79	3·79	35
40	2·977	3·44	7·17	3·106	3·59	7·38	40
45	3·172	3·17	10·34	3·412	3·41	10·79	45
Endowment Assurances maturing at Age 55							
Age at Entry 20							
25	2·817	5·99	5·99	2·858	6·08	6·08	25
30	2·976	5·65	11·64	3·103	5·89	11·97	30
35	3·164	5·34	16·98	3·400	5·74	17·71	35
40	3·387	5·07	22·05	3·760	5·63	23·34	40
45	3·655	4·82	26·87	4·202	5·54	28·88	45
50	3·979	4·59	31·46	4·750	5·48	34·36	50
55	4·376	4·38	35·84	5·439	5·44	39·80	55
Age at Entry 30							
35	2·817	4·76	4·76	2·858	4·83	4·83	35
40	2·976	4·45	9·21	3·104	4·64	9·47	40
45	3·167	4·17	13·38	3·404	4·49	13·96	45
50	3·395	3·92	17·30	3·773	4·35	18·31	50
55	3·674	3·67	20·97	4·234	4·23	22·54	55
Age at Entry 40							
45	2·817	3·71	3·71	2·858	3·77	3·77	45
50	2·977	3·43	7·14	3·105	3·58	7·35	50
55	3·171	3·17	10·31	3·410	3·41	10·76	55

TABLE B.—*Bonuses from Annual Surplus Margin*, 10s.

Age	Cash Bonus	Reversionary Bonus		Cash Bonus	Reversionary Bonus		Age
		New	Total		New	Total	

| EXPERIENCE HᴹM 4 PER-CENT Cash Bonus from Surplus Margin 2·8165 | | | | EXPERIENCE HᴹM 4½ PER-CENT Cash Bonus from Surplus Margin 2·8584 | | | |

ENDOWMENT ASSURANCES MATURING AT AGE 65

Age at Entry 20

Age	Cash Bonus	New	Total	Cash Bonus	New	Total	Age
25	2·817	7·00	7·00	2·858	7·10	7·10	25
30	2·975	6·68	13·68	3·102	6·96	14·06	30
35	3·160	6·40	20·08	3·393	6·87	20·93	35
40	3·376	6·15	26·23	3·742	6·82	27·75	40
45	3·630	5·92	32·15	4·162	6·79	31·54	45
50	3·931	5·73	37·88	4·672	6·82	41·36	50
55	4·287	5·57	43·45	5·290	6·87	48·23	55
60	4·716	5·42	48·87	6·052	6·95	55·18	60
65	5·244	5·24	54·11	7·011	7·01	62·19	65

Age at Entry 30

Age	Cash Bonus	New	Total	Cash Bonus	New	Total	Age
35	2·817	5·70	5·70	2·858	5·79	5·79	35
40	2·975	5·42	11·12	3·103	5·65	11·44	40
45	3·162	5·16	16·28	3·396	5·54	16·98	45
50	3·381	4·93	21·21	3·750	5·47	22·45	50
55	3·610	4·73	25·94	4·179	5·43	27·88	55
60	3·950	4·54	30·48	4·704	5·40	33·28	60
65	4·331	4·33	34·81	5·363	5·36	38·64	65

Age at Entry 40

Age	Cash Bonus	New	Total	Cash Bonus	New	Total	Age
45	2·817	4·60	4·60	2·858	4·67	4·67	45
50	2·976	4·34	8·94	3·103	4·53	9·20	50
55	3·164	4·11	13·05	3·399	4·42	13·62	55
60	3·387	3·89	16·94	3·759	4·32	17·94	60
65	3·658	3·66	20·60	4·208	4·21	22·15	65

Age at Entry 50

Age	Cash Bonus	New	Total	Cash Bonus	New	Total	Age
55	2·817	3·66	3·66	2·858	3·71	3·71	55
60	2·976	3·42	7·08	3·104	3·56	7·27	60
65	3·168	3·17	10·25	3·406	3·41	10·68	65

Average Table.

Table a.—*Founded on Experience* H^M 4 *per-cent—all Loading spent.*

Policy Age	Cash Bonus			Reversionary Bonus		Policy Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
ENDOWMENT ASSURANCE MATURING IN 15 YEARS						
5	·870	...	·870	1·14	1·14	5
10	2·447	·019	2·496	2·88	4·02	10
15	4·355	·200	4·555	4·56	8·58	15
ENDOWMENT ASSURANCE MATURING IN 25 YEARS						
5	·458	...	·458	·77	·77	5
10	1·257	·026	1·283	1·91	2·68	10
15	2·189	·102	2·291	3·01	5·69	15
20	3·292	·247	3·539	4·08	9·77	20
25	4·626	·485	5·111	5·11	14·88	25
ENDOWMENT ASSURANCE MATURING IN 35 YEARS						
5	·293	...	·293	·61	·61	5
10	·782	·017	·799	1·49	2·10	10
15	1·346	·064	1·410	2·34	4·44	15
20	1·999	·152	2·151	3·18	7·62	20
25	2·756	·293	3·049	3·99	11·61	25
30	3·652	·505	4·157	4·78	16·39	30
35	4·739	·813	5·552	5·55	21·94	35

Table a.—*Experience* H^M $4\frac{1}{2}$ *per-cent—all Loading spent.*

Policy Age	Cash Bonus			Reversionary Bonus		Policy Age
	1, from Policy	2, from Bonus	Sum of 1 and 2	New	Total	
ENDOWMENT ASSURANCE MATURING IN 15 YEARS						
5	1·314	...	1·314	1·73	1·73	5
10	3·702	·113	3·815	4·40	6·13	10
15	6·591	·460	7·051	7·05	13·18	15
ENDOWMENT ASSURANCE MATURING IN 25 YEARS						
5	·692	...	·692	1·16	1·16	5
10	1·902	·060	1·962	2·92	4·08	10
15	3·314	·236	3·550	4·67	8·75	15
20	4·985	·575	5·560	6·41	15·16	20
25	7·005	1·139	8·144	8·11	23·30	25
ENDOWMENT ASSURANCE MATURING IN 35 YEARS						
5	·443	...	·443	·92	·92	5
10	1·183	·038	1·221	2·27	3·19	10
15	2·038	·147	2·185	3·63	6·82	15
20	3·028	·353	3·381	5·00	11·82	20
25	4·174	·687	4·861	6·36	18·18	25
30	5·530	1·197	6·727	7·75	25·93	30
35	7·177	1·948	9·125	9·13	35·06	35

AVERAGE TABLE.

TABLE β.—*Bonuses from Annual Surplus Margin*, 10s.

		FOUNDED ON EXPERIENCE HM 4 PER-CENT Cash Bonus from Surplus Margin 2·8165		FOUNDED ON EXPERIENCE HM 4½ PER-CENT Cash Bonus from Surplus Margin 2·8584			
Policy Age	Cash Bonus	Reversionary Bonus		Cash Bonus	Reversionary Bonus		Policy Age
		New	Total		New	Total	
ENDOWMENT ASSURANCE MATURING IN 15 YEARS							
5	2·817	3·70	3·70	2·858	3·76	3·76	5
10	2·977	3·43	7·13	3·105	3·58	7·34	10
15	3·170	3·17	10·30	3·409	3·41	10·75	15
ENDOWMENT ASSURANCE MATURING IN 25 YEARS							
5	2·817	4·73	4·73	2·858	4·80	4·80	5
10	2·976	4·43	9·16	3·104	4·62	9·42	10
15	3·166	4·16	13·32	3·403	4·48	13·90	15
20	3·394	3·91	17·23	3·771	4·35	18·25	20
25	3·671	3·67	20·90	4·229	4·23	22·48	25
ENDOWMENT ASSURANCE MATURING IN 35 YEARS							
5	2·817	5·85	5·85	2·858	5·94	5·94	5
10	2·976	5·54	11·39	3·103	5·77	11·71	10
15	3·163	5·25	16·64	3·398	5·64	17·35	15
20	3·384	5·00	21·64	3·755	5·55	22·90	20
25	3·648	4·78	26·42	4·191	5·49	28·39	25
30	3·965	4·57	30·99	4·727	5·44	33·83	30
35	4·354	4·36	35·35	5·401	5·40	39·23	35

DISCUSSION.

The PRESIDENT (Mr. W. Sutton, M.A.) having invited discussion,

Mr. GEORGE KING said that the question was one that involved a great deal of thought, and it was not easy to go through a paper of the kind in a hurry and analyze it with any degree of confidence. However, the subject was one that most actuaries had studied, and he (Mr. King) had given it very close attention during the last few years. He referred principally to the question of bonuses on endowment assurances as compared with bonuses on whole-life policies. If Mr. Sunderland had been able to bring out more distinctly a comparison of the results now obtained with those of his

former paper, it would have been an advantage.   His experience
showed that hypothetical tables of the kind Mr. Sunderland had
given were not altogether satisfactory.   That statement was not
meant in disparagement of Mr. Sunderland's work, because he did
not think it possible to prepare tables that were entirely satisfactory.
He would point out one source of surplus in companies making
stringent valuations, and that was the gain in respect of what
Mr. C. D. Higham had called the death strain.   An office not only
obtained the surplus interest while the policy remained in force, but
when a claim arose the difference between the experience reserve and
the actual reserve remained in its hands as surplus, and that was a
very important item.   He thought it was Mr. Higham, Sen., who
suggested that that difference should go as a final bonus to the
policyholders.   But such was not the custom, and therefore a
company valuing on a stringent basis had a fund over and above the
gain in interest, which an office with a less reserve did not possess.
He had examined the returns of a great many companies in the
Blue-Books, and he had always found that the bonuses they had
actually paid were in excess of what one might expect from accumu-
lating surplus interest and loading.   When Mr. Sunderland read his
former paper, he (Mr. King) pointed out that the effect of his figures
was to show that the method of allotting bonus by way of reversion,
at a certain rate per-cent per annum upon the sums assured and
existing bonuses, was a fair one, and this view had been confirmed.
The figures now given showed that as regards endowment assurances
the same rule held.   He had occasion recently to go very carefully
into this question, and while it was impossible to lay down general
rules, because one company differed so much from another, yet he had
arrived at certain results for a particular company, and believing
that those actual results might be useful to the profession, he had
ventured to give them.

I.—*Ordinary Whole-Life.*

Age at Entry	Office Premium	Bonus Loading	Expense Loading
20	2·104	·463	·250
25	2·233	·500	·224
30	2·463	·538	·211
35	2·771	·583	·196
40	3·179	·633	·185
45	3·725	·688	·186
50	4·433	·742	·203
55	5·400	·796	·246
60	6·713	·854	·318

II.—*Endowment Assurances.*

At 50.

Age at Entry	Office Premium	Bonus Loading	Expense Loading
20	3·200	·671	·311
25	3·913	·742	·347
30	5·025	·821	·435
35	6·900	·908	·631

II.—*Endowment Assurances*—(continued).

At 55.

Age at Entry			Office Premium			Bonus Loading			Expense Loading
20	.	.	2·775	.	.	·608	.	.	·310
25	.	.	3·279	.	.	·675	.	.	·318
30	.	.	4·021	.	.	·746	.	.	·359
35	.	.	5·146	.	.	·825	.	.	·451
40	.	.	7·033	.	.	·913	.	.	·646

At 60.

Age at Entry			Office Premium			Bonus Loading			Expense Loading
20	.	.	2·458	.	.	·558	.	.	·285
25	.	.	2·871	.	.	·617	.	.	·315
30	.	.	3·413	.	.	·683	.	.	·333
35	.	.	4·171	.	.	·750	.	.	·380
40	.	.	5·317	.	.	·829	.	.	·472
45	.	.	7·258	.	.	·913	.	.	·679

At 65.

Age at Entry			Office Premium			Bonus Loading			Expense Loading
20	.	.	2·300	.	.	·521	.	.	·325
25	.	.	2·608	.	.	·571	.	.	·322
30	.	.	3·029	.	.	·629	.	.	·338
35	.	.	3·592	.	.	·692	.	.	·350
40	.	.	4·388	.	.	·763	.	.	·400
45	.	.	5·600	.	.	·838	.	.	·504
50	.	.	7·604	.	.	·921	.	.	·723

III.—*Endowment Assurances Averaged.*

Term			Average Office Premium			Average Bonus Loading			Average Expense Loading
45	.	.	2·300	.	.	·521	.	.	·325
40	.	.	2·533	.	.	·565	.	.	·303
35	.	.	2·892	.	.	·618	.	.	·318
30	.	.	3·371	.	.	·680	.	.	·328
25	.	.	4·123	.	.	·750	.	.	·372
20	.	.	5·275	.	.	·828	.	.	·466
15	.	.	7·199	.	.	·914	.	.	·670

The figures showed, first of all, the effect of discounting certain bonuses as regards whole-life policies. The bonus discounted was at the rate of 27*s*. 6*d*. per-cent per annum, and allowance was made for the incidence of interim bonus and for various other things. It was assumed in the tables that the bonus was surrendered when allotted, so that, apparently, it was a simple bonus, but, for all that, because the bonus was allotted by means of the H$^{M(5)}$ 3 per-cent reversion values, it really represented a compound bonus, or a bonus upon bonus. The figures showed that, taking the actual rates employed by the best companies, such a bonus worked out satisfactorily, and left a sufficient loading for management expenses. The method was not perfectly equitable, but nearly so. The premiums were not theoretical rates prepared for the purpose of producing certain bonuses, but competition rates, calculated with a view to compare well with those of other companies. Their further effect was to show that, as regards endowment-assurance policies, practically the same reversionary bonus, as in the case of whole-life policies, should be

paid by an office which valued at a low rate of interest. He knew some great authorities took a different view, and held that endowment assurances should not receive as much as whole-life policies. The figures rather tended to show that, if any distinction was to be made, it should be in favour of endowment assurances. On the whole, the short-term endowment assurances ought to get the higher rate of reversionary bonus, the margin of loading being considerably greater in respect of a short than of a long period; for instance, with a 40 years' policy the margin of loading was 6s., while with a 15 years' policy it was 13s. 6d. Mr. Sunderland showed, and his (Mr. King's) figures entirely confirmed the same, that it was of comparatively little consequence at what *age* an endowment-assurance policy was effected. The chief point was, *how many premiums* were contracted for. If a life were rated up, say 10 years, it was surprising how few pence addition was made to the premium. These points coming out so clearly in the paper, and being confirmed by the figures he had given, and by the experience of actuaries generally, actuaries would possibly value endowment assurances by discarding the question of age, and by only taking into account the endowment term.

Mr. F. Schooling asked whether, in the average office rates given on page 262, the premiums from which they were formed were in all cases payable by n instalments and not $n+1$, it being known that offices differed in this respect. The symbol "$_nB_{jj}$" was not explained in this paper, but on reference to the former paper it was seen that it denoted the total reversionary bonus, and it would be well to repeat this explanation. Mr. Sunderland asked at the commencement of the paper, whether endowment assurances, regarding them as a whole, did not get more than their fair share of the surplus. This question was not directly answered in the paper, but tables were given from which it was suggested that each company could, for itself, answer the question. By means of Table α the profit from the difference between the rate of interest realized and the rate assumed at the valuation could be determined, and by means of Table β the profit from the surplus margin was ascertained. Mr. Sunderland also showed, on page 266, that the reversionary bonus obtained from profit arising from favourable mortality could be found from Table β. By that method all profit arising from this source was assumed to have been earned equally by new and old entrants, and was equally distributed amongst them. That should not be absolutely taken for granted. Thus, the three large sources of profit were dealt with, but nothing was said as to how extraneous sources of profit should be dealt with. Profits from these sources should probably be added in some way to Table α, because they belonged more particularly to policies of long duration, and should not be divided equally amongst entrants of all durations. From the table given on page 263, Mr. King's remarks were fully borne out. At the end of 10 years the amounts were 10·41 for the 15-year table, 10·00 for the 25-year table, 10·08 for the 35-year table, and 9·97 for the 45-year table. That seemed to point to the fact that the uniform reversion bonus system could be applied with a certain amount of equity. It did not follow that the same rate of reversionary bonus should be given to endowment-assurance policies and whole-life policies, but it was an important point gained if so easy a method could be equitably

employed. The point whether the same rate of reversionary bonus should be given to endowment-assurance policies and whole-life policies was probably one which only investigation by each individual company could accurately discover, but the table of specimen bonuses threw some light on the subject. He noticed that the final total bonus in all the tables was about equal to a uniform bonus of £1. 2s. per-cent per annum, allowing for bonus on sum assured and previous bonus additions. In no case was there a difference in the total of £2. In the tables, however, only profit from interest and surplus margin was included. It might be that other profits would raise the percentage to £1. 10s. per annum—a rate about equal to that given on all classes of policies by many companies whose investment rate of interest was about 1 per-cent higher than that assumed in the valuation. It therefore appeared that the advocates of the uniform bonus plan had received strong support.

Mr. A. W. Tarn suggested the adaptation of the bonuses on endowment assurances to the method of distribution in vogue in the office rather than to adopt a special method for endowment assurances. He pointed out that the interest on the reserve values of endowment assurances was far greater than upon whole-term policies, because the reserves bore a larger proportion to the premiums paid. The profit on surrenders and lapses must be greater on endowment assurances than on whole-term policies.

Mr. H. W. Manly thought that the opinions he had previously expressed with regard to the lines on which such investigations should proceed were correct, namely, that the opposite course to that taken by Mr. Sunderland should be pursued, and that they should first consider what form of bonus was to be given, secondly, estimate the future bonuses, and finally ascertain what premiums would provide such benefits. They were continually asked to give estimates, and they should be careful to see that the premiums charged would produce the estimated result. There were many different ways of dividing the surplus, and it was not advisable that each office should charge the same premium. At the present time the variations between the premiums and the bonuses were very great, and the public were getting wiser than formerly and could distinguish those differences, and the tendency was for the public to go to the office which charged a very low premium and gave a very large bonus. If the premium was too small to stand a large bonus the bonuses must be reduced, and the estimates put forward would not be realized. On the other hand, it was possible that the larger premiums had been calculated with a view to producing the results which were anticipated. The office with higher rates, unfortunately perhaps, did not get its fair share of business; nevertheless, what it obtained was profitable, and the assured would not be disappointed with the results. Therefore, a set of tables like Mr. Sunderland's was after all not exactly what was required. They wanted to see whether the premium charged would produce the results anticipated. Competition was exceedingly severe as to the rates charged, and he was sorry to say that many offices were departing from their published rates. He heard the other day of a person, of nearly 50, who wanted an endowment policy payable in 10 years, with profits, and by setting one office against another he obtained a quotation of £9. 18s.

per-cent, no commission to be paid. Assuming that it was a purely banking transaction, and that the premiums were to be accumulated at $3\frac{1}{2}$ per-cent, a premium of £8. 5s. per-cent would be required. Therefore, the office had to run the risk for the period, pay all the office expenses, and give a large bonus for merely £1. 13s. per-cent. One matter which Mr. Sunderland had brought out clearly, and to which Mr. King had referred, he had himself found out some time ago, namely, that the term the policy had to run was more important in the valuation than the ages ; and as he had to make an annual valuation, he had adopted a plan of classifying endowment policies according to the year of maturity. He found the average age by multiplying the sum assured by the age and dividing the sum by the total sum assured, and then valued the whole of the policies in each class as an endowment assurance running for the unexpired term, the results agreeing very closely with those obtained by valuing each policy separately. With the evidence Mr. Sunderland had given they could follow the plan with confidence.

The PRESIDENT, in moving a vote of thanks to Mr. Sunderland, said that endowment assurances on the other side of the Atlantic had taken a very conspicuous part in ordinary insurance business, and he had taken some trouble to accumulate information with regard to how insurance business generally was conducted in America. He did not get so much assistance as he expected with regard to the subject under discussion, but found, to his astonishment, that a large amount of business was apparently being carried on in that country upon lines more or less resembling that fallacious principle which was exploded in this country many years ago. Instead of business conducted on that principle showing signs of falling off in America, it appeared to show signs of increase; and if the Council would grant him the opportunity, during the present session, he would endeavour to place upon record his opinion with regard to the innovation.

Mr. SUNDERLAND, in reply, said the paper was to be regarded as to a great extent a continuation of that read in April 1887 (*J.I.A.*, xxvi, 357). That fact would explain some questions which had been raised with regard to it. For that reason it was mainly concerned with the question of profit from interest and profit from loading. Mr. King said the effect of a stringent reserve on bonuses upon endowment assurances had not been considered. [Mr. KING—I said you had not considered the question of the difference between what we may call the experience reserve and the actual reserve which is left when the policy becomes a claim. I do not think that has been touched upon.] Although he had not made any direct reference to the point mentioned by Mr. King, he had done so indirectly, because he had considered the question of a low rate of interest in office calculations, and also of a high rate of mortality. He had not discussed in any complete manner the effect of favourable mortality, but had merely considered the way in which the effect might be estimated. It was satisfactory to hear from Mr. King that, as the result of his calculations, endowment-assurance policies could be given as large bonuses as life policies, and it came as a surprise that the bonuses on short-term endowment policies might be at least as great as those on policies which had a long period to run. Some of Mr. Schooling's questions had been answered by the

explanation that the paper was a continuation of the one read a few years ago. Mr. Manly's view that they should first of all determine what bonuses were to be given, and then calculate the premiums accordingly appeared to him to run counter to the modern competitive system.

The Income Tax. *Address by* BENJAMIN NEWBATT, F.I.A., F.S.S., F.R.G.S., *Actuary of the Clerical, Medical and General Life Assurance Society.*

[Delivered before the Institute, 27 January 1890.]

MR. PRESIDENT and gentlemen,—Since Mr. Bailey, some four years ago, read his interesting paper on the subject of the income tax (*J.I.A.*, xxv, 314), a good deal has happened to throw light upon, or, as some possibly may think, to obscure the question of the real liability of insurance companies towards this impost; and the Council of the Institute being of opinion that a further discussion would be useful and instructive, have, for reasons which will occur to most of you, asked me to open it. The moment is certainly not inopportune, for, as you are aware, during the last two or three weeks—since even my notes were put together—it has been literally raining income tax cases in the law courts. Two of these recent cases—Colquhoun *v.* Heddon, and the Gresham Office *v.* Styles—are both interesting and important; but as neither materially affects—the Gresham case seems even to strengthen—my argument, I have not thought it necessary to re-cast my remarks.

I would say, by way of preamble, that the remarks I am about to make have been framed more with a view to their fitness for debate, than on the narrower and less elastic lines of a formal essay. Apart from the desire to acquire the freedom which such a mode of treatment gives me, I have thought it the preferable course to adopt in dealing with a question more or less on the border line between the abstract or theoretical, which can be fully considered here, and the concrete or practical, which probably could be better dealt with elsewhere. To-night, therefore, we court opinion rather than invite action, and I shall not, I hope, be exceeding my privilege if I say that opinion will be welcomed from any quarter and from any person having matter to offer. The question is essentially one on which fresh minds may throw new lights, and even do that which some of us older men have failed to do, namely, reconcile judgment with justice. And it is above all a question in regard to which we seek the truth.

We are not, in this room or out of it, a body of men afflicted

with an ignorant impatience of taxation. After making all allowance for the utmost diversity of view among the members of this Institute, I think I may say that we recognize the obligations which insurance companies owe to the State for the safe and secure conduct of their business, and are prepared to pay in reasonable manner for the law and order which is as necessary to them as to other trading bodies, but not more necessary.

If I start, as I propose to do, from the case of Last *v.* The London Assurance Corporation, I shall, in effect, begin at the beginning. For however true it may be that there have, in days long past, been understandings and possibly also arrangements between the Board of Inland Revenue or their surveyors and the offices, there has never been a clear and uniform method of dealing with all offices alike. In many respects, sometimes in regard to matters of importance, different offices have been differently treated by different surveyors, and even the same office by successive surveyors, each surveyor, in fact, having been more or less a law unto himself. In the London Assurance case (as for better identity I propose to call it) solid ground seemed at last to have been reached, and a mode of assessment laid down which, while applicable to all companies and societies alike, might be acquiesced in by all. In saying this, I of course assume a willingness to accept the definition of "profits" which was arrived at in the House of Lords in that case. Personally I was, and am, so prepared, for reasons which I will give later.

But before going into the details of the judgments—the combined judgments, I mean—in the London Assurance case, it will, perhaps, be convenient to recall to your minds what at the time these judgments by general consent seemed to be. In saying "general consent", I hope I am not putting it too high. When two litigants disputing about money agree, after the battle has been fought, on a mode of reckoning which both think expresses the results of the action, those results, I submit, are removed from the region of doubt and academic discussion to the surer basis of fact and figures. The pecuniary results being undoubted, the judgments on which they rest should be undoubted also; and I say this all the more strongly, because, on the best opinion I can form, the pecuniary results of the judgments and the language of the judgments in the London Assurance case are in accord.

That these pecuniary results may be thoroughly appreciated as we go on, I have had printed on the slips which have been circulated among the members, the " Form of Return, after final

" judgment of House of Lords, as agreed upon betweeen Mr. Last
" (on behalf of the Board of Inland Revenue) and the London
" Assurance Corporation", and, alongside it, a return, in like
form, by the Clerical, Medical and General Life Assurance

(A)

FORM OF RETURN after Final Judgment of House of Lords, as agreed upon between Mr. LAST (on behalf of the Board of Inland Revenue) and the LONDON ASSURANCE CORPORATION.   YEAR........

MARINE PROFITS, viz., Premiums, less Losses, Commission, and Bad Debts £

FIRE   Do.   do.   do.   £

LIFE †   Do.   viz.:—
Surplus at Quinquennial Valuation on 31 December, 1880 . £
Less Balance undivided at previous Quinquennium   .   £
                    1/5th) £

Interest, all, except on Life Funds   .   £
Profit on Realization of Securities (except on Life Account included in Surplus) £
                                            £

Deduct Expenses of the Year   .   .   .   .   £

Net Profit   .   £
Taxed Interest   .   £

Amount remaining to be Assessed, or Amount on which the Tax has been overpaid, as the case may be   .   £

† All Life Profits.

(B)

CORRESPONDING RETURN OF CLERICAL, MEDICAL AND GENERAL LIFE ASSURANCE SOCIETY on basis of (A).

YEAR 1885-6.

MARINE PROFITS   .   .   .   .   *Nil*

FIRE   Do.   viz.:—   .   .   *Nil*

LIFE †   Do.   viz.:—
Surplus at Quinquennial Valuation as at 30 June 1881 .   £366,693 19 5
Less Balance undivided at previous Quinquennium   .   7,940 16 4
                    £358,753 3 1
                    1/5th) £71,750 12 7

Interest, all, except on Life Funds.   £2,500 0 0
Profit on Realization of Securities (except on Life Account included in Surplus)   *Nil*
                                            £74,250 12 7

Deduct Expenses of the Year: as the above is *net* Profit, the deduction is   .   .   *Nil*

Net Profit £74,250 12 7
Taxed Interest 107,378 13 8

Amount on which the Tax has been overpaid   .   £33,128 1 1

† All Life Profits.

Society. It is manifest on these figures that, as a matter of pounds, shillings, and pence, while the London Assurance Corporation was worsted in its contention that the proportion of profits paid to policyholders was *not* liable to tax, it succeeded in its other contention—its main contention, as I regard it— that the interest of the life fund was *exempt* from taxation; and it is not a little remarkable, though I do not wish to make more of it than it is worth, that in settling this return the Inland Revenue authorities seemed to have admitted that there might be such a thing as tax "overpaid." Now, I put this return in the fore-front, because it expresses in arithmetical symbols, if I may so say, the view which both litigants took of the effect of the combined judgments, and because, I think, it establishes the fact that those of us who agreed with Mr. Bailey, in the paper already referred to, "that the Crown has not the right to levy the tax upon life insurance funds", and that "the " claim to duty on untaxed interest which has been made of late " years was thus disposed of", did not act without reason.

This matter being of importance, as having formed the starting point of the litigation which followed, should be made clear. The words of Mr. Justice Day's judgment, which were relied on for the contention that the interest income of a life assurance society is not liable to tax—a judgment, it will be remembered, which was not appealed from—are these: "The " third question is as to the right of the Crown to levy the duty " upon what has been termed the life fund. This should, in my " opinion, be answered in the negative"; and while, of course, I admit that the insertion of two, or at most three, other words would have made the meaning of the whole sentence so clear as to be beyond all possibility of dispute, I yet affirm that as the words stand they are capable of only one construction. What can be the right of the Crown here referred to? It cannot mean the right to levy duty on the life fund in the literal sense—*i.e.*, on the accumulated capital of the fund. Not only was no such claim made, but it is scarcely conceivable that it could be made by rational beings. This being so, there is only one other aspect in which the life fund could be the subject of taxation, namely, as in every other case, and in regard to every other body or person, on the annual produce of the fund; and, therefore, the words of Mr. Justice Day must be held to mean, as in subsequently dealing with the London Assurance Corporation the Inland Revenue authorities admitted they meant, that the claim of the

Crown to levy duty on the annual produce—*i.e.*, the interest income—of the life fund did not exist. Rightly or wrongly, at all events, that was the view held by many competent authorities, and when, emboldened by the judgment of the House of Lords that all profits were taxable, the Inland Revenue became more than usually aggressive, it was resolved by a number of companies to make common cause with the Clerical Office, which happened to be the first office attacked, and to support that society in resisting a claim for tax on interest which had escaped taxation at its source.

Now, the particular action of the Clerical and its associates has been attacked in more than one quarter, where I think wisdom followed rather than preceded the event, on the ground that it was illogical and at best only founded on "abstract justice." As is well known, the amount of interest which had escaped taxation and which formed the subject of the action— £165 only—was trivial to a degree by comparison with the sum taxed at the source, which amounted to £107,378. 13*s*. 8*d*. "What a paltry contention", said some. Mr. Justice Manisty, even, said, "Now what an extraordinary thing it would be, and " how contrary to all one's notions of sense would it be, that the " whole of the income, or the whole of the interest is liable to be " taxed, and has been taxed, and yet a mere sum like this, where " the interest was not taxed, is to escape taxation." But I would remind those who take this view that unconsciously they are comparing unlike things. If we had acquiesced in the taxation of the larger sum, then I admit that, if not actually illogical, it would have been petty and even childish to refuse to acquiesce in the taxation of the smaller sum, which I admit freely was of precisely the same nature as the larger one. But need I say here to this well-informed assembly, that so far from acquiescing, insurance offices have been accustomed to protest vigorously that in being so taxed they have been wronged? They could not help themselves in regard to the larger sum, but in regard to the smaller sum they thought they could, and this constitutes the essential difference between the two which some critics have overlooked. If while I am asleep and helpless, 19*s*. be abstracted from my pocket, leaving only 1*s*. behind, would it be illogical, or out of harmony with abstract justice, if I strenuously declined to accede to a demand made on me in my waking hours that I should surrender the remaining 1*s*., on the ground that, being in exactly the same position as the 19*s*., it might, and but for an

accident would, have been taken from me while helpless? I think
the want of logic would be on the side of those who should raise
such a contention.

As to what is " abstract justice"—whether it is something
more or less than justice—I confess to being unable to under-
stand, and I refer to it only because amongst the things which
have come out plainly from the litigation in which I have been
personally interested, the absence of justice, as plain men
understand justice, and still more of equity, in the conduct of the
State, has been one of the saddest. I do no one wrong when
I say that the business of the State is carried on on principles
which would speedily destroy the repute and deservedly wreck
the affairs of any body of traders. In saying this I desire
to cast no imputation on those who, in administering hard
law, only perform what they deem to be their duty. Mr. Dowell,
the Assistant-Solicitor of Inland Revenue, in his work on the
Income Tax Laws, after stating, and stating rightly, that
" nothing is more important to the department having the
" administration of taxes than to have before them, as a guide,
" a clear rule as to the construction to be put on the text of
" the law to be administered", says, " such a rule has at last
" been laid down by two judges of the highest authority in the
" ultimate Court of Appeal in the House of Lords in cases where
" the question was clearly raised, as follows: 'As I understand
" 'the principle of all fiscal legislation,' said Lord Cairns in
" Partington *v.* The Attorney-General, 'it is this: If a person
" ' sought to be taxed comes within the letter of the law he must
" ' be taxed, however great the hardship may appear to the
" ' judicial mind to be.   .   .   . In other words, *if there be*
" ' *admissible in any statute what is called an equitable con-*
" ' *struction, certainly such a construction is not admissible in*
" ' *a taxing statute, where you can simply adhere to the words*
" ' *of the statute.'*   Again—'No tax can be imposed on the
" ' subject', said Lord Blackburn in Coltness Iron Company *v.*
" Black, 'without words in an Act of Parliament clearly showing
" ' an intention to lay a burden on him. But when an intention
" ' is sufficiently shown it is, I think, vain to speculate on what
" ' would be the fairest and most equitable mode of levying that
" ' tax.   .   .   . The object is to grant a revenue at all events,
" ' even though a possible nearer approximation to equality may
" ' be sacrificed in order more easily and certainly to raise that
" ' revenue, and I think *the only safe rule is to look at the words*

" ' of the enactments and see what is the intention expressed by
" ' those words.' " I do not know how that strikes you, but it
seems to me to put into legal and sonorous phrase the ancient
maxim of the Quaker, addressed to his son on entering life:
" My son, get money; get it honestly if thou can, but if not—
get money."

How thoroughly this doctrine is understood and acted on in
the office of the Inland Revenue I have some reason to know from
what passed between the Clerical Office and the Board prior to
legal hostilities. It is of course manifest that if the view taken by
the Clerical Office and its associates of the result of the London
Assurance case had been the true one, it would have been in
equity, and by the case of the Clerical Office *v.* Carter it was hoped,
inferentially at least, to establish that it was in law, a necessary
consequence that tax over-paid must be returned; and it is in this
connection that the admission of the word " over-paid " into the
form of return in your hands, to which I have already drawn
attention, is so significant. Acting on this pretty obvious con-
sequence, the counter suggestion to the demand for more tax was
the natural one that the tax already over-paid should be refunded.

Nothing will be gained by taking you through the lengthened
correspondence which ensued. The result of it will be sufficiently
gathered from the statement that not only was this claim for
return of over-payments not recognized, but that, in addition
to legal action against the Clerical, it was followed by the
issue of instructions from the Board of Inland Revenue to
their surveyors to claim tax on interest or on profits, whichever
might be the larger. Where the authority for such an inter-
pretation of the law as it then stood, or as it now stands, is
to be found, it will be very difficult to discover. I regard it as
both disingenuous and unworthy. I do not propose, either, to
detail at any great length the course of the litigation in the
Clerical case which ensued. The point at issue was very simple.
In the Appeal Court it was thus stated by Mr. Finlay, Q.C., on
behalf of the appellants : " Where the profits of a company are
" derived from interest on investments, is the interest liable to be
" taxed as such, and are they liable to pay income tax upon
" interest received in full upon a portion of their investments,
" although they have already paid income tax, by deduction at
" the source, on an amount which exceeds their annual profits?"
It was answered with equal directness in both Courts. In the
Divisional Court, on a technical difference between the London

Assurance case and the Clerical case, the London Assurance case was put on one side, and in the words of Mr. Justice Charles, the Clerical case was "considered and decided upon apart from the authority of that case", while in the Appeal Court the London Assurance case was treated with even less consideration. In the Divisional Court, Mr. Justice Charles said that the intention of the Income Tax Acts seemed to be to tax interest of money in all cases as being a profit or gain, and that the language of the Acts makes no distinction between investments necessary to carry on the business and other investments. Of Mr. Justice Manisty's judgment it is not necessary to say much. It seems to have been grounded on the assumption—I need not say how entirely erroneous, except as regards their paid-up capital—that assurance societies are owners of property, the income of which they are free to enjoy and to dispose of according to their own pleasure.

In the Appeal Court, the fight was even fiercer than in the Divisional Court, but was more concentrated. It turned mainly on the interpretation of the 3rd clause in Schedule D of the 16th and 17th Vict., c. 34, s. 2. Schedule D is, of course, one only of what are called the charging schedules. The whole of them are as follows :

THE INCOME TAX ACT, 1853.

16 & 17 VICTORIA, CAP. 34.

An Act for granting to Her Majesty duties on profits arising from property, professions, trades, and offices.

*   *   *   *   *   *

Sec. 2.   .   .   .   .

Schedule (A).

For and in respect of the property in all lands, tenements, hereditaments, and heritages in the United Kingdom, and to be charged for every twenty shillings of the annual value thereof:

Schedule (B).

For and in respect of the occupation of all such lands, tenements, hereditaments, and heritages as aforesaid, and to be charged for every twenty shillings of the annual value thereof:

Schedule (C).

For and in respect of all profits arising from interest, annuities, dividends, and shares of annuities payable to any person, body politic or corporate, company or society, whether corporate or not corporate,

out of any public revenue,—and to be charged for every twenty
shillings of the annual amount thereof:

Schedule (D).

For and in respect of the annual profits or gains arising or
accruing to any person residing in the United Kingdom from any
kind of property whatever, whether situate in the United Kingdom
or elsewhere, and for and in respect of the annual profits or gains
arising or accruing to any person residing in the United Kingdom
from any profession, trade, employment or vocation, whether the
same shall be respectively carried on in the United Kingdom or
elsewhere, and to be charged for every twenty shillings of the annual
amount of such profits and gains:

And for and in respect of the annual profits or gains arising or
accruing to any person whatever, whether a subject of Her Majesty
or not, although not resident within the United Kingdom, from any
property whatever in the United Kingdom, or any profession, trade,
employment or vocation exercised within the United Kingdom,—and
to be charged for every twenty shillings of the annual amount of such
profits and gains:

And for and in respect of all interest of money, annuities, and
other annual profits and gains not charged by virtue of any of the
other schedules contained in this Act,—and to be charged for every
twenty shillings of the annual amount thereof:

Schedule (E).

For and in respect of every public office or employment of profit
and upon every annuity, pension, or stipend payable by Her Majesty
or out of the public revenue of the United Kingdom, except annuities
charged to the duties under the said Schedule (C),—and to be charged
for every twenty shillings of the annual amount thereof.

It will be seen that in Schedule A, which deals with the
ownership of lands, tenements, hereditaments, and heritages in
the United Kingdom, the charge is made in respect not of
profit but of property, *i.e.*, on the annual value thereof; and that
in Schedule B, which deals with the occupation of such lands, &c.,
the charge is made in respect not of profit but of occupation, *i.e.*,
as before, on the annual value thereof, the fact being that real
property has always been put on a different footing from any
other property. Friendly societies, and industrial and provident
societies, for example, which approach most nearly to insurance
societies, have their exemption from tax limited to the stocks,
dividends and interest belonging to them under Schedule C,
and to interest and other profits or gains under Schedule D.
According to Dowell in his *History of Taxation*, there has always
been a clear intention " to impose an additional charge upon

" the owners of land, ... justified on the ground
" of the superior interest acquired by landed property as com-
" pared with capital invested in any other property producing the
" same profit." So much with regard to Schedules A and B.
But when we come to Schedule C, we find a change in the
language. Though dealing with the fundholder—the person who
is, *par excellence*, the very *beau ideal* of a man of property—the
words of charge are, not in respect of the dividends or interest
received by him, but "*for and in respect of all profits* arising from
" interest, annuities, dividends, and share of annuities, payable to
" any person, body politic or corporate, company or society,
" whether corporate or not corporate, out of any public revenue."
" Profits ", you will observe, arising from interest : something
manifestly different from the interest out of which it arises.
Similarly, in Schedule D, but with clearer purpose, the words of
charge are "*for and in respect of the annual profits or gains*
" *arising or accruing from any kind of property whatever ;* and for
" and in respect of the annual profits or gains arising or accruing
" from any profession, trade, employment or vocation." This
clause deals with a resident in the United Kingdom. Then
there comes a clause dealing with a non-resident, the language
being practically identical ; and then follows the famous third
clause—the sweeping-up clause, as it has been called—over which
the fight took place, where there is a curious and, as it proved, a
fatal change of phrase. The words are :—" And for and in
" respect of *all interest of money,* annuities, and other annual
" profits or gains not charged by virtue of any of the other
" schedules contained in this Act."

The appellants contended that the words must be read as if the
words in Schedule C, which, if there appropriate, are still more
appropriate here, had been inserted :—" For and in respect of
all profits arising from interest", &c. Apart from the language
of the first clause of Schedule D, where the charge is generally
on the *annual profits or gains from any kind of property
whatever*—thus apparently changing under certain circumstances
the incidence of taxation in respect even of real property—the
words in this particular clause—*and other annual profits and gains*
—governed, it was maintained, the previous words " interest of
money ", and were necessary to make the whole read consistently
and coherently,—" And for and in respect of the annual profits
" and gains arising from all interest of money and annuities, and
" other annual profits and gains." As the words stand, either

the word "other" is superfluous, or they literally mean for and in respect of all *interest* of money, *interest* of annuities, and *interest* of other profits or gains, which is absurd.

The judges, with some hesitation and reluctance I think, came, however, to the conclusion that interest was a separate item of charge, and that interest was to be taxed wherever found—Lord Justice Fry even going so far in the course of the argument as to say, "that it might be that the Legislature had intended to tax interest specifically as it had taxed tobacco"; while the Solicitor-General was driven—under press of a suggestion from the Master of the Rolls that insurance companies could not carry on their business without having large funds—to make the following nonsensical statement: "They can carry on business " quite easily with smaller invested funds than they have. " The fact is, those funds, invested as they are invested, and " advertised to all the world as lying there to answer the possible " responsibilities of the business, afford, of course, the best " advertisement to the world that they can be trusted with these " risks." One would suppose that the learned gentleman was, as "Solicitor-General" in the American sense of the word, the advocate of an assessment society, for certainly these are not the words of truth and soberness which I think should have fallen from a law officer of the Crown. The result of this appeal—that all interest, *quâ* interest, is taxable—seems to me to be much wider than was intended, and even, perhaps, than was understood —much wider, at all events, than the concerns of insurance companies; and involves the startling assumption that bankers and money dealers of all descriptions, who retain for their own profit only a small portion of the interest they receive, paying out the larger portion to their depositors or creditors, are liable for tax on all they receive, whether they retain it or not. Of this judgment I am entitled to say that it is, at all events, contrary to the usage of every-day life.

Before, however, discussing the judgment on its merits, as— after the admission of the Master of the Rolls that, " all arguments " about the income tax are puzzling, enough nearly to puzzle one's " head off "—I may, without disrespect, be allowed to do, I would briefly glance at one or two other judgments which have been lately given, so that in taking stock of our present position we may have all the necessary elements before us. The first of these cases is the case of Colquhoun *v.* Brooks, where it was decided that interest derived from foreign investment—if it remained abroad and was

re-invested there—was not liable to tax here. How that may affect some of our great northern insurance companies, who have large funds invested abroad, I am not in a position to say. The next case at first sight does not seem to be germane to our enquiry, but, as I think you will see later on, it is very much so: I refer to the case of Goslings and Sharpe *v.* Blake. In the Divisional Court it had been ruled, on the authority of a very ancient case, known as Bebb *r.* Bunny, that interest received *de die in diem* was in the same nature as interest received *de anno in annum.* But the judges of the Divisional Court came to this conclusion reluctantly; and when the case came into the Appeal Court, this view was over-ruled, and it has now been declared that in the case of interest payable for less than a year the tax cannot be deducted. The effect of that judgment you will see later on. The third case is Colquhoun *r.* Heddon, which I may remind you is a denial, on what seems to be a protectionist plea, of the right to exemption from tax in respect of premiums which have been paid to an American insurance company. There is also the case of the Nizam State Railway, which I only mention because it raises sharply that unpleasant question that all the proceeds of an annuity, principal and interest, are liable to taxation. Then there is the most recent case—the Gresham Office *v.* Styles— which is, of course, very fresh in your minds, and of which, as it is *sub judice,* I will only now say that in my opinion it supports practically every contention sought to be established in the remarks which will follow. In an interesting article in the *Juridical Review* for July, 1889, since reprinted as a pamphlet, Mr. McCandlish has with great lucidity brought together some other decisions of the Courts useful for our purpose. He quotes the case of Smiles *v.* The Australasian Mortgage and Agency Company, in which a decision favourable to the company was given as the result of an argument running on all-fours with that of the insurance companies. He describes the case in this way: " It was held that the company was not liable to be assessed on " interest received from foreign and colonial securities, except in " so far as it entered into their profits and gains under the first " case of Schedule D. A distinction was drawn between companies " whose business it is to make such investments and companies " transacting other business, but making investments as an " incident of such other business. The judges recognized the " principle that such a company might earn interest from its loans " and at the same time suffer loss from other parts of its business;

" that it was entitled to set the loss against the profits, and that
" consequently its assessable profits could only be reckoned under
" the first case of Schedule D, and, in the words of the Lord
" President, that it is impossible to distinguish one part of the
" business from another, and to hold that one part falls under the
" fourth case and the other under the first."   "This", says
Mr. McCandlish, " is very much what the insurance offices claim
" to be their case.   They invest the premiums they receive as
" a necessary incident of their insurance business, and (quoting
" Lord Shand) entirely ancillary to that business."   I refer to
these cases not so much for their direct bearing as for the purpose
of strengthening the argument I shall shortly address to you
based on the usages of trade.

More immediately important are the cases of the Scottish
Union and National Company and that of the New York Life
Office *v.* Styles.   The first, however, is interesting, not so much
positively as negatively, in that a successful resistance was offered
to an attempt to make the balance of receipts over disbursements
the measure of profit, one of the reasons given for this desired
change being that the valuations of life assurance companies were
designedly made on principles tending to withdraw profit from
observation.   In this room, I imagine, a not uncommon object of
a valuation would be stated in quite another fashion.   The case
is further interesting as showing the aggressiveness of the depart-
ment, and the need for readiness to resist attempts to disturb
settlements, however solemnly arrived at.   And in this connection
I think the case of Colquhoun *v.* Heddon comes in.   That is a
denial of the right to deduct the tax on premiums paid to an
American office.   I do not think that the position of that question
has been quite rightly understood.   I do not pretend to say that
on the strict letter of the clause which has governed the decision
of the judges, the judges were wrong.   It would seem, at first
sight, as though certain companies were indicated as the companies
alone in which a man might make an insurance with the certainty
of getting back the tax on his premiums.   But I rather hold to
the view that that was not intended as a restrictive statute, but
as pointing out the class of offices in which a man might insure,
to make it certain that there should be no bogus insurance.
That section of the Act has been since enlarged twice—by the
introduction, first, of friendly societies, as offices in which a man
might insure, and be entitled to get back the tax on premiums;
and secondly, the Commissioners for the reduction of the National

Debt, because it is not only premiums on life insurance, but premiums on deferred annuities which are capable of going in diminution of the amount of tax paid by the individual. But I base my notion that the judgment is not really right on this ground. It is very commonly supposed that the Act of 1853 created the right to deduct the tax on premiums of insurance, but that is not so. It was in the Act of 1798. In that Act—Pitt's Act—there are two elaborate schedules, one of charges and one of deductions, and the final deduction allowed to be made under that Act was a general deduction of premiums on insurance on life. The privilege granted by Pitt's Act was continued in the Act of 1803, and also in the Act of 1806, with this difference—that it was then limited to persons whose incomes were under £150. These Acts are not, of course, in force now. I speak of them to show you that the Act of 1853 did not create the exemption, and that it may be fairly held that the intention of the latter Act was not to restrict the companies with which a man might insure, but rather to point out the sort of companies with which assurances might be made.

The second case—that of the New York *v.* Styles, in the House of Lords—has a wider as well as a more permanent importance. Three law lords, differing from five judges in the Courts below and from two of their own body—*i.e.*, three against seven in all, and illustrating in that particular the proverbial uncertainty of judge-made law—have, so far as purely mutual offices are concerned, reversed the decision in the London Assurance case, and declared that what even mutual offices call profits are, in reality, not profits at all. Even if this judgment remain undisturbed by legislation, it rather adds a new inconsistency to the income-tax decisions than ameliorates the position of assurance companies; for, in spite of the London Assurance case, the Board of Inland Revenue has never attempted to put in force the one decision which, until the New York case, seemed to be certain of all assurance societies, and is still undoubted as regards the majority of them—of all proprietary companies, that is to say—that profits are taxable.

For the reason which made me welcome the decision of the House of Lords in the London Assurance case, I regret the more recent judgment of that illustrious body. And here comes in a very convenient opportunity for saying—as I promised in the earlier part of my remarks to do—why I approve of the House of Lords judgment in the London Assurance case about profits. In

defending that judgment, I may seem to put myself in conflict with the majority of the law lords who decided the New York case, as well as with the minority who thought with them in the London Assurance case. If the matter in question were one of pure law, I should shrink from a position which would argue great temerity, if not great presumption; and the more so because I should have been antecedently prepared to accept the dictum of Lord Herschell, in whose judgments I recognize a breadth of view and an alertness of intelligence not always exhibited in high places. But, in truth, the point at issue is rather economical than legal. Let me first read to you what Lord Herschell says. " Let us", he says, " see how the so-called profit arises. It is " due to the premiums which the members are required to pay " being in excess of what is necessary to provide for the requisite " payments to be made upon the deaths of members, and not " being, as the case states they were intended to be, commensurate " therewith. This may result either from the contributions having, " owing to an erroneous estimate or over-caution, been originally " fixed at a higher rate than was necessary, or from the death-rate " being lower than was anticipated. Can it be properly said " that, under these circumstances, the association of mutual " insurers has earned a profit? The members contribute for a " common object to a fund which is their common property; it " turns out that they have contributed more than is needed, and, " therefore, more than ought to have been contributed by them " for this object, and accordingly their next contribution is " reduced by an amount equal to their proportion of this excess. " I am at a loss to see how this can be considered as a 'profit' " arising or accruing to them from a trade or vocation which they " carry on. It is true the alternative is allowed them of leaving " the excess in the common fund, and so increasing their repre- " sentatives' claim upon it in case of death, but I cannot think " that this makes any difference."

Now you will see that Lord Herschell rests his contention on three things, the truth in regard to all of which he assumes— 1st. That the premium charged was excessive from erroneous estimate; 2nd. That it was also excessive from over-caution; and 3rd. That the death-rate proved lower than was anticipated. It will be seen at once that the third of these items is, in reality, only a branch of the first—being one of the erroneous estimates that lead to the excessive premium. I had prepared some detailed reasons for differing from Lord Herschell on each of these assumptions;

but I shall save your time and pass them over, because they are
not essential from my point of view.

For the purposes of my argument I can afford to concede the
whole of Lord Herschell's premises. In that case I fall back on
analogy and ask in what respect does the *modus operandi* of an
assurance company differ from that of a trader? The trader,
equally with the assurance company, has his pure premium—*i.e.*,
the prime cost of the article he sells; equally, also, he makes
estimates, usually erroneous and in his own favour, as to his
expenses—*i.e.*, the cost of distribution; he, too, has his expecta-
tion of mortality, which he rarely under-estimates—though he
calls it delay in realization, and depreciation of value by damage
of material or loss of fashion; and, finally, he, like the assurance
company, adds an arbitrary percentage—his loading, in fact—for
profit. Now, supposing, in such a case, that the trader were to
undertake to return to his customers a fair proportion of all the
items (his own loading for profit excepted) charged in excess on
the prime cost; to make, in other words, a return of over-
payments arising from erroneous estimate or excess of caution,
would not such over-payments while in his hands be taxable
profits, and must not tax be deducted from them before going
into the hands of his customers? Assuredly so, according to the
doctrine laid down in the famous case of the Mersey Docks *v.*
Lucas, where it was held that the profits or gains of a concern,
for purposes of Income Tax, meant the earnings of the concern
after deducting the expenses of earning and obtaining, before you
came to the application of them. And, personally, I deprecate
any other view being taken of the profits of an assurance company.
I thought at the time, and still think, that the final judgment in
the London Assurance case furnished the best basis for an
equitable settlement; and I agree with a writer in the *Post
Magazine*, when commenting on Lord Herschell's judgment, that
" the wiser plan would be for the whole of the offices, proprietary
" and mutual alike, to join forces and elect, if possible, to be
" taxed on the common ground of profits. . . . On the
" whole there is more reason to regard the members of an
" ordinary mutual office as associated adventurers, with Lord
" Halsbury, than as communists, with Lord Herschell."

I am now in a position to indicate my grounds of objection to
the decision in the Clerical case, which, as declaring all interest
to be taxable, is by far the most important as—again speaking
economically—I think it the most erroneous of all the recent

Income-Tax judgments. It can hardly have been left to these latter days to make what, if interest be really taxable wherever found, would be in the nature of a discovery; and the fact that no such pretension has ever been set up by the authorities, and that interest in a thousand hands escapes taxation with their full knowledge and acquiescence, is in itself almost proof enough that the judgment is somewhere wanting in soundness. Much of the confusion which has arisen in dealing with Assurance Companies has been caused by their operations having been treated as more or less of a mystery, and altogether differing from those of trading bodies. The position I take up is that we are trading bodies and ought so to be treated. As regards proprietary companies, it is no longer open to doubt that they are trading companies. The judgment of the House of Lords in the New York case, equally with that in the London Assurance case, starts from that premiss. Then what remains to be done is to claim community of treatment with trading bodies. Counsel in the Clerical case insisted on this strenuously, and the stress of the argument was manifestly felt by the judges, though ultimately they escaped from it by taking refuge in a technicality, and declared that the Act ordained for taxation, while in the hands of an assurance company, a commodity which, when in the hands of a money dealer at all events, commercial usage exempted from taxation. The case of a bank or financial association—a dealer in money—was used by counsel as the basis of comparison. Now what happens to a dealer in money in this connection? Does he pay on all his interest receipts? Assuredly not. A dealer in money, whether a banker or discounter, in addition to employing his own money, on the one hand borrows money and pays interest on it, and on the other hand lends money and takes interest for it. The tax which he pays is not on his total interest receipts, but on the balance which remains in his hands after deducting his losses and expenses—in fact, his enjoyable income. To make my meaning clearer, I have had printed in the paper already in your hands the figures (altered as to amounts but not as to proportion) which formed the basis of an actual return made to Income Tax by a large dealer in money, which were placed at my disposal for the purposes of the Clerical case. Though we failed to get the figures before the Court, they will, I think, be found highly instructive here:

FIGURES forming Basis of an Actual Return of Net Profit liable to
Income Tax, made by a Money Dealer.

	£
Full Receipts from Interest on Investments and Loans, and from Discounts, &c.	20,000
Less Interest Allowed, Re-Discounts, &c.	13,000
	7,000
Less Expenses	1,000
	6,000
Less Interest on which the Tax has been paid	2,000
Balance	£4,000

Being Net Profits only, less the Interest on Investments already taxed.

In the first place it will be seen that the interest actually
brought into account for the purposes of taxation is little more
than a third only of the amount received, and that that third,
again, is subject to a deduction for expenses before the profit is
arrived at. If the Clerical judgment be right, tax should have
been paid on the £20,000, instead of, as it was ultimately, on
£4,000 only. It cannot be argued that tax should have been
paid on the £20,000, and deducted on the £13,000, for—such is
the departmental inconsistency—the Inland Revenue authorities
have in the case of "Goslings and Sharpe *v*. Blake" successfully
opposed a claim of the bankers to do this very thing, on the
ground that sec. 102 of the Act of 5 & 6 Vict., and sec. 40 of the
16 & 17 Vict., which makes it obligatory on mortgagees, for
example, to allow deduction of tax from interest paid by their
mortgagors, applies only to interest in the nature of yearly
interest of money—interest received *de anno in annum*, and not
de die in diem. But, even had the judgment in Gosling's case
been different, and in the result it had been held that bankers
ought to deduct tax on all interest paid by them and allow
tax on all interest received by them—in connection with
loans and discounts for days or weeks it might be, thus
creating a disturbance of trade in itself almost intolerable—
though no injustice would in the result be done them from that
point of view, injustice would arise from another point of view,
for then the undoubted right to deduct expenses before declaring
taxable profits would be gone. This would be a clear denial of
justice. Assuming, what I have reason to believe to have been the
actual case, that the £2,000 of interest included in the gross
receipts from interest and subsequently deducted was in respect
of capital, it will be seen that £18,000, *i.e.*, $\frac{9}{10}$ths of the whole

interest earnings, arose from money used in the business which belonged to others; and the close analogy of this case with that of an assurance company—whose funds consist in small degree only of its own capital and in large degree of its life fund, which is the money of other people—is clear and complete.   In both cases alike the interest on the capital is taxable, and in both cases alike the interest from the funds borrowed in the one case, and held in trust in the other, should be taxable only when, and in the degree, in which it becomes profit.   Though I have no real need of the case for the purposes of my argument, I would point out that the Australasian case to which I have already referred carries the argument a stage further, for there the interest from capital, even, was held to be not all profit, but if used in trade was profit only after deduction of losses and expenses.

We have now reached a convenient point for testing the combined judgments in the income tax cases—which, I again remind you, mean for the bulk of the assurance societies that all interest and all profits are liable to tax.   Though it does not affect my abstract argument, I would say, to show that I have not overlooked the point, that if the taxable profits of an assurance company come to be enquired into, the judges in the Appeal Court in the Clerical case intimated pretty clearly that so much of them as, in the shape of interest, had already paid tax must be left out of the account.   Subject to this then, I say that whereas assurance societies are subject to be taxed on their interest receipts, though such receipts are in excess of their profits, and are, besides, liable to tax on their profits also, other trading bodies dealing with the moneys of other people and even, to some extent at all events, when dealing with their own money, are taxed on their profits only, and not on their interest at all, unless it be profit. These two positions are irreconcilable, and, if irreconcilable, then either ordinary trading bodies are unduly favoured, or assurance societies are unduly taxed.   The first of these propositions being in the nature of a *reductio ad absurdum*, the latter must be right, and so I hold it to be.

Before establishing this proposition by way of argument, let me first point out what is the position as determined by practice. It will probably surprise most of you to learn that in spite of the overthrow (as the result of the Clerical case) of the interest portion of the London Assurance case, the Inland Revenue authorities still assess the London Assurance Corporation in (practically) the same manner as they did after the decision in the Divisional Court—

when, that is to say, it was held on both sides that the interest on
the life fund was not taxable. This is inconsistency No. 4. I
would further point out the fact that in the case of life offices
simpliciter, to whom, as far as possible, has been denied the
benefit of the London Assurance judgment, the bulk of the
interest from the life fund is, in fact if not in form, similarly
exempted. Using the figures of the Clerical case, it is manifest
that so long as the £74,000 of profits is not taxed by direct
assessment, the interest of the life fund is, to that extent, in
reality exempted from taxation. And to carry the matter one
step further, it scarcely admits of doubt that if the operations of
a life office resulted, as usually do the operations of a composite
company, in profits exceeding the interest from the life fund, the
whole of such interest could be deducted from the profits, as it is
still deducted by the London Assurance Corporation, thus, in
effect, exempting the whole of the life fund from tax, as
Mr. Justice Day declared it ought to be exempted.

How comes it, then, that a position admitted partially in the
case of all assurance societies, and wholly in the case of others,
should be contested, and should be successfully contested, as a
matter of law? The position is consistent with what happens in
the case of money dealers, as I have shown. It is equally con-
sistent with the judgment in the Australasian Mortgage case,
where trade losses—and not merely interest paid out as in the
case of the money dealer—were allowed to be deducted from
interest on its loans, although such loans might have been made
with its own property. Now, is there anything in the nature of
the business of an assurance society which, for purposes of tax,
rightly distinguishes it from other trading bodies? I confidently
answer in the negative. What is the principle of the Income Tax?
Dowell, in his work on the *Income Tax Laws*, says—he is speaking
primarily of Schedules A and B, though the principle is obviously
general—" Originally the occupier pays both taxes where they are
" chargeable, and subsequently, if a tenant, deducts the tax under
" Schedule A from the next payment of rent to his landlord.
" Landowners, in their turn, deduct a proportionate amount of
" the tax on payment of any rent-charge, quit-rent, or annuity,
" charged on the land, and on payment of interest to any
" mortgagee. Income is thus traced to its source and charged
" where it arises, and the burden is subsequently distributed, so as
" to fall on the persons who are in the enjoyment of the income."
Though taxed at its source, he says, the burden ultimately falls

on the person enjoying the income. Does, then, an assurance society enjoy the whole of its interest income? It receives the whole of it as does a banker, but, also like a banker, passes on a portion of it to third persons. If it receives 4 per-cent, and needs 3 per-cent to meet its obligations, it returns that 3 per-cent in effect, if not in form—passing it on to the credit of its assured, who, for this purpose, are its creditors, just as much as are depositors the creditors of a banker. The remaining 1 per-cent it retains, letting it fall into its profits, where it is rightly taxable.

I am quite conscious that there is a weak spot in this analogy, and I do not desire to evade the difficulty. The interest which I have assumed to be passed to the creditor assured does not come into his hands in the same direct way as does the interest passed by a banker to his creditor depositor. As a consequence, it is not directly taxable in the hands of the assured as it is in the hands of the depositor. As regards that I would say two things—1st. That if a remedy be needed for this, it ought not to take the form of taxing the debtor, who is under a contract which needs the whole of the interest in question to meet his engagements. Such a procedure might easily result in failure to meet the engagement. This fact was recognized and strongly emphasized by Lord Herschell in the judgment in the New York case. "It is of the very essence of such an enterprise", he said, "that a portion of " the income should from time to time be invested in order to " create and maintain a fund capable of meeting the liabilities that " have been and are being created. To treat and distribute as " profits all the income in excess of the costs, and expenses of " receipt and collection, would soon land such an undertaking " in hopeless insolvency." If the valuation rate and rate earned be identical, how is an office to carry on its business if mulcted in a tax which, in time of war, might easily be 2s. in the £, or (should history repeat itself, and we go back to the days of the old annual Land Tax, which Pitt's Income Tax superseded) even 4s. in the £, or even possibly ¼ per-cent. And I would say, 2nd, That in the case of an assurance company it would be quite in accordance with the avowed objects of the Legislature to allow such interest to be exempt from tax as a matter of right. By law, ⅙th of a man's income is exempted from tax, if used in payment of life assurance premiums on assurances on his own or his wife's life or in payment of a deferred annuity for the benefit of his wife or children. Mr. Bailey used this argument, and it was employed

with great force and effect in the Scotch Courts in the Scottish
Union and National case. Mr. McCandlish thus states it: " Into
" all the finance of a life office interest enters very largely. It
" was stated in one of the cases before the Scotch Court, as an
" illustration, that a man of a certain age might secure a policy
" for £1,000, payable at his death, by making an immediate
" payment of £450. To enable the company to pay £1,000 in
" return for £450, they must obviously in the course of his life
" convert the £450 into £1,000 by investing it at compound
" interest." Now, as between the office and the policyholder, the
interest earned by the premium is as much a part of the con-
sideration for the assurance as is that which we technically call
premium. In the absence of interest, or on the basis of a lower
rate of interest, the premium (technically so-called) becomes more,
and in that shape, however large it might be—subject only to its
not being, as I imagine it rarely is, more than $\frac{1}{6}$th of the man's
income—it would be exempt from tax. Why then should not
the interest which, *pro tanto*, makes the premium smaller, be
similarly exempt ? The two together, taken on an average over
a number of years, would probably never exceed the $\frac{1}{6}$th of the
assured's income, even individually, and certainly not collectively.
But another consideration arises out of this. Let us take
Mr. McCandlish's figures, where a sum of £450 is assumed to be
paid down for a policy of £1,000. How does this differ from
the purchase of an ordinary reversion ? If I go into the market
and pay £450 for the right to receive £1,000 at the death of a
life-tenant—buy a reversion, in fact—of what is the difference
between these two sums composed ? That which enables my
£450 to grow into £1,000 is interest. Is such interest—which
here amounts to £550, it will be observed—taxed ? And if not,
why is it taxed when the reversion bought is a policy of assurance ?
And thus by a variety of processes we arrive—I hope with
some strictness and some conclusiveness—at the conclusion that
interest, *quâ* interest, on the life funds of an assurance company,
while in the hands of an assurance company, ought not to be
taxed ; and I hope also that I shall have carried with me a strong
concensus of opinion that the true basis of taxation should be
profits—those, I mean, which we ourselves call profits, however
they may have got into that category.

With this conclusion, my part in this debate might properly
be allowed to end. But, if I have not already too much wearied
you, I would ask to be allowed to submit one further—one last

point for your consideration. It is this : suppose that my
contention of to-night were to be admitted, and the Government
were to agree to make profits the basis of taxation, would the
settlement be a permanent one? I am not myself quite sure that
it would. This doubt arises out of the consideration that there is
a tendency showing itself in modern practice to work on narrower
lines, for the express purpose of diminishing or extinguishing
bonuses or profits. With such a tendency I am in profound
sympathy. More than twenty years ago I wrote, in the *Journal
of the Institute* (*J.I.A.*, xx, 54):

> "Bonuses, we are convinced, have worked a moral mischief, by
> withdrawing the attention of the assured from the safety of his office,
> on which it should be chiefly fixed, and concentrating it on the
> accident of gain. .   .   .   . We are inclined to think that nothing
> would purify and benefit the cause of assurance so much as a well-
> considered reduction of the premiums."

And if the economic millennium here indicated should ever be
reached, would the State be satisfied with its bargain to make
profits alone the basis of taxation, if the profits themselves were
to largely disappear? I fear not.

I was originally opposed to the attempt made in the London
Assurance case to get both interest on the life fund and profits
paid to policyholders out of the grip of the tax-gatherer, as I
felt that success would probably be followed by worse things.
Mr. Bailey himself quotes—I presume with approval—the dictum
of Adam Smith, "that the subjects of every State ought to
" contribute towards the support of the Government as nearly as
" possible in proportion to their respective abilities, on account of
" the protection they enjoy from the State." Can it be expected
—ought it even to be desired—that assurance companies, or
rather, as I would prefer to put it, the associated adventurers who
form an assurance society, whether proprietary or mutual, should
be allowed to escape the cost of the protection which they derive
from good government so directly and so largely? Well then,
what is to happen if profits by their diminution or disappearance
cease to provide a fitting measure of taxation? My answer is—
the suggestion is, perhaps, at the present crude—that a new
measure of taxation might be substituted, to take the form of a
percentage deduction on whatever is paid under the policy,
whether as bonus or surrender-value or claim. In this way, the
ultimate beneficiary—the creditor—the true owner of the profit—
and not the assurance company, who is a mere debtor, or agent, or

trustee—-whichever term may be preferred—would be, as he should be, the person to bear the tax. Such a tax would, at least, be in close accord with the fiscal canon which, while exempting the raw material from impost, and leaving the processes of manufacture unfettered, would fix the tax on the manufactured article.

DISCUSSION.

The PRESIDENT (Mr. W. Sutton, M.A.) having invited discussion,

Mr. A. H. BAILEY said, that as the corporation with which he was connected (London Assurance Corporation) had been mentioned not infrequently by Mr. Newbatt, perhaps he might be expected to offer a few remarks on this occasion. To his mind, this was an eminently suitable subject for the Institute, and, as one of its oldest members, he recollected something like a generation ago it was thought that all actuaries ought to have an opinion with regard to the income tax. Having had some experience in this particular matter in the law courts, he had come to the conclusion that, as it was the work of the judges to interpret Acts of Parliament—and in the judgment of the Master of the Rolls there was considerable difficulty in doing that when the income tax was the subject—any influence the members of the Institute could exert ought to be with the Legislature. He was exceedingly pleased to read Mr. Bartley's long letter in the newspapers, in which he incidentally mentioned how hardly the tax pressed on insurance companies. So high an authority as Mr. Gladstone had said that the Act was so full of inconsistencies that the only thing now left was to get rid of it, but as it had lasted since 1842, he thought he must be a very sanguine man who believed that the income tax would disappear during the lifetime of any man in the room. All they could do was to improve its working. The object of the officials seemed to be to extract, *per pas et nefas*, as much money out of the subject as they could. What was wanted was a more just administration of the tax according to the spirit of the Act of Parliament. One of the injustices which they in particular felt was a confusion between the terms "income" and "profits." There was no doubt that an individual was fairly taxed on the income he made from his profession and his investments, and in companies generally the income they made from the reserves and the like was fairly profits, but in life insurance companies the case was very different. They had to do two things. They granted an insurance which tended to equalize life, and they also undertook to invest the premiums at compound interest. There was no more justice in taxing the income from interest, than the income from premiums, therefore, what was wanted was a clause to exempt the interest on life insurance investments, just as there was a clause exempting the investments of hospitals, and other charities. He did not mean that they could avoid the deduction of the tax at its source, but they should have the means of recovering it. This interest was not a profit or gain: it was wanted

to meet the ordinary liabilities.   The recent case about annuities appeared to him to be a different matter altogether.   To his mind, their position, in reference to life annuities, was somewhat analogous to that of the Bank of England and the fundholders.   They acted simply as agents for the Government.   What tax they deducted from the annuities should be a mere matter of account, and nothing more. It had nothing to do with the return made under Schedule D.   Then, about the deduction of life premiums from income in making the return for the tax.   In the last case—the New York Life—it was said to be an encouragement to thrift, but if that took the form of insuring in a foreign company, it was not to be encouraged at all. The absurdity of the matter was the supposition that premiums of insurance were always matters of thrift.   Within the last few years, there had been four or five private Acts of Parliament passed whereby sums of money borrowed on life interests were allowed to be charged on the fee of the estates, and the insurances originally effected to secure the repayment of those loans, were to be kept up. He supposed that several gentlemen present had signed certificates to enable deductions of income tax to be made in such cases.   It was difficult to see how anyone could suppose that those premiums were an encouragement to thrift.   They all knew that life insurances were effected for other purposes, and therefore this deduction should be abolished.   What was wanted was a tax varying with the sources of income.   In the last session of Parliament, the Chancellor of the Exchequer made a remark with reference to the increased death duties, when it was observed how hardly these fell upon the owners of personal property as compared with real estate.   A man who succeeded to real estate was assessed on the value of an annuity during his life, whereas, if the will directed it to be sold, he was taxed on the *corpus*. Mr. Goschen said the owner of real property paid on the gross rental of his estate, but there were always material deductions for repairs, &c., and therefore, if the income tax was 6*d*. in the £ to a fund-owner, it would be 8*d*. to the landowner.   He knew from his own experience that the death duties sometimes pressed unfairly and unjustly, because, in the case of a sickly family, where the deaths recurred very frequently, the payments were more frequent and when they were least able to bear them.   What was wanted was to carry out the maxim of Adam Smith, that every man should pay according to his ability.   The late Lord Addington had endeavoured, without much success, to force upon public attention the need that a different rate should be levied upon the different schedules.   " B " was the farmer's schedule, but why a farmer could not determine his profits as well as a manufacturer, he could not tell.   The fundholder got his income clear, and had 6*d*. deducted from it, but a landowner had to submit to considerable deductions, and he therefore thought the tax on Schedule A should be lower than on Schedule C.   There was another anomaly that he thought might be remedied.   Schedule A referred not only to land, but also to houses.   He himself occupied a house on a repairing lease.   He paid no premium for the lease, but when the representative of the Chancellor of the Exchequer sent him a " Demand Note", his demand was not on the rent that was paid, but on £25 more, and the explanation given was that it was on the

gross rental. How that could be profit or income was wholly beyond his comprehension. If the tax was to be perpetual, the recipient of weekly wages also should pay. One insurance company collected over three millions annually, mainly from the recipients of weekly wages, in premiums which averaged 2*d.* a week. If that could be done by one insurance company, what an enormous income there must be from that source as a whole, and why should the recipients of weekly wages be exempted? Of course, the system of collection must be altered, but when the expenses were all paid, the result would be very considerable.

Mr. T. G. ACKLAND, referring to the case in which his society had lately been interested (*Gresham Life Assurance Society* v. *Styles*), explained that the surveyor, in assessing the society to income tax, took first of all the amount of the profit arising in the triennium as actuarially ascertained, and added thereto the amount of the annuity payments made by the society during the triennium. He then allowed a deduction, in respect of the interest which had been charged with tax at its source over the same period, and the difference, being the amount upon which the society was deemed to be liable to tax for the three years, was divided by three to obtain the amount of the annual assessment. The effect of this was that the society, having already paid tax by deduction at its source upon an amount in excess of their ascertained profits, had to pay on a further annual sum of about £76,000.

In the case stated for the consideration of the Court, the surveyor on behalf of the Crown contended, "that the exception of the annuities " and the income tax paid by the society was by way of deduction from " the gross income of the society in arriving at the amount on which " income tax should be charged, and that the same was not allowable " under 5 and 6 Vic., cap. 35, sees. 100 and 159, and 16 and 17 Vic., " cap. 34, sec. 40; and further, that the society was entitled to deduct, " and should deduct, the amount of the duty on payment of each " annuity under 5 and 6 Vic., cap. 35, sec. 102, and 16 and 17 Vic., " cap. 34, sec. 40."

The society, on the other hand, contended: " First, that they had " treated, and were entitled to treat, the payment of annuities in the " same manner as the payment of sums payable on the death of the " assured, and contended that the same cannot be treated as a " deduction from or be brought into account so as to increase profits, " or be treated as paid out of profits or gains either within the meaning " of Schedule D, 4th rule, 1st case, sec. 100 and sec. 102 of the 5 and " 6 Vic., cap. 35, or as otherwise brought into charge by virtue of " such Act, or as sums on which income tax is payable by the society " under any of the provisions of the 16 and 17 Vic., cap. 34; and, " Secondly, that the amount of the society's profits and gains for the " triennium being £93,056. 16*s.* 8*d.*, and they having paid tax on " £120,722 as aforesaid, they were not liable for any further " assessment."

The questions for the consideration of the court were—

" 1st. Whether the society is liable to be assessed in respect of the amount paid by them for annuities, or any part thereof."

" 2nd. Whether the society is, in any event, liable to be assessed

" for any sum in excess of the amount on which tax has already been
" paid, by way of deduction at its source, as aforesaid."

Turning to the Income Tax Acts, it was provided by sec. 102 of
1842 Act (5 and 6 Vic., cap. 35), "That upon all annuities, yearly
" interest of money, or other annual payments   .   .   .   .   there
" shall be charged" so much, " provided that *in every case where the
" same shall be payable out of profits or gains brought into charge
" by virtue of this Act*, no assessment shall be made upon the person
" entitled to such annuity, interest, or other annual payment, but the
" whole of such profits or gains shall be charged with duty on
" the person liable to such annual payment without distinguishing
" such annual payment, and the person so liable to make such annual
" payment   .   .   .   .   shall be authorized to deduct out of such
" annual payment   .   .   .   .   and the person to whom such payment
" liable to deduction is to be made shall allow such deduction."   The
fourth rule under Schedule D, in sec. 100 of the same Act, provides
that " in estimating the amount of the profits and gains arising as
" aforesaid, no deduction shall be made on account of any annual
" interest or any annuity or other annual payment *payable out of
" such profits or gains.*"   In the later Act of 1853 (16 and 17 Vic.,
cap. 34) it is provided in sec. 40 that " every person who shall be
" liable to the payment of any rent, or any yearly interest of money,
" or any annuity, or other annual payment   .   .   .   .   *shall be
" entitled and is hereby authorized* on making such payment, to
" deduct and retain thereout" the amount of the tax   .   .   .   .
" and the person to whom such payment as aforesaid is to be made
" shall allow such deduction."   It therefore appeared, speaking
generally, that while there was some evidence of an intention in
these enactments to tax annuities as such, the tax was to be assessed
upon the payer of the annuity in a particular case only, namely,
where the annuity was payable "out of profits and gains brought
into charge" by the original Act.   The 40th section of the 1853 Act
was only permissive and not compulsory, and it did not appear that
there was in either Act any provision that the payer of an annuity
was to be charged with the tax, or was bound to make the deduction
in paying over the annuity, except in the case where such annuity
was payable out of profits or gains.   It might, therefore, be fairly
assumed that it was the intention of the Legislature that in the case
of all other annuities the tax was to be assessed upon the receiver of
the annuity or the annuitant.   It was noteworthy in this connection
that the Inland Revenue authorities had, in a recent enactment, "The
Customs and Inland Revenue Act, 1888" (51 Vic., cap. 8) inserted
a provision in sec. 24, subsection 3, that "upon payment of any
" interest of money or annuities charged with income tax under
" Schedule D, and not payable, or not wholly payable, out of profits
" or gains brought into charge to such tax, the person by or through
" whom such interest or annuities shall be paid shall deduct thereout
" the rate of income tax in force at the time of such payment, and
" shall forthwith render an account to the Commissioners of Inland
" Revenue of the amount so deducted, or of the amount deducted out
" of so much of the interest or annuities as is not paid out of profits
" or gains brought into charge, as the case may be; and such amount

" shall be a debt from such person to Her Majesty, and recoverable
" as such accordingly."

This clause would seem to have an important bearing on the
future assessment of annuity payments to income tax, but did not
affect the particular case of Gresham *v.* Styles, which raised the
question of a prior assessment; and the main question in that case
seemed to be, are the annuity payments of the society paid "out of
profits and gains" in the sense of the earlier Acts? It was con-
tended on behalf of the society that they could not be held to be so
payable, being obligations under contracts specifically entered into in
consideration of certain purchase-moneys received, and payable out of
a special annuity fund made up of such purchase-moneys and interest
thereon, and that though profits might be made from the annuity
transactions, the payments themselves were clearly not paid out of
profits. It was further contended by Sir Horace Davey on behalf of
the society, that the annuities contemplated in the Acts as payable
out of profits and gains were such special annuities as, for example,
an allowance of £500 a year to a retiring partner in a firm of
solicitors, payable, with possible continuance to his widow, out of the
profits of the business. In the case of Gresham *v.* Styles the
annuities were largely contracted abroad, and the question arose
how far such foreign annuitants were liable to British income tax,
and whether the society could, in such cases, properly deduct the tax,
bearing in mind also that in many cases the annuity was already
subject to a considerable deduction in respect of income tax in the
foreign country. Also, in the case where the annuity represented the
foreigner's sole source of British income and was within the limits of
exemption: how (in the event of deduction of tax by the society) was
the annuitant to recover from the British Crown?

The decision of the judges was, however, adverse to the society,
and they were very clearly of opinion that the annuity payments in
question were paid "out of profits and gains" within the meaning of
the Income Tax Acts. In the judgment delivered by Baron Pollock
(in which Mr. Justice Hawkins fully concurred), the learned judge
said: "Now it is said that this is not payable out of profits or
" gains. I think the true answer to that is, that it is payable
" out of profits or gains, upon the principle with which I
" commenced my judgment, namely, that unless the lump sums
" received from time to time by the appellants' office are chargeable
" with those annuities, those lump sums so paid would be profits or
" gains. That, to my mind, is absolutely clear, and to my mind it is
" equally clear that, although they are not net profits or gains, if they
" are received from time to time by the insurance office in considera-
" tion of the granting of the annuities, they are as much gains as
" any sum which is brought into the account of a merchant, being a
" payment to him in respect either of goods sold, or services rendered,
" or in respect of some special contract. It is for him to discharge
" himself of that. It is not that they are taxed as gains or profits in
" this particular case, but that they would be taxable, unless there
" was a set off, by the payment of those annuities. Therefore, when
" we come to see what is the intention and the language of the
" Legislature in this section, it seems to me clear, that the intention

" of the Legislature was to do in their case what it has done
" with respect to the Bank of England and other companies, in
" other cases, to find at its source a taxable item, and to tax it while
" it is in the hands of the assurance company, and that those who are
" the annuitants may have the benefit if they should be called upon to
" account for the same sum."

This, so far as intelligible to the non-legal mind, would appear to
imply that the annuities in question must be considered as paid out
of profits and gains, because if the society had not to pay the annuities
granted in consideration of certain lump sums received, those lump
sums would have been in the nature of profits and gains.   As a
commentary on this, it might be said on behalf of the society that
if it had not to pay the annuities it would not have received the lump
sums, and that therefore no question could, in such case, arise of
profits or gains at all, so far as the annuities were concerned.   There
was every probability that the case would be carried to appeal by the
society.

In Italy the question of income tax generally had recently been
before the courts and the Legislature.   For some years past life
assurance societies operating in Italy had been assessed on the excess
of receipts over payments in the revenue account, and the courts had
invariably decided against the contention of the companies that they
should be taxed on the proportion of their profits as actuarially
ascertained.   By a Royal Decree of 1886, however, it was ordered
that detailed valuation returns and account statements be rendered
annually by life assurance companies operating in Italy ; and as these
returns involved an annual valuation and statement of profits, it
seemed very probable that the logical outcome of these regulations
would be the assessment of the income tax on more equitable
principles.   This had, in fact, taken place, and it had now been
decided that the tax was, in the case of life assurance companies, to
be assessed upon the annual profits as actuarially ascertained in respect
of operations in Italy.   And, wonderful to relate, certain companies
had actually obtained a return of excess amounts overpaid on account
of tax for some years past, in addition to the material reduction of
future assessments.   It would thus appear that the Italian courts
took a more liberal and equitable view than our own courts of the
proper treatment of life offices in this respect.

Mr. F. Bell said there was one point worthy of mention, as
supporting the contention that profits and not interest should be
taxed, namely, that unless profits as such were liable to tax, heavy
assurance might be a means of avoiding the payment of income tax
on a small portion of income.   A man escaped income tax on the
life assurance premiums which he paid ; some part of those payments
would be returnable to him as profits, and if profits were not liable to
tax, it would seem that a small part of the assured's income, but an
appreciable part possibly, might escape taxation.   The position taken
up in the New York Life case was, that a mutual office could not
make profits.   He (Mr. Bell) knew of a purely mutual association
engaged in ordinary trade—the *employés* in a large business having
combined together to establish shops ; only the members who had
supplied the capital could deal there, and the profit was from time to

time divided among the members. No one, he thought, would say that this purely mutual trading concern could not make profits.

Mr. AUGUSTUS HENDRIKS said that he would not refer to the legal cases, the subject matter of which had been already explained by Mr. Newbatt, and by Mr. Bailey and Mr. Ackland, but suggested that in any approach which might be made to Parliament, as recommended by Mr. Bailey, it must be from the point of view of asking relief on the broad ground of the State encouraging thrift. In his opinion it would hardly do to claim exemption from taxation of the investments of the life reserves as a matter of right or abstract justice, for it might readily be urged on the other side that if a man of saving habit were to put by a certain sum each year he would make an investment upon which the interest would not escape taxation. If, instead of saving the money himself, he confided it to an insurance company by way of annual premium, it was difficult to see why the investments derived therefrom and constituting the life reserve should be exempt from income tax *per se*, or unless as a set off against taxation of the profits, as shown by an actuarial valuation. He hardly concurred in the view that there could be any positive right to claim such exemption upon the ground that any deduction of income tax from the rate of interest earned by a company might jeopardize its solvency if the rate earned happened to be less than or precisely equal to the rate assumed in the calculations, for the answer would be that a prudent company would fix such rate sufficiently low to provide against all contingencies, and if it made a loss the company must bear it just as it would have derived a profit therefrom if the interest earned had exceeded the rate assumed. Upon the general question of levying income tax upon the full payment of the annuity, such a course is manifestly an injustice, as all annuities consisted partly of interest and partly of a return of capital, which latter ought not to be taxed as income. Mr. Gladstone might possibly have had this and other inequalities in view when he talked of the varying incidences of the tax and the injustices that were committed. As regards Mr. Ackland's contention respecting annuities payable to foreigners, he (Mr. Hendriks) entirely sympathized with the view that it was altogether wrong in principle to tax such payments, especially when derived from investments made abroad out of capital obtained from foreigners. He thought that they should be exempt upon a declaration of the same character as that habitually made with reference to foreign debenture coupons held by foreigners, instead of through the complicated machinery of Somerset House. The effect of the recent judgment was to throw the loss upon the insurance company, as, in most instances, it would not be found practicable to deduct the income tax from non-British annuitants; and he trusted that some steps might still be taken to modify the late decision.

Mr. R. C. TUCKER said that the Surveyor of Taxes at Great Winchester Street had expressed surprise that the amount of "untaxed interest" in the case of the Pelican was so much smaller for 1889 than it was a year or two ago. He had told the Surveyor that since the decision in the case of the Clerical, Medical and General Life Assurance Society *v.* Carter, the Pelican had determined to allow tax upon *all* interest paid to them, including

that upon loans on personal security and loans upon policies which had hitherto not been so dealt with. The Surveyor pointed out that the company might be wrong in adopting this course, as in many cases the tax so deducted would not find its way into the hands of the Government, as it should do: and gave as an example the case of a man whose income was under £150 a year, borrowing upon security of his policy. The man would, of course, be exempt from income tax. The Surveyor thought that, before long, the Inland Revenue Authorities would consider the advisability of compelling the companies to pay the tax direct, instead of allowing the borrowers to account for it (so far as loans on personal security and upon policies were concerned). In these cases, the Surveyor advised that a note to the following effect should be made upon the official notices: "Tax can be deducted as above, only if the borrower pays income tax. "In other cases, the full interest is payable, and the company will "account to the Revenue Authorities."

Mr. A. G. MACKENZIE said he had been struck by a remarkable unanimity of opinion with regard to one of the main points in Mr. Newbatt's address, namely, that the proper way of assessing income tax was upon the profits which were earned by a company, and not upon the interest which was annually payable, and which, as a matter of fact, might be much greater than the profits which the company earned. He hoped that the discussion would not end in a purely academic manner, and that steps would be taken to give the Legislature an idea of the views which the profession generally held on this question. The present time was advantageous, because it was probable that when the next budget was presented, there would be a considerable re-adjustment of income tax. A large surplus was expected, and he thought that if an influential body of members of the profession submitted their views to the Chancellor of the Exchequer, he might be induced to give them full consideration.

The meeting terminated with a few words, in reply, from Mr. Newbatt.

The following letter was addressed by Mr. Sprague to the Editor of the *Insurance Record:*

SIR,—I had occasion some time ago to consider carefully most of the points raised in the recent discussion at the meeting of the Institute of Actuaries, as to the liability of the life offices to income tax; and the conclusions I arrived at, differ very considerably from those of Mr. Newbatt, as reported by you. As my conclusions were unfavourable to the contention of the offices, I did not at the time publicly state them; but, as a full discussion of the whole question has now been commenced, I think there is no longer any good reason for keeping silence.

Mr. Newbatt's conclusions are that interest, *quâ* interest, on the life funds of an insurance company, ought not to be taxed; and that the true basis of taxation should be profits—that is to say, in the case of life offices, what they themselves "call profits, however they

may have got into that category." These conclusions appear, from your report, to rest partly upon Mr. Newbatt's understanding of the Income Tax Acts and the decided cases, and partly on his ideas of what I call "abstract justice". As he has taken exception to this phrase, I should explain that by "abstract justice" I mean justice independent of the law of the land. If two persons have a dispute and go to law, and a decision is given in accordance with the law of the land, then each of them has, in the true sense of the word, got justice done him; but if the defeated party declares that the decision given is not a fair one, that it may be according to law, but is not according to justice, then he appeals to what, for the sake of distinction, I call "abstract justice". I think it will conduce to clearness of view if the conclusions that rest upon considerations of "abstract justice", are carefully kept apart from those which rest upon judicial decisions; and this is the course which I will take in the following remarks:

Taking account then simply of the decisions of the law courts, I hold that their combined effect is (1) that all insurance offices are liable to pay income tax, either direct or by way of deduction, upon all the interest of money that they receive; (2) that, if their net profits exceed that interest, they are also liable to pay income tax on the excess. This, it will be seen, agrees exactly with the instructions given by the Board of Inland Revenue to their surveyors, as quoted by Mr. Newbatt. As he says that it is very difficult to discover where the authority for such an interpretation of the law is to be found, I will state my reasons for believing that it is correct. When I first entered the insurance business, and for many years afterwards, it was the practice of the life offices, instead of making an income tax return in the ordinary way, to insert in their returns a statement that they had already paid, by way of deduction, a larger amount of income tax than they would have been liable for if they had been assessed upon their profits; and that, consequently, they had no return to make; and I have always understood that this statement was made from year to year in pursuance of an arrangement between the life offices and the income tax authorities, that was come to soon after the present income tax was first imposed. That this arrangement, whatever may have been its nature, was not considered a final settlement of the question, but as rather keeping it open for future discussion, is clear from the fact stated by Mr. Newbatt, that, notwithstanding the arrangement, there never was a clear and uniform method of dealing with all offices alike. He says,—as I believe—with perfect truth, that in many respects, sometimes in regard to matters of importance, different offices have been differently treated by different surveyors, and even the same office by successive surveyors; each surveyor, in fact, having been more or less a law unto himself. This was, of course, a most unsatisfactory state of affairs; and the Board of Inland Revenue cannot, I think, be justly blamed for taking steps to put an end to it. I do not know what particular circumstances (if any) caused them to take action in the matter, nor is this at all material to the discussion. It was their duty to carry out the law in its integrity; and if they thought they had good reason for believing that the insurance offices were legally liable

to pay a larger amount of tax than they were in the habit of paying, it was their duty to claim that larger amount; and when the offices disputed their liability, it then became the duty of the Board of Inland Revenue to take the proper steps to obtain a judicial decision as to the liability of the offices. They may have thought that, as the insurance offices disputed their liability, it was not worth while, so long as the amount of revenue at stake was comparatively unimportant, to fight the point with them; but now that the offices have such large sums invested in the colonies and abroad, on which they receive interest without deduction of income tax, the point has become of more practical importance. However this may be, it is the fact that a few years ago the Surveyors of Income Tax began to make demands upon the insurance offices, one after the other, for payment of income tax on various items of income on which they had not been in the habit of paying any tax. It was claimed (1) that the insurance offices are liable to pay income tax upon all the interest which they receive without deduction of tax, and (2) that they are liable to pay upon the miscellaneous profits of the business. As far as I can ascertain, the latter claim has not been pressed when it has been resisted by any office, although the income tax authorities have taken steps to keep the claim alive. They have, apparently, thought it desirable to confine their attention to one point at a time; and they have now for several years past pressed the former of the two above-mentioned claims, with the result that a considerable number of income tax cases have within the last few years been decided by the law courts. I am aware that some persons contend that these decisions are inconsistent, and not reconcileable with each other; but that is not my opinion. I hold, on the contrary, that the decisions are perfectly consistent with each other, and that it is possible by means of a careful study of the principles on which they are based, as stated in the judgments delivered in the different courts, to define exactly the liability of the different classes of insurance offices.

The old form of return made by the insurance offices, as above mentioned, implied that the offices contended that they ought to be assessed simply upon their profits; and that, in consequence of their receiving the interest on their investments under deduction of income tax, they virtually paid more income tax than they were liable for. This was the ground taken up by the *Clerical and Medical* Office; and everything that could be said in favour of it, was most ably urged by their counsel. After full consideration of the whole question, the Court decided that that ground was untenable; and the decision they gave establishes the principle that the offices are liable to pay, either by way of deduction, or direct to Government, tax upon the whole of the interest they receive. This is the decision of which Mr. Newbatt says it is, "speaking economically, the most erroneous of all the recent income tax judgments." I do not know what ground he has for the statement which he makes immediately afterwards, "that a " very large proportion of interest receipts are not taxed at their " source, but pass from hand to hand untaxed", and again, "that " interest in a thousand hands escapes taxation with the full " knowledge and acquiescence of the authorities"; but, if these

statements are correct, they cannot, in my opinion, fairly be considered as tending to prove that "the judgment is somewhat wanting in soundness", but they seem rather to suggest that the income tax authorities are not discharging their duties properly. In my opinion, the decision in the *Clerical and Medical* case has finally disposed of the old contention of the offices. I had, however, long before that decision was given, satisfied myself that the position taken up by the offices could not be successfully defended. In 1883, and again in 1884, a claim was made against the *Scottish Equitable* Life Office for income tax upon its untaxed interest, and upon certain items of profit, in particular, profit on reversions fallen in; and I then went into the matter very thoroughly, with the result stated in my letter which you printed on 16 July 1886.* In order to argue the case before the Commissioners of Income Tax, I carefully studied the judgments in the Court of Queen's Bench in the *London Assurance* case, and I had before me various opinions of counsel; and the conclusion I finally came to was that the contention of the offices was not admissible unless it was supported by some provision in the Income Tax Acts, and that the Acts contained no provision of such a nature. I was not, however, sorry to learn that other persons took a different view, and that the *Clerical* Life Office had resolved, in conjunction with other offices, to have the point submitted to the decision of the law courts. I consider that in the *Clerical* case the question as to the liability of the offices to pay income tax upon untaxed interest received by them, was raised in the simplest and most direct way; and that a decision was given which was not confused by any collateral issues. The office, being called upon to pay income tax on £165 which it had received without deduction of income tax, this being interest on sums at deposit with its bankers during the year, pleaded that it had already paid, by way of deduction, income tax on £170,000; whereas its annual profits were only £74,600; and that therefore it was not liable to pay anything further (Law Reports, Q.B.D., 1888, p. 339, and 1889, p. 444). The case of the company rested upon considerations of abstract justice, but these had little (if any) weight with the judges, who decided the point at issue by an examination of the exact wording of the Income Tax Acts. They held that these Acts distinctly enact that all interest on money is chargeable with income tax; that the position of insurance companies as dealers in money, makes no difference in their liability; and that, consequently, each insurance company is liable to pay tax upon the whole of the interest it receives. The judgments delivered by Mr. Justice Charles in 1888, and by Lord Esher and Lord Justice Fry in 1889, place this beyond all possibility of doubt.

Mr. Newbatt maintains that the decisions in this case are inconsistent with the decision of Mr. Justice Day in the *London Assurance* case, which, according to him, lays down the principle that the income of the life fund is exempt from taxation. As this is the basis upon which Mr. Newbatt's argument principally rests, it will be worth while to examine it carefully. The following are the words of Mr. Justice Day's decision on which Mr. Newbatt relies:—" The

* The substance of the letter here referred to is given on p. 315.

" third question is as to the right of the Crown to levy the duty upon
" what has been termed the life fund. This should. in my opinion,
" be answered in the negative." I admit that these words, taken by
themselves, are obscure; but their meaning becomes perfectly clear
when we read them in connection with other passages contained in
the authorized report of the trial. This report states that the
Surveyor of Taxes contended, that if the profits of the life branch of
the *London Assurance* were to be brought into account, those profits
should be determined by taking the difference, between the life
premiums on the one side, and the claims and expenses on the other
side; and that the balance remaining would be the profits chargeable
to the income tax for the life branch. On this point Sir Henry
James contended that the additions made to the life fund from
income, were subject to income tax as profits of business. It was
this contention that Mr. Justice Day had in view when he used
the above-quoted words; and if any doubt could possibly remain as
to his meaning, it would be dispelled by the words which he uses
further on. " In no sense whatever can the life fund, as such, be
deemed to represent profit." The inference is therefore clear, that
the decision of Mr. Justice Day simply means that the additions to
the life fund from premiums and interest are not profits, and are not,
as such, liable to income tax. Lest it should be supposed that this is
only my own interpretation of the judgment, I may add that the
summary of the decision prefixt to the report contains the following
paragraph:—" Held also that additions made to the life funds of the
" company out of receipts of the year, were not subject to income tax
" as profits." There can thus be no doubt, I think, that Mr. Bailey
and Mr. Newbatt have misunderstood Mr. Justice Day's judgment;
and that this judgment did not lay down the principle that the
income of the life fund is exempt from taxation. This being so, we
get rid of the supposed contradiction between the decision in the
London Assurance case and the decisions in some other cases.[1]

The law was laid down with great distinctness in the Scotch case
of the Scottish Mortgage and Land Investment Company of New
Mexico, 19 November 1886. In this case the company was assessed
under the 4th case of Schedule D, which charges tax upon interest
on foreign securities; and the Court held that the assessment was
rightly made, and that the Crown had the option of assessing the
company, either under the first case, which relates to profits or gains,
or under the 4th case. A decision on which Mr. Newbatt, following
Mr. McCandlish, lays great stress, is that given in the case of the
Australasian Mortgage and Agency Company. Mr. Newbatt speaks
of the argument in that case as running on all-fours with that of the
insurance companies. This, however, is not the case; for the decision
of the Court was based on the ground that the interest which that
company received, was not upon permanent advances, but upon
fluctuating balances of account due by its customers; that these
advances were part of its proper trading, and not investment of
money upon securities; and that it was impossible to distinguish one
part of the business of the company from another, and to hold that
one part fell under the 4th case, and the other under the 1st case, of
Schedule D. The money on which interest was received was money

used by the company in the ordinary carrying on of its business; and the return from it was neither more nor less than trade profit, and was to be assessed accordingly. This case is of importance to bankers and other dealers in money, who do not, like the insurance companies, make permanent investments: but it has, in my opinion, no bearing upon the case of the insurance companies.

Having got rid, as explained above, of the supposed inconsistency between the decision in the *London Assurance* case and those in other cases, there remains no room for doubt that, under the provisions of the Income Tax Acts, the insurance offices are liable to pay income tax upon the whole of the interest they receive; and, this being admitted, there seems no reason for speaking of the action of the income tax authorities as oppressive, or, indeed, in any way open to objection. It still remains to consider the case of the life insurance offices from the point of view of the general policy of the Income Tax Acts; but, in consequence of the length to which this letter has already extended, I will reserve my remarks upon this part of the subject for another occasion.

I am, Sir, your obedient servant,

Edinburgh.   T. B. SPRAGUE.
    11 *February* 1890.

The substance of Mr. Sprague's letter to the *Insurance Record*, dated 16 July 1886, is as follows:

In the year 1883 a demand was made upon the Scottish Equitable Life Assurance Society for income tax upon the profit realized upon reversions and upon interest on certain Colonial investments previously untaxed. On appeal, the Commissioners decided in favour of the Society as regards the profit on reversions, but against it as regards the untaxed Colonial interest. This decision was acquiesced in by both parties. In 1884 the income tax authorities reopened the question, but the Commissioners affirmed their previous decision. The Crown then gave notice of appeal against the decision of the Commissioners as to the profit on reversions, and the Society gave notice of appeal as regards the untaxed Colonial interest. A case was accordingly stated for the opinion of the Law Courts upon both points, and was forwarded to the Board of Inland Revenue for their instructions. After some delay the Society was informed that the Board would not proceed with their appeal, whereupon the Society resolved not to proceed with its appeal, the result being that the Society pays income tax upon its untaxed Colonial interest, but not upon its profit on reversions.

In 1883 the income tax authorities also claimed income tax upon the "fines and fees" received by the Society, but the Commissioners decided in favour of the Society on this point, and the decision was not questioned.

On the Further Development of Gompertz's Law.   By WILLIAM
MATTHEW MAKEHAM, *Fellow of the Institute of Actuaries.*

REFERRING again to Art. 4 of Gompertz's treatise, let us
denote by a_0dx the actual probability of dying in the infinitely
small time dx, at the initial age of the mortality table, and by
a_xdx the abstract chance of death (in the time dx) at the end of
x years; that is to say, the chance considered "independently of"
(as Gompertz expresses it) or as "abstracted from" the deterio-
ration resulting from increased age, then, according to Gompertz's
law, the actual probability of dying, in the time dx, at the latter
age will be a_xq^xdx, in which expression q only is an arbitrary
constant denoting the rate of deterioration.   If, in a_xq^xdx, we
put $x=0$, it becomes a_0dx, which coincides with the expression
first assumed.   Hence, Gompertz's law, which supposes that "the
" vital force or recuperative power loses equal proportions in equal
" times", is expressed, *in its general form*, by the equation
$\mu_xdx=a_xq^xdx$, or, which is the same thing, by $\mu_x=a_xq^x$.

Although Gompertz does not *explicitly* make any assumption
whatever as to the form of the function denoted by a_x,—which
may be said to represent the "force" of the abstract chance of
death at the precise age x,—yet *implicitly* he assumes it to be
constant for all ages, for he deduces the expression aq^x for the
value of μ_x, where a and q are both arbitrary constants.   Nor
could he well have done otherwise considering the very inadequate
data available at the time, for the observations of John Finlaison
which first established the existence of a disturbing force at the
ages of early manhood (since found also in so many independent
observations) had not then been published.   But this assumption
of a constant for the function denoted by a_x (or a) is, as we have
just seen, in no wise essential to the application of the physiological
law propounded by Gompertz as governing the gradual exhaustion
of the recuperative power, for, evidently, quite consistently with
that law, a_x may be assumed to vary with x in any way that may
be required, in order to adapt the supposed law of mortality to
the facts observed: which consideration, I think, will be found
to afford a sufficient answer to the otherwise not unreasonable
criticisms of Mr. Young in vol. xxii of this *Journal* (p. 139).

But although I have thought it necessary to advert to the
above circumstance for the purpose of showing that in any case

Mr. Young's objections are, for the reasons stated, quite inapplicable to Gompertz's law in its general form, it is, nevertheless, a fact that the most reliable observations tend to support Gompertz's implied assumption that (generally speaking) the abstract chances of death may, for practical purposes, be supposed to be equally distributed throughout life—excepting always the periods of infancy and childhood,—and that the occasional deviations from this normal law, which are found in some observations at particular ages, are merely "disturbances" resulting either from artificial causes (such, for instance, as the unequal effect of selection in Mr. King's H$^{\text{M}}$ Table, see *ante*, page 190) or from the various causes enumerated by Mr. Young, and which, for the sake of distinction, I have termed "natural" causes—an example of which we have had in the H$^{\text{M(5)}}$ Table of decrements, also given at page 190. Assuming that "the principles of human nature, even morality and social habits, &c." (to quote Mr. Young's words), should be found to act *uniformly* with greater effect at certain specific ages of the mortality table than at others (and unless their action is uniform it ought clearly to be disregarded), the disturbances thus produced can always be allowed for, with all necessary precision, by the formation of complementary (or supplementary) series, according to the method adopted in the construction of the H$^{\text{M}}$ and H$^{\text{M(5)}}$ series of decrements above referred to.

Gompertz's notion of the chances of death, considered "independently of the deterioration" which results from the gradual exhaustion of the recuperative power, or, as I have termed them, the "abstract" chances of death, is probably deserving and capable of much further development than I have herein given it. But to say the least, we should by means of it be enabled, if necessary, to give effect to the distinction which Mr. Young draws between questions of purely material science and others which may be supposed to have a wider scope. On the present occasion I shall confine myself to an examination of the form of the function in question, as applicable to Gompertz's original law and to its first development.

In Gompertz's formula $l_x = dg^{q^x}$ put $x = 0$, and suppose l_0 the radix of the table to be unity. We then have $dg = 1$ or $d = \dfrac{1}{g}$, and therefore $l_x = g^{q^x - 1} = \epsilon^{\log_\epsilon g (q^x - 1)}$ and $\log_\epsilon l_x = \log_\epsilon g . (q^x - 1)$. Also $\mu_x = -\dfrac{d \log_\epsilon l_x}{dx} = -\log_\epsilon g . \log_\epsilon q . q^x$; and, assuming μ_x to be

expressed by aq^x, it follows that $-\log_e g \cdot \log_e q = a$ or $\log_e g = -\dfrac{a}{\log_e q}$. Substituting this value in $l_x = \epsilon^{\log_e g\,(q^x - 1)}$, we have finally, $l_x = \epsilon^{-a\frac{q^x - 1}{\log_e q}}$.

If, in this expression for the value of l_x we eliminate the effect of deterioration by assuming $q = 1$, we obtain a function for determining the abstract chances of death in the same way that l_x enables us to determine the actual probabilities of death. But the substitution of unity for q in $\dfrac{q^x - 1}{\log_e q}$ gives $\log_e l_x = -a \times \dfrac{0}{0}$. When a result of this kind is obtained by giving a particular value to the variable, it is termed a "singular value" of the function; but when, as in the present case, it follows from giving a particular value to a constant, the resulting singularity may be said to be not of value merely, but of "form." The method of procedure, however, is in both cases the same, namely:

In the expression $\dfrac{q^x - 1}{\log_e q}$ differentiate separately, with respect to q, the numerator and denominator upon the assumption that x is constant. The fraction obtained by substituting the results for the actual numerator and denominator respectively (if not also of the singular form), will be the solution required. Thus, $\dfrac{d(q^x - 1)}{dq} = x q^{x-1}$ and $\dfrac{d \log_e q}{dq} = q^{-1}$. Hence, dividing the former by the latter we have $x q^x$, which becomes x for the particular case of $q = 1$.

The result, therefore, obtained by substituting unity for q in $-\dfrac{d(q^x - 1)}{\log_e q}$ is $-ax$, and the corresponding value of l_x becomes ϵ^{-ax}, which is the form required of the function for determining the abstract chances of death according to Gompertz's original law. Again, if in $-\dfrac{d \log_e l_x}{dx} = \mu_x = aq^x$ we put $q = 1$ we have $\dfrac{d \log_e l_x}{dx} = -a$, and integrating, $\log_e l_x = -ax + c$, or $l_x = \epsilon^{-ax} \cdot \epsilon^c$, which coincides with the preceding result by taking $\epsilon^c = 1$.

Proceeding in the same way with the first development of Gompertz's law, namely, $l_x = d g^{q^x} s^x$, which becomes $l_x = \epsilon^{-a\frac{q^x - 1}{\log_e q}} \epsilon^{-bx}$ where $-b = \log_e s$, we have for the final result $l_x = \epsilon^{-(a + b)x}$ for the series representing the abstract chances of death in this case.

Hence, we see that the form of the function required is the same for the original law, and for its first development also.

The question whether Gompertz's hypothesis that the recuperative power loses equal proportions in equal times, is the true expression of a physiological law actually existing in nature, is one which necessarily does not admit of absolute demonstration like a proposition in Euclid, for instance; and, therefore, like most other questions of a similar nature, it will be answered differently by different individuals. When, as so frequently happens, absolute certainty is unattainable, the amount of evidence necessary to produce conviction in different minds is a very variable quantity. That the strongest evidence in support of Gompertz's hypothesis should be found in the mortality experience of assurance societies, that is to say, among almost the only bodies available whose members admit of being accurately classified according to their true ages, and who, by a process of natural selection, are, to a great extent, weeded from the admixture of individuals whose mode of life is not conducive to a strict conformity with the normal law is, in itself, I think, a strong argument in favour of the hypothesis; but the weight of this evidence is, to my mind, greatly increased by the following considerations.

More than one writer on Gompertz's law has been struck by the remarkable fact to which I called attention so far back as 1867 (*J.I.A.*, xiii, 347), that the value of $\log q$ is found to be very nearly the same in so many independent observations. Thus, from the H^{MF} observations, Woolhouse deduces the following values of $\log q$, namely:

First curve—ages 20, 40, 60, 80, $\log q = \cdot 0402225$

Second curve—ages 30, 50, 70, 90, $\log q = \cdot 0395573$

Mean curve, $\log q = \cdot 0400008$.

And for the 17 Offices' Experience the same writer deduces $\log q = \cdot 03956$ for the mean curve obtained in the same way. That is to say, the two values of $\log q$ deduced from the same observations differ from each other by $\cdot 00067$, while the two values obtained from the mean curves of the two different sets of observations differ by $\cdot 00044$ only. The values of $\log q$ deduced from the observations of the American Life Offices and from the observations of the German Gotha Life Office are $\cdot 041280$ and $\cdot 039625$ respectively.

Here, then, we see that not only do these extensive observations on assured lives resident, be it observed, in widely different parts of the globe, give strong independent evidence in favour of Gompertz's law, but the fact last mentioned suggests an important extension of that law. For, evidently, the practically identical agreement in the value of $\log q$ in these several instances could only result from the rate of deterioration of the vital force being the same for each individual. Thus extended, Gompertz's law may be stated as follows: The vital force or recuperative power of each individual loses equal proportions in equal times; and the proportion of vital force so lost by each *is universally the same,* being approximately represented by $\log q = \cdot 04$.

Now for this addition to, or amplification of Gompertz's law, suggested, it will be observed, entirely by the unlooked-for results of observation, there are also very strong *a priori* grounds, as may be easily shown.

Let us suppose that there are two individuals who have precisely the same probability of living n years (say, for instance, $\cdot 5$), but that the rate of deterioration is large in the one case, being expressed, say, by $q - 1 = \cdot 02$; while in the other it is inappreciably small—the average rate in the two cases being $\cdot 01$ nearly. Now the probability of living n years being $\cdot 5$, the most probable number of survivors will be 1, and it is about an equal chance (if n be not taken too large) whether the survivor would be the one whose rate of deterioration is equal to $\cdot 02$, or the one whose rate of deterioration is assumed to be practically *nil;* but *in either case* the rate of deterioration of the vital force of the survivor would differ materially from that required for conformity with Gompertz's law. On the other hand, if instead of making the assumptions above stated we suppose the rate of deterioration to be the same (namely, about $\cdot 01$) in each case, then the requirements of the law will be complied with, whichever of the two given lives may happen to be the survivor. From this it follows that without the assumption of a constant rate of deterioration for all individuals, Gompertz's law cannot possibly hold good for the aggregate body.

It is always to me a source of considerable satisfaction to find myself in agreement with Mr. Sprague—a writer who has done so much by his numerous contributions to this *Journal,* and in other ways, to disseminate sound views in almost every department of actuarial science—for in that case I can always calculate

upon being able to enforce my arguments with greatly increased effect by the simple expedient of "saying ditto " to Mr. Sprague. My readers, I think, will thank me for reproducing here the following extract from one of his admirable addresses (in this instance to the Actuarial Society of Edinburgh), although this address in question has already appeared in the pages of the *Journal* (*J.I.A.*, xviii, 407).

" I come next", says Mr. Sprague, "to the theory of " probabilities upon which the whole theory of life contingencies " ultimately rests. Although little or no use is made of the " theory of probabilities in deducing the various formulas for " single and annual premiums, yet that theory explains to us the " reason for the use we always unhesitatingly make of the ordinary " mortality tables, and shows us how far we may safely rely upon " the conclusions drawn from them. If this theory were better " understood by actuaries, we should not so often meet with the " mistaken views that are frequently embodied in the reports and " the prospectuses of insurance companies. . . . It would take " too long to explain the inaccuracies of these various forms of " speech. I must, therefore, content myself with saying that " they all indicate fundamentally erroneous notions on the subject " of probabilities. To take only one simple instance : Suppose " that a coin has been tossed 20 times, and that each time ' head ' " has come uppermost. Suppose, further, that the coin is " accurately shaped so that there is no reason why head should " fall uppermost rather than tail. Then, if the coin is tossed " again 20 times, is the fact that head has fallen uppermost " 20 times any reason for believing that tail is more likely to " turn up than head in the second series of throws ? This is an " idea that habitual gamesters at the Continental gaming tables " find it almost impossible to divest themselves of, and arguments " of some plausibility may be adduced in favour of it. Thus, it " may be said, since there is no reason why head should come " uppermost rather than tail, there will, when the coin is tossed " a large number of times, be as many heads as there are tails. " If, then, there has been a run of heads, it is necessary, in order " that this equality may hold, that there should be subsequently " a run of tails. This reasoning, however, is altogether unsound. " In the first place, we may remark that if head has come upper- " most 20 times running, this affords a very strong reason for " believing that there is something in the construction of the coin " that renders head more probable than tail. If, however, we

" suppose the coin is an evenly-balanced one, the correct
" conclusion is that the occurrence of head for any number of
" times in succession, however large, is utterly without effect
" upon the subsequent throws. How, then, is this to be recon-
" ciled with the admitted equality in the number of heads and
" tails that will prevail in the long run? Simply by observing
" that the theory informs us that the equality will only prevail
" when the number of trials is increased indefinitely; and when
" this is done any finite number of heads that have been observed
" in succession—20, 50, 100, or any larger number—will vanish
" in comparison with the larger number of trials.

   " The theory of probabilities", continues Mr. Sprague, "is an
" extremely difficult study in its highest branches, but it is one
" well deserving the attention of the actuary. It teaches us how
" to calculate, not only the number of claims that we may expect
" out of a known number of lives under observation, but also the
" probability of the number of deaths falling short of or exceeding
" the expectation. When we find claims exceeding the expecta-
" tion, it will enable us to judge whether this is to be attributed
" to accidental fluctuations, or whether it indicates that the lives
" have been badly selected. It also teaches us what are the real
" advantages to be gained in life assurance companies by an
" increase in the number of lives at risk, and shows the actuary
" how he may best make use of the practice of re-insurance to
" spread his risks over a wider area."

   These just and exceedingly apposite remarks may be fittingly
capped by the following extract from the dedication ("à Napoleon-
le-Grand") of Laplace's great work on the theory of probabilities,
wherein that illustrious mathematician assigns to the calculus a
still far wider scope even than that which Mr. Sprague has claimed
for it. Laplace says: " Ce calcul délicat s'étend aux questions
" les plus importantes de la vie, QUI NE SONT EN EFFET, POUR
" LA PLUPART, QUE DES PROBLÈMES DE PROBABILITÉ."

   To nothing, perhaps, are the considerations so forcibly brought
out in these extracts more particularly applicable than to the
process of testing the adjustment of the mortality table, which
table Mr. King has defined, very happily, as " the instrument by
" means of which are measured the probabilities of life and the
" probabilities of death" (*Text-Book*, Part II, p. 1). Nevertheless,
it is the fact that hitherto scarcely any use has been made by
English actuaries (for the purpose in question) of the calculus of
probabilities, to the perfecting of which calculus so many eminent

mathematicians of the Continent have devoted abilities of the highest order. Woolhouse, it is true, has given a very interesting application of the method of least squares to the purpose of investigating the possibility of the division of the law of mortality into geometrical series (*J.I.A.*, ii, 150), and he has also contributed a valuable paper "On the Philosophy of Statistics" (*J.I.A.*, xvii, 37); but I am not aware of a single contribution by any other English actuary (in connection with the errors of mortality observations) in which any use is made of the calculus of probabilities in the whole of the 27 volumes of the *Journal*.*

In Woolhouse's "Philosophy of Statistics", above referred to, we find a full and lucid explanation of the law of the "facility of error" (or of the "frequency of error" as it is sometimes called), which is expressed mathematically by the formula $\sqrt{\dfrac{c}{\pi}} \cdot \epsilon^{-cx^2}$. This function is, in some respects, analogous to the force of mortality (μ_x), with which all are now perfectly familiar; and just in the same way as $\mu_x dx$ denotes the probability of a life aged x dying in the infinitely small time dx, so $\sqrt{\dfrac{c}{\pi}} \cdot \epsilon^{-ck^2} \cdot dk$ denotes the probability, in a single observation, of the occurrence of an error (usually said to be "included between k and $k + dk$", but which may be more definitely described as) "of the precise magnitude k"; and as $l\mu_x dx$ denotes the expected number of deaths, in the time dx, among l living aged x, so $n \sqrt{\dfrac{c}{\pi}} \cdot \epsilon^{-ck^2} \cdot dk$ denotes the expected number of errors of the precise magnitude k in a number (n) of independent observations. In all these expressions it is assumed that the errors are what are termed "graduated errors", that is to say, that they may be of any value whatever, whole or fractional; but they become applicable to the case where integer errors only are possible, by the substitution of unity for dk. (See Wittstein on "Mathematical Statistics, &c.", *J.I.A.*, xvii, 185.)

In the application of the law of error to observations on mortality, let l be the number exposed to risk at any given age, p the true probability of surviving a year, and $q(= 1 - p)$ the true

---

* This reproach will, no doubt, before very long be wiped out, for I observe that among the optional questions at the last Institute Examination (Part II), there are two relating to the theory of errors of observations. (Since this article was in type my attention has been called to Mr. G. F. Hardy's ingenious solution of a "Question in Probabilities",—*J.I.A.*, xxvii, 214,—which has an important bearing upon the subject here referred to.)

probability of dying within the year at the same age. Then, as Dr. Wittstein shows in the paper before referred to, $c = \dfrac{1}{2\,lpq}$ (Woolhouse's constant c denoting the same as Wittstein's constant h^2). Hence, we may denote the force of error $\left(\sqrt{\dfrac{c}{\pi}} \cdot \epsilon^{-ck^2}\right)$ by $\dfrac{1}{\sqrt{\pi}\,.\,\sqrt{2lpq}}\,\epsilon^{-\frac{k^2}{2lpq}}$, an expression from which all arbitrary or indefinite constants are eliminated.

As Mr. Sprague, in the preceding extract from his Edinburgh address, has very truly observed, "the theory of probabilities is an extremely difficult study in its highest branches", and it is a fact that more mistakes have been made in connection therewith by writers of the highest eminence, than in any other branch of mathematics. It is not surprising, therefore, to find (as pointed out by Mr. Sprague in *J.I.A.*, xvi, 303) that in a comparatively new application of the calculus, Dr. Bremiker and the late Dr. Kanner should be so greatly at variance "in their views as to the proper measure of risk"—a fact which strongly illustrates the extreme necessity for caution in the applications which may be made of this invaluable calculus. But it affords no reason whatever for questioning the reliability of the calculus itself, which astronomers have found by long experience to be, not only perfectly trustworthy, but of the greatest utility also in their own extremely delicate and intricate calculations. Some more fortunate, but, probably, not more able investigator, profiting by the errors of his predecessors, will, no doubt, be found who will show us conclusively which is the true method of procedure in this case, and until he appears we must be content to suspend our judgment upon the questions at issue.

The following application of the calculus, for the purpose of testing the reliability of Mr. King's H^M Table, is of the most elementary nature possible, as I think any such application (to our pursuits) must necessarily be which is at all likely to be generally appreciated at present in this country; and, therefore, although my processes may possibly be objected to as "unscientific" (compared with others which have been proposed), there may also, for that very reason, be less risk of my conclusions being vitiated by any oversight, and a greater probability that they may be better adapted to the purpose of producing conviction than those more scientific and recondite processes have, apparently, hitherto been found to be.

The unadjusted H^M Table is deduced from a series of observations upon the rate of mortality at consecutive ages, undertaken for the purpose of enabling us to determine as accurately as possible the true probabilities of dying in a year among assured lives in this country. Every actuary will now admit that owing to the limited number exposed to risk at each separate age, the observations in question in their crude or unadjusted state would form an exceedingly imperfect instrument for the purposes specified in Mr. King's definition. Nevertheless, as Herr Lazarus has well observed (in a letter published in the *Journal* in 1872—*J.I.A.*, xvi, 424), these unadjusted results are the most probable that can possibly be deduced from the observations unless we suppose that a law of some kind exists connecting the probabilities at consecutive ages with each other. By supposing that such a law does exist, but without making, any *a priori* assumption whatever as to its nature, Woolhouse has deduced a beautiful and exceedingly valuable formula of adjustment, the principle of which is not likely, I think, soon to be superseded by anything better adapted to the purpose required. The beauty and value of the formula, to my mind, consists chiefly in the fact that each individual observation is treated exactly like all the rest—a very great advantage indeed over the old method of selecting, arbitrarily, isolated terms of the mortality table, and deducing therefrom the intervening values by the ordinary processes of interpolation, as all who have been engaged in investigating the normal law of mortality will very well know. By his ingenious contrivance, Woolhouse enables us to take, at the outset, a most important preliminary step in approximating to the true probabilities, and thus to facilitate greatly the subsequent investigation of the *nature* of the law—respecting which, as I have said, he makes no assumption whatever, but the knowledge of which must certainly prove of the greatest service in the determination of the best possible substitute for the true probabilities of life at successive ages.

Taking the unadjusted H^M observation for age 23, it appears that the number exposed to the risk of dying in the year was 8,534, and that the number of deaths observed was 66. Suppose, now, that this observation were repeated in another group of 8,534 lives similarly composed, and the number of deaths occurring among them recorded, which in all probability would differ more or less from the results of the original observation. Further, suppose that the same observation were repeated again and again

in the same way to the number of n times, n being supposed to be
of considerable magnitude. Again, if we assume the probabilities
of life to be, for practical purposes, truly represented by King's
H^M Table, the expected or most probable number of deaths among
8,534, the number observed, would be represented by 57; and if
we deduct this number from the actual number of deaths occurring
in each of the groups observed, we should have a series of n
numbers, being the actual errors of the several observations; and
taking the numerical sum of these n errors (irrespective of the
signs of the terms) and dividing it by n, we should get the mean
actual error for the particular age supposed.

Next, suppose precisely the same operation to be performed
with respect to each of the other ages of the H^M unadjusted
observations, and, finally, let the mean actual error for each age
be tabulated in the same way as the actual errors (of the original
observations) have been tabulated by Mr. King, in the *Text-Book*,
Part II, p. 94. Comparing, age for age, the two series with each
other, that is, the series consisting of the actual errors of the
original observations only, and the series consisting of the *mean*
actual errors of the whole of the supposed n repeated observations,
we should, of course, find considerable discrepancies between the
corresponding terms of the two series. But if we take the
numerical *sum* of each series (irrespective of the signs) for a
number of consecutive ages sufficient to form an average, we
should expect to find a comparatively close agreement between
them.

Practically, of course, it would be quite impossible to repeat
the observations in the way supposed, and the operation in
question has been described merely for the purpose of affording
an illustration of what is meant by the term "mean error" in
this paper. By means of the law of the facility of error, we are
able to calculate accurately the mean "expected" error, and
therefore to compare the sum of the "actual" errors for con-
secutive ages with the corresponding sum of the mean "expected"
errors; and by their approximate agreement or divergency, to
draw inferences sufficiently trustworthy as to the degree of
reliability to be placed upon the table adopted as representing
the true probability of life at each age.

The expected *number* of errors of any given magnitude (k) in
n independent observations being $n \sqrt{\dfrac{c}{\pi}} \cdot \epsilon^{-ck^2} . dk$, it follows that

the *sum* of the expected errors of the magnitude k will amount to $n \sqrt{\dfrac{c}{\pi}} \cdot \epsilon^{-ck^2} . k . dk$. Hence, the expected sum of all possible errors of positive sign is $n \sqrt{\dfrac{c}{\pi}} \cdot \displaystyle\int_0^{\infty} \epsilon^{-ck^2} . k . dk$. Now the indefinite integral $\displaystyle\int \epsilon^{-ck^2} . k . dk = -\dfrac{\epsilon^{-ck^2}}{2c}$ which, taken between the limits 0 and ∞, reduces to $+\dfrac{1}{2c}$. Hence, the sum of the expected positive errors in n observations is $\dfrac{n}{2\sqrt{\pi c}}$. And the sum of the expected negative errors being the same, the sum of the total possible errors (taken irrespective of their signs) is $\dfrac{n}{\sqrt{\pi c}}$, and dividing by n, we have for the mean expected error

$$\frac{1}{\sqrt{\pi c}} = \sqrt{\frac{2lpq}{\pi}} = \sqrt{\frac{2}{\pi}} \cdot \sqrt{lpq}.*$$

It is necessary, I should here point out, that the signification I have attached to the term "mean error" agrees precisely with that which Sir George Airy (the late Astronomer Royal) has given to it, in his work on the *Theory of Errors of Observations*. It expresses in fact what Woolhouse very appropriately designates the "average divergence." I gather from Todhunter's *History of the Theory of Probabilities* (p. 574), that Laplace also used the term "mean error" in the same sense as Airy has done, for in speaking of the function $\sqrt{\dfrac{c}{\pi}} \cdot \displaystyle\int_0^{\infty} \epsilon^{-ck^2} . k . dk$, he terms it "la valeur moyenne de l'erreur à craindre en plus" (*i.e.*, the mean value of the expected positive error). Apparently, however, there is now a different practice prevailing, to some extent at least, in this respect, for I find that Dr. Thiele and most others whose writings on this subject have appeared in this *Journal* use the term "mean error" to signify what is, strictly speaking, "the square root of the mean square error", the value of which is $\sqrt{lpq}$.

---

* Although the above demonstration is strictly applicable to graduated errors only, it may easily be shown that the result obtained is practically true for integer errors also. For y_x being any function of x, we have $\Sigma_0^x y_{,x} = \displaystyle\int_0^x y_{,x} dx - \frac{1}{2}(y_{,x} - y_0) + \dots$ But in the present case $y_x = \epsilon^{-cx^2} . x$, a function which vanishes both for $x = 0$ and $x = \infty$. Hence, $\Sigma_0^{\infty}(\epsilon^{-cx^2} x) = \displaystyle\int_0^x \epsilon^{-cx^2} x\, dx$ (approximately). (The expression in the text for the mean expected error agrees with that obtained by Mr. Hardy in the note before referred to—*J.I.A.*, xxvii, 214.)

The function $\sqrt{\dfrac{2lpq}{\pi}}$, it will be observed, denotes the *absolute* mean expected error of the observed number of deaths. If we divide it by l it becomes $\sqrt{\dfrac{2pq}{\pi l}}$, which expresses the ratio of the former to the number under observation, and may therefore be termed the proportional mean expected error. We see, then, that the "absolute" *mean* error to be expected varies *directly*, and the "proportional" *inversely*, as the square root of the number under observation. The proportional mean error is, of course, identical with the mean error of the probability of dying.

The following results (p. 329) of the process above described, as applied to Mr. King's H^M Table, are given for the purpose of showing how far the latter can be relied upon as representing the true probabilities of life according to the H^M observations. In order to avoid inconvenient decimal fractions, I have taken the *absolute* in lieu of the *proportional* errors, the relative ratios of the actual and expected errors at each age being, of course, precisely the same in both cases.

The close general agreement in this table between the sums of the "actual" errors and the sums of the mean "expected" errors is remarkable, and indicates, I think, that King's H^M Mortality Table has a very high degree of probability in its favour as a sufficiently true representation of the probabilities of life among healthy assured lives. But in order to bring out the full significance of the figures of the preceding table, let us suppose this same test to be applied to the *unadjusted* H^M Mortality Table—that is to say, let us assume the unadjusted series to represent the best approximation which it is possible to make to the true probabilities (as, in fact, they *would*, upon the supposition of there being no "law" of mortality). In the case supposed we should have for the sums of the mean "expected" errors, a series not differing much from the series contained in the last column of the preceding table, in consequence of the numbers under observation being the same in both cases. But the discrepancies between the mean "expected" errors and the "actual" errors would evidently be very great, for the latter series would necessarily be represented by *zero* at all ages. In fact, it is evidently only when the number under observation at each age is infinitely large that the unadjusted probabilities can be accepted as a satisfactory approximation to the true probabilities,

Age	"Actual" Errors	Mean "Expected" Errors	Sum of "Actual" Errors	Sum of Mean "Expected" Errors	Age	"Actual" Errors	Mean "Expected" Errors	Sum of "Actual" Errors	Sum of Mean "Expected" Errors
10	+ 1	1	1	1	54	−23	17	452	471
11	− 2	1	3	2	55	+38	17	490	488
12	0	1	3	3	56	+ 9	17	499	505
13	0	1	3	4	57	− 6	17	505	523
14	0	1	3	5	58	−15	17	520	540
15	− 1	1	4	7	59	−41	17	561	557
16	− 4	2	8	9	60	+20	17	581	574
17	0	2	8	11	61	+ 7	17	588	591
18	+ 2	2	10	13	62	+ 1	17	589	607
19	+ 4	3	14	16	63	+ 2	17	591	624
20	0	3	14	19	64	+ 1	16	592	640
21	+ 4	4	18	23	65	+ 3	16	595	657
22	− 1	5	19	28	66	0	16	595	673
23	+ 9	6	28	34	67	−12	16	607	688
24	− 1	7	29	41	68	+ 5	15	612	704
25	−26	8	55	49	69	+12	15	624	719
26	− 5	9	60	58	70	−45	15	669	733
27	−16	9	76	67	71	−36	14	705	747
28	+ 8	10	84	77	72	+19	14	724	761
29	− 6	11	90	88	73	−11	13	735	775
30	+15	11	105	100	74	+54	13	789	788
31	−15	12	120	112	75	− 5	12	794	800
32	+ 9	13	129	124	76	+ 5	12	799	812
33	+ 3	13	132	137	77	− 8	11	807	822
34	+10	14	142	151	78	0	10	807	833
35	−14	14	156	165	79	+ 4	10	811	842
36	0	14	156	179	80	− 4	9	815	851
37	+17	15	173	194	81	+ 3	8	818	859
38	+35	15	208	209	82	+ 2	7	820	867
39	+36	15	244	224	83	+11	7	831	873
40	− 5	16	249	240	84	− 6	6	837	879
41	+ 3	16	252	255	85	0	5	837	884
42	− 4	16	256	271	86	− 3	5	840	889
43	−23	16	279	287	87	− 4	4	844	893
44	+ 3	16	282	304	88	+ 1	3	845	896
45	+ 4	16	286	320	89	− 6	3	851	899
46	−10	16	296	336	90	− 4	2	855	901
47	+23	17	319	353	91	− 1	2	856	904
48	− 2	17	321	370	92	+ 2	2	858	905
49	+11	17	332	386	93	+ 3	1	861	907
50	+22	17	354	403	94	− 2	1	863	907
51	+21	17	375	420	95	− 1	1	864	908
52	−16	17	391	437	96	+ 1	1	865	909
53	−38	17	429	454					

for in that case alone will the proportional mean "expected" errors $\left(\sqrt{\frac{2pq}{\pi l}} \right)$ be reduced sufficiently to agree with the "actual", in conformity with the condition required by the proposed test.

I have said that no actuary would *now* be found who considers the unadjusted probabilities to form a trustworthy representation

of the *true* probabilities, but I am not at all sure that everyone would be prepared to give a satisfactory reason for considering them inadmissible for that purpose. The most generally prevalent idea would seem to be that their inadmissibility is owing to their supposed violation of the law of "continuity of value", by their somewhat irregular mode of progression from age to age. But this charge is certainly not well founded. There is no more reason to assume a discontinuity of value between the rate of mortality, say, at age 18 and that at age 19 in the unadjusted observations, than there is in Woolhouse's adjusted table, where the rate of mortality at the same ages makes a somewhat sudden jump from 4·79 to 5·75 per thousand. We are not to suppose that during the whole of the first of the two years the mortality remains constant at the lower rate, and that it changes instantaneously to the higher immediately upon the termination of the first year. I have not the least doubt that, by the skill he has acquired in the manipulation of his "graphic" method of adjustment, Mr. Sprague, after dividing each year into its component months, and drawing his curve accordingly, would be able to construct a table for monthly intervals which should be free from all traces of discontinuity or abrupt changes of value—the rate of mortality for each entire year being at the same time in precise agreement with the actual observation. It is clear, therefore, that the proof of the inadmissibility of the unadjusted observations cannot be based upon a supposed breach of the law of continuity, nor do I think that any such proof can be given, unless it be based, virtually, upon the principle of the test herein proposed.

Mr. King has shown that the total number of "expected" deaths among the numbers actually under observation is 20,496, against the total number of "actual" deaths 20,517—a difference of 21 only—the total positive errors being 443, and of negative errors 422. It appears, also, that the total number of observations was 87 (from age 10 to age 96, both inclusive), and the number of agreements between the "expected" and "actual" deaths being 9, the number of errors properly so called is 78. The "expected" number of errors of each sign is therefore 39—from which the "actual" numbers differ by 2 only, both for positive and negative errors, the former being 37 and the latter 41. Again, if we examine the number of sequences of the same sign, we find 1 case of *six* sequences, 1 case of *four*, 10 cases of *three*, 7 of *two*, and 24 of *one*—if it be allowable to speak of a *sequence of*

one.   Tabulating these results, together with the corresponding
" expected" sequences, we have the following results :

COMPARISON OF "ACTUAL" AND "EXPECTED" SEQUENCES IN THE
SIGNS OF THE ERRORS OF THE OBSERVATIONS

Sequences of	Number of Cases			
	Actual		Expected	
1	24	24	21·5	21·5
2	7		10·7	
3	10	17	5·4	16·1
4	1		2·7	
5	0	2	1·3	4·7
6	1		·7	
7	0		·4	
8	0	0	·2	·7
9	0		·1	
Totals	43	43	43	43

Now it is evident that if there had been an undue proportion
of sequences of the higher orders in the " actual" cases, that fact
would have indicated the *probable* existence of disturbances which
had not, but which should have been, taken into account in the
construction of the series representing the true probabilities of
life.   Nothing of the kind, however, is shown in the last table;
but, on the contrary, the evidence, so far as it goes, is decidedly
in the opposite direction.

POSTSCRIPT.

That I am not mistaken in my conjecture of the term " mean
error" being sometimes used, not according to the strict accepta-
tion of the words, but in the sense which I have indicated in
the body of this paper, is proved by the following extract from a
paper by Dr. Bremiker which appeared in *J.I.A.*, xvi, 286.
" This *mean error*", says Dr. Bremiker, " is the square root of
" the squares of the errors divided by their number; and the
" squares of the errors themselves are formed from the deviations
" of all the single cases from the average or most probable value."

Now the mean square error is $2 \sqrt{\dfrac{c}{\pi}} \int_0^x \epsilon^{-ck^2} . k^2 . dk$, and the

indefinite integral $\int \epsilon^{-ck^2}.k^2.dk$ is equal to $\dfrac{1}{2c}\left(\int \epsilon^{-ck^2} - \epsilon^{-ck^2}.k\right)$.

Hence, $\displaystyle\int_0^\infty \epsilon^{-ck^2}.k^2.dk = \dfrac{1}{2c}\int_0^\infty \epsilon^{-ck^2}.dk$, for $\epsilon^{-ck^2}.k$ vanishes both

for $k=0$ and $k=\infty$. But $\displaystyle\int_0^\infty \epsilon^{-ck^2}.dk = \dfrac{1}{2}\sqrt{\dfrac{\pi}{c}}$ (see De Morgan,

Diff. and Int. Cal., p. 294). Hence, $\displaystyle\int_0^\infty \epsilon^{-ck^2}.k^2.dk = \dfrac{1}{4c}\sqrt{\dfrac{\pi}{c}}$, and

the mean square error $= 2\sqrt{\dfrac{c}{\pi}}\cdot\dfrac{1}{4c}\sqrt{\dfrac{\pi}{c}} = \dfrac{1}{2c} = lpq$. The value of

the "mean error" (so called) as given by Dr. Thiele and others,
is the square root of this or $\sqrt{lpq}$.

It will be observed that the expression, $\sqrt{lpq}$, as also the

corresponding expression, $\sqrt{\dfrac{2}{\pi}}\cdot\sqrt{lpq}$, are deduced in accordance·
with the general theory of errors, which assumes that positive and
negative errors are equally probable, and that such errors may
have any magnitude from $-\infty$ to $+\infty$, which assumptions are
not true in the case of mortality observations.

These conditions would be applicable if l, the number exposed
to risk, were infinitely great—in which case, however, the errors
themselves (that is, the *proportional* errors, or the errors of the
probabilities) would disappear. But Mr. Hardy has shown, by a
numerical example, that even when l is small the above expression
for the mean error is abundantly accurate enough for all practical
purposes. (See the "Actuarial Note" before referred to, *J.I.A.*,
xxvii, 214).

Errata in Erskine Scott's Logarithms.

Mr. Thomas Arnold, Jun., points out that the following
corrections should be made in the list of errata given on
page 249, *ante:*

    Page 40—When 462 is altered to 463, the following 0's do
             not require thickening.

   ,, 305—The number to be altered to 406 is 506, *not* 306.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXVIII.—PART V.

JULY 1890.

CONTENTS OF NO. CLVII.

NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

INSTITUTE OF ACTUARIES.

On some of the Practical Questions raised during the Discussion that followed the reading of my Paper printed in the "Journal", vol. xxvii, p. 362; and on a Further Use for my Tables in applying Profits to making Annual Premiums cease at an Earlier Age than fixed in the Policy. By HENRY WILLIAM MANLY, *Actuary of the Mutual Life Assurance Society, and Fellow of the Institute of Actuaries.*

[Read before the Institute, 21 February 1890.]

I BELIEVE there was a general concensus of opinion that the discussion which followed the reading of my paper at the sessional meeting of the Institute on 17 December 1888 (*J.I.A.*, xxvii, 362) as an eminently practical one, and that another evening might usefully be devoted to a further discussion of many of the questions which were then raised.

With the object, therefore, of starting the discussion again, I propose to examine, in a general way, some of the views which were then expressed; and to better carry out that object, I shall consider the subject under four heads. (1) Whether, in the formula $_1t_{-z-1}a_z = {}_{t-z-1}a_z + \dfrac{b}{Sd + P_x}$, P_x should be the pure premium according to the mortality table and rate of interest used in the valuation, or some other value. (2) Whether d should be taken at the rate of interest used in the valuation, or

at some other rate. (3) To what extent special refinements can be introduced, and how far they may be neglected in practice. (4) The superiority of my tables for practical use over other methods.

The subject might easily be enlarged to cover the very wide field of the proper methods of ascertaining and distributing the surplus of a life office, and the proper way of loading the premiums for different classes of assurance; but I shall endeavour to confine myself as closely as possible to the one method of applying profits, which was the subject of my paper: assuming that the surplus has been correctly ascertained, and the loadings of the different classes of premiums have been accurately adjusted.

No one suggested that the tables themselves should be calculated on any other data than those used in the valuation: unless an observation " that the tables would be more useful if based on a higher rate of interest than $3\frac{1}{2}$ per-cent" could be so interpreted. I do not see how the use of any different data could be justified; and as I do not believe in the pulpit form of argument of setting up supposititious cases in order to knock them down, I shall not trouble you with any observations on that point.

(1). *Should* P_x *be the pure premium according to the data used in the valuation, or some other value?*

In order to discuss this, and the following question properly, I think it would be desirable to have before us a re-arrangement of formula (22) in a manner similar to that proposed by Mr. F. Bell, so that we may be better able to comprehend the significance of the treatment of P and d in the general expressions. Making z the age of the life assured at the date of the first valuation, and z' the age at the date of the second valuation; and t, ^1t, the ages at which the policy is made payable by the first and second bonuses, $\mathbf{b}^1$, $\mathbf{b}^2$, respectively; we have for the first application of the bonus,

$$(P_x + d)(a_z - {}_{t-z-1}a_z) = \mathbf{b}^1 \quad . \quad . \quad . \quad . \quad (39)$$

which, being interpreted, means that the company in consideration of retaining $\mathbf{b}^1$, foregoes the receipt of P_x after the life assured has attained the age of $t-1$, and also d, the value of the interest on the sum assured which, under the new conditions, is then to be handed over to the policyholder.

At the second distribution the formula is

$$(P_x + d)({}_{t-z'-1}a_{z'} - {}_{^1t-z'-1}a_{z'}) = \mathbf{b}^2 \quad . \quad . \quad . \quad (40)$$

which means that the company foregoes the receipt of P_x between

the ages of $t-1$ and $t-1$, and also the interest on the sum assured for the same period, in exchange for retaining $\mathbf{b}^2$.

Now, as regards the value that is to be adopted for P_x, I have given it as my opinion, that as the policy runs off the books at, say, age t, as completely as any other endowment assurance, " it " would be unjust to charge the policyholder for profits he will " never receive, and to debit him with expenses he will never " occasion, so the pure premium ought to be used." But, no ! say others : because 1st. Although such an argument may be fair as between policyholders, there are in many offices other people to be considered, namely, the shareholders : 2nd. The policy having been originally taken out for the whole of life, therefore the policyholder should be made to contribute towards the expenses for the whole of life ; 3rd. The company ought to recoup itself, for the large initial expenditure incurred in obtaining the business, by a contribution from the loading spread over the whole of life, and therefore the company's interest requires to be protected by the use of a pure premium slightly loaded.

With respect to the first of these statements, I do not think we need trouble very much. A shareholders' capital is of very little use to an assurance company, for it very seldom produces much at the time when it is most needed, that is, when a winding-up order has been made ; and it is very handsomely remunerated by its share of the current profits, without requiring any share of deferred prospective profits.

The second statement is founded on the legal doctrine of a contract, and is one which I think we should be very careful of introducing into an abstract question of what is fair and equitable as between members of a mutual association. When the company itself offers to alter the contract, by giving the policyholder the option of taking his profits in this manner, it cannot be considered reasonable and just for that company to turn round and say, " *You* are going to break the contract and must therefore pay something towards the expenses after you leave us."

The third proposition is one suggested by a correspondent, and is worthy of some attention in these days of high procuration fees, branch establishments, and all the elaborate organizations for procuring new business. I consider it, in many cases, exceedingly desirable to distribute a portion of this expenditure over the whole period of a policy, but the principle should be applied to all assurances alike, and not to one particular class alone. Now the best way to do that, and I think the only satisfactory way, is

to adopt a method of valuation proposed by Mr. Sprague (*J.I.A.*, xix, 10), namely, to value the pure premium P_{x+1} instead of P_x, omitting from the valuation all policies on which only one year's premium has been paid. In that way so much of the initial expenses as is represented by the present value of $P_{x+1} - P_x$ is distributed over the whole term of the policy. But the adoption of such a method should involve other considerations, such as that no surrender-value should be given in respect of the first year's premium, and also that no bonus should be given in respect of the first year of assurance. If such a method of valuation be in use, then undoubtedly P_{x+1} should be substituted for P_x in my formulas in order to make the calculations consistent throughout. Similarly, if the valuation has been made by deducting a percentage from the gross premiums, then the gross premiums less that percentage should be substituted for P_x in the formulas. But, on the other hand, if the whole of the initial expenditure has been paid out of previously accumulated profits, then one class should not be charged for it any more than another.

I would therefore lay it down, as a general rule, that whatever value for P_x has been adopted in the valuations, that same value should be used in the formulas for applying profits to making a policy payable during lifetime.

(2). *Should d be taken at the rate of interest used in the valuations, or at some other rate?*

The arguments against my proposition, that the same rate should be used, are all practically summed up in the general formula,—that as the policyholder will take his money before the time originally contracted for, therefore he ought to be made to pay the rate of interest for the remainder of the term which the company could earn on that money, and not the rate adopted in the valuation. A writer in the magazine called *The Insurance Spectator of London,* very ingeniously put the case of those who would adopt a higher rate of interest for *d*, thus : " The interest assumed in valuation is really interest which " the office *pledges itself to pay* to its policyholders. The interest " assumed in this calculation as part of the annuity to be valued " is interest which it is exacting from its policyholders upon " capital advanced to them, and the plainest common sense " requires that it should exact better terms than it will bind " itself to give. . . . . There is, we think, no reason for " dealing with the policyholder on any other footing than that of " a borrower upon absolute security, and that being so, we think

" that $3\frac{1}{2}$ per-cent is a distinctly improvident rate to charge.
" Life assurance companies, as a rule, get from 4 per-cent to
" $4\frac{1}{2}$ per-cent for their investments, and some such rate as this
" should be made the basis of these bonus commutations."
Although the writer of this article is exceedingly well versed in his
subject, and knows what he is writing about, he is certainly not
an old literary hand, or he would not introduce into his writings
that doubtful virtue "the plainest common sense"; because he
must know that, in our business especially, what is described as
the plainest common sense is very often nothing but nonsense.
Now, the principle which should govern a proper valuation is the
application of accurate data to the calculation of future contingent
liabilities; and involves, therefore, the use of that table of mortality
which will best represent the experience of the future, and that rate
of interest which the company can obtain in the future on the best
securities.   The office does not *pledge itself to pay* any interest:
it pledges itself to pay the sum assured when it matures; and the
actuary has to see that a sufficient reserve is made which, with the
future premiums and accumulations of future interest, subject to
proper allowances for future profits and expenses, will prove
sufficient to meet all the claims as they arise.   If, then, you say,
that at the fag-end of these assurances you are going to make
a higher rate of interest on the best securities than you are
assuming in your valuation, you are actually stultifying yourself,
and acknowledging that you have adopted a wrong rate of
interest in the valuation; and that you are in effect making a
reserve out of existing profits for the benefit of future entrants.
I am not now arguing as to what is the proper rate to use in a
valuation; but I do think it is unwise to say, " We are making
" a large reserve now because we anticipate a fall in the interest
" in future", and then to turn round and say to the policyholder,
" We are going to charge you a higher rate for the advance of
" your sum assured 40 years' hence, because we expect to make
" a higher rate then than we are now calculating on."   I really
think that we should endeavour to be consistent in these matters.
As I said before, in replying to the discussion on my paper, there
are two general ways of distributing profits, that is, either in cash
or as a reversionary addition to the sum assured.   If the surplus
is distributed in cash, an option is generally given of taking an
equivalent reversionary addition, and if it is allotted as a rever-
sionary addition, an option is given of taking an equivalent
cash value, and these options are given without any medical

examination or special loading for expenses. Why, then, should
this additional option be treated on a different basis ? It is a
reversionary benefit, with a special advantage to the office,
because only those will benefit who are good lives; while in the
case of the reversionary addition to the sum assured, the bad lives
benefit the most. This, then, being a reversionary benefit, I
consider that the fairest plan as between policyholders is to adopt
the reversionary addition as a basis, and use the net premium and
the rate of interest adopted in the valuation for P_x and d in the
formulas.

An argument for the plea of consistency was well put by
Mr. King, when he said that " he thought there were great
" reasons for employing the same data [throughout], because, if
" not, then at the next valuation there would be a kind of sup-
" posititions loss or profit as the case might be."

Before leaving this part of the subject, I may as well refer to
another view of the question that was mentioned, namely, that
when the policy matured the office gave the outgoing policyholder
the reserve value of his policy plus the cash value of the bonus,
as if that was some reason why he should not have the full value
of his option. I really cannot see why he should not have it.
When an endowment assurance matures, the office pays the full
value of the sum assured; and I have never found any endowment
assurance premiums yet that were loaded with expenses for the
after-lifetime of the assured, nor any charge made for the loss of
interest by the payment of the policy. If the office is willing to
convert a whole-life policy into an endowment assurance, it
should do it fairly and honestly, and place the new assurance on
exactly the same terms as the other endowment assurances.

However, if anyone after this wishes to use any other values
for P_x and d, it will not take long to make a table of the
reciprocals of $(P_x + d)$, and such values can be substituted for
$(1 + a_x)$ in the formulas.

(3). *To what extent may special refinements be introduced, and
how far may they be neglected in practice?*

In discussing this part of the subject, we are met, at the
outset, with the peculiar difficulty that the very conditions on
which our formulas are founded do not obtain in practice. Take
our very first example, for instance. On the date of the valuation
(say 31 December 1889) we are supposed to be dealing with a
policy taken out on the person's 20th birthday, on which five
premiums have been paid, and the sixth premium is just due but

not paid, and that a bonus is allotted in respect of five years of assurance. On what date was the policy effected? If we say on 31 December 1884, we shall find, in consequence of the enticing " Notice: all policies effected before the end of the year will be entitled to an additional year's bonus at the next valuation", that the policy will be entitled to six years' bonus. If we say that the policy was effected on 1 January 1885, then, according to the rule, which I believe is frequently followed in classifying policies for valuation purposes, of reckoning the age of the assured on the last day of that year as the same as on the day the policy was issued, we shall set down the age of the person in our example on the day of the valuation as 24 instead of 25. We must therefore suppose it was effected on a kind of 32 December 1884, that is, after the year had closed, but before 1885 had commenced. But then everybody is not born on that peculiar date, nor are all the policies effected on that date.

Suppose, however, that we resolve to be extremely accurate, and assume that two policies were effected on that date, but that one life was exactly aged 20 and the other 19 years and 3 months, both having to pay the premium for age 20 next birthday; so that at the date of valuation, when we come to apply the bonus to making the policy payable during lifetime, the first will be of the exact age of 25 and the other 24 years and 3 months. Then let us see what the difference will be in the maturing ages.

To be exact, we must use formula (15),

$$N_{z+n-1} = N_{z+m-1} + \frac{b \cdot D_z}{S \cdot d + P_x}.$$

Let us now suppose the cash bonuses declared at the successive valuations to be the same as on page 377, vol. xxvii, so that we have

AGES OF ASSURED AT DATE OF VALUATION				Bonus for 100	$\frac{b}{Sd + P_x}$
A		**B**			
Years	Months	Years	Months		
25	0	24	3	2·1	·509
30	0	29	3	3·5	·712
35	0	34	3	4·4	·933
40	0	39	3	5·5	1·166
45	0	44	3	6·8	1·412
50	0	49	3	8·6	1·823

Let us also assume that the D and N columns have been interpolated for months. The calculations, reduced to their smallest proportions, will be as follows:

A	$z+n-1$ Years	Months	Age when payable Years	Months	B	$z+n-1$ Years	Months	Age when payable Years	Months
$\cdot509\ D_{25} = 39{,}379 \times \cdot509 = 20{,}044$	70	8	71	8	$\cdot509\ D_{24\cdot3} = 40{,}616 \times \cdot509 = 20{,}673$	70	6	71	6
$\cdot742\ D_{30} = 32{,}017 \times \cdot742 = 23{,}756$					$\cdot742\ D_{29\cdot3} = 33{,}044 \times \cdot742 = 24{,}519$				
$N_{z+n-1} = 13{,}800$					$N_{z+n-1} = 45{,}192$				
$\cdot933\ D_{35} = 25{,}882 \times \cdot933 = 24{,}147$	64	9	65	9	$\cdot933\ D_{34\cdot3} = 26{,}734 \times \cdot933 = 24{,}943$	64	7	65	7
$N_{z+n-1} = 67{,}947$					$N_{z+n-1} = 70{,}135$				
$1\cdot166\ D_{40} = 20{,}783 \times 1\cdot166 = 24{,}232$	60	9	61	9	$1\cdot166\ D_{39\cdot3} = 21{,}492 \times 1\cdot166 = 25{,}058$	60	6	61	6
$N_{z+n-1} = 92{,}179$					$N_{z+n-1} = 95{,}193$				
$1\cdot442\ D_{45} = 16{,}570 \times 1\cdot442 = 23{,}894$	57	7	58	7	$1\cdot442\ D_{44\cdot3} = 17{,}156 \times 1\cdot442 = 24{,}738$	57	2	58	2
$N_{z+n-1} = 116{,}073$					$N_{z+n-1} = 119{,}831$				
$1\cdot823\ D_{50} = 13{,}022 \times 1\cdot823 = 23{,}739$	55	0	56	0	$1\cdot823\ D_{49\cdot3} = 13{,}520 \times 1\cdot823 = 24{,}647$	54	7	55	7
$N_{z+n-1} = 139{,}812$	52	8	53	8	$N_{z+n-1} = 144{,}578$	52	4	53	4

One object I had in selecting these examples was to show how accurately the maturing ages were obtained by inspection from my tables. I have omitted from the above the long process of multiplication, so that the actual working is not quite so easy as it looks.

The difference in the final maturing ages is exceedingly small, amounting to between four and five months, and not sufficient to justify all the extra labour.   A practical injustice, however, would be done to B if it should happen that he had to pay one more full year's premium in consequence of being reckoned of the exact age of 20 when he entered; but I think that would be fairly overcome by the suggestion that only a proportion of the premium for the interval between the renewal date and the date of the maturity of the policy should be paid.   My plan would be strictly accurate, because in the formulas it is assumed that the premium due from the date when the policy is made payable has been valued.   I was twitted with being hypercritical in suggesting that a portion of the last premium should be returned, and saying nothing about the current profits for the time between the last valuation and date of maturity.   I agree that the assured is entitled to the current profits—in fact, I stated as much on page 375; but if the assured cannot get both, it is no reason why he should not have one of the benefits.   Half a loaf is better than none.

The other case, which will certainly be the more common, is where two policies on lives of exactly the same age at entrance are effected at different times of the year.   Here again, if we want to be minutely accurate, we shall have to make z in, say, formula (28) fractional, so that, if one policy was effected in the middle of the year, we should have to use

$$_{t-z+\frac{1}{2}-1|}a_{z+\frac{1}{2}} = \mathbf{b}(1 + a_x).$$

Now a simple glance at the tables will show that very little would be gained by interpolating for months between the columns, as the differences are exceedingly small.   By my method of ascertaining the *age* at which the policy is payable, no injustice is done by treating both policies alike at the valuation.   For instance, suppose one policy to be effected on the proposer's 20th birthday, on 31 March, and the other on the proposer's 20th birthday, on 30 September, and by the tables it is found that both policies finally mature at age 60.   Each policy will be payable exactly 40 years from the date it was effected, and both will have been treated alike.   If, on the other hand, you say that both policies will mature 40 years hence, and then calculate the due date from the date of valuation, there would be a certain amount of injustice.   Mr. Todd said, " when he had determined the " *age* at which the policy became payable, he went back to the

" last anniversary of the premium becoming due, and counted
" from that in order to determine the *date* of maturity." That,
I agree, is a refinement which is only just and fair to the policy-
holder, and worth all the trouble, *if* you intend to exact the full
yearly premium on the day the last premium becomes due; but if
you only charge him the proportion of the premium from the due
date to the date when the policy matures, then I consider that
such a refinement is not worth the trouble. Generally, therefore,
I consider that all the delicate refinements may be ignored on the
one condition that you shall not exact the full premium on the
renewal date preceding the date of maturity.

(4). *The superior advantages of my tables over other methods.*

I have read Mr. Todd's observations several times, and I
cannot understand how he considers his process superior to one
by which he could obtain the same results by little more than
simple inspection. He said: " One great value of a formula of
" this kind [Mr. Sprague's] was that it was continuous; there
" were at each valuation only two elements to consider—the age
" at entry and the duration—and they were not hampered, as in
" Mr. Manly's system, by the results of the previous valuation."
By that he meant, I presume, that I had to consider the age at
which the policy was made payable at the last valuation. Well,
that age is certainly an immutable fact; for whatever happens
you cannot alter that. On the other hand, is there any great
value in a formula that is continuous in the sense which Mr. Todd
means, and which is illustrated in the example on page 340 where
the N_{z+n-1} of the last valuation becomes the N_{z+m-1} of the new
valuation. Supposing you wanted to alter the rate of interest in
your valuation, the N_{z+n-1} of the last valuation will no longer
be the same as the N_{z+m-1} of the new valuation, and the con-
tinuity will immediately cease; whereas, if you have my tables
calculated on the new basis, the new age can be found just the
same by inspection as if no alteration in the basis of valuation had
been made, because my formula is wholly independent of the
elements used in previous calculations.

I had the problem recently presented to me of quoting the
ages at which 8,000 policies could be made payable by the
application of a new bonus, when the basis of valuation had been
entirely altered, and 3,800 of those policies had already been
made payable during lifetime by the application of previous
bonuses. The new bonus was a uniform percentage on the sum
assured for each year the policy had been in force since the last

valuation. Table **A** took about a couple of hours to make, and, judging from the results produced in the example on page 377 (vol. xxvii), I considered it unnecessary to go into the refinement of quoting the age in months for any age over 60, so that in all such cases the next higher complete age was quoted. The whole of the work was performed by a rapid manipulator in 20 hours, giving an average of 400 an hour. I think I may safely defy anyone to produce the same results by any other process in anything like the time.

Use of the Tables for applying Profits to make the Payment of Premiums cease at an Earlier Age than fixed in the Policy.

This problem does not present in its solution so many difficulties as that which we have just left, but it is interesting as requiring a different treatment of the most important factor, P_x.

The necessary formulas could be deduced by the same reasoning as was used in the former paper (*J.I.A.*, xxvii, 370), thus:

A policy (S) was originally taken out on a life then aged (x), at an annual premium (P_x), payable until age (t). The present age of the life assured is (z), and a cash bonus (**b**) has been declared, which is to be applied towards making the premiums cease at an earlier age $({}^1t)$. Find ${}^1t$.

It is evident that the present value of the new assurance,

$$(S \times A_z) - P_x(1 + {}_{{}^1t-z-1}a_z),$$

must be equal to the old assurance and the bonus now declared,

$$= (S \times A_z) - P_x(1 + {}_{t-z-1}a_z) + \mathbf{b}.$$

Hence
$$ {}_{{}^1t-z-1}a_z = {}_{t-z-1}a_z - \frac{\mathbf{b}}{P_x}, $$

and
$$ {}_{{}^1t-z-1}a_z = {}_{t-z-1}a_z + \frac{\mathbf{b}}{P_x}, $$

which, in the case of a whole-life policy and the first application of the bonus, becomes

$$ {}_{{}^1t-z-1}a_z = \frac{\mathbf{b}}{P_x}, $$

since $t = \omega$, and ${}_{t-z-1}a_z = 0$.

This method of reasoning, however, excludes from our notice a most important element in this problem, namely, that P_x must not be taken as the pure premium. As the policy is to continue for the whole of life, the expenses connected with it will likewise continue for the whole of life; and as the policyholder will certainly expect to participate in the profits after his premiums have ceased, he must be made to contribute the loading for profit for the whole of life; so that, in applying the profits to reduce the number of payments, we must apply them to the purchase of the gross premiums payable after a certain age. It is necessary, therefore, to set out the problem in another form, thus:

A whole-life policy taken out at age (x), at an annual premium (P'_x), has had a cash bonus (b^1) declared upon it, which the policyholder desires to be applied to reducing the number of his payments. The present age of the assured is (z); at what age (t) will his payments cease?

b^1 must evidently be equal to the present value of an annuity-due of P'_x on a life aged (z), deferred $t - z$ years.

Hence $b^1 = {}_{t-z} a_z . P'_x$; and, since ${}_{t-z} a_z = {}_{t-z-1|} a_z$, we have

$$ {}_{t-z-1|} a_z = \frac{b^1}{P'_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (41) $$

and, by entering my Table **B** for Cash Bonuses with $\dfrac{b^1}{P'_x}$, t can be found by inspection.

When a second bonus (b^2) has to be applied, it is to a policy on which the premiums already cease at age (t); and we have to find a new age $({}^1t)$, so that the present value of an annuity-due of P'_x on a life now aged (z') say, deferred ${}^1t - z'$ years, and to continue for $t - {}^1t$ years, shall equal b^2. We have, therefore,

$$ {}_{{}^1t - z'}{}_{t - {}^1t} a_{z'} . P'_x = b^2 $$

$$ ({}_{{}^1t - z'|} a_{z'} - {}_{t - z'} a_{z'}) P'_x = b^2 $$

$$ ({}_{{}^1t - z' - 1|} a_{z'} - {}_{t - z' - 1|} a_{z'}) P'_x = b^2 $$

$$ {}_{{}^1t - z' - 1|} a_{z'} = {}_{t - z' - 1|} a_{z'} + \frac{b^2}{P'_x} . \quad . \quad . \quad (42) $$

and as this formula is general, and we can use (z) to denote generally the age of the assured at the date of valuation; (t) the age at which the premiums were made to cease at the last valuation; and $({}^1t)$ the age at which the premiums can be made

to cease by the application of the new bonus ; and (**b**) the cash bonus now declared ; we again arrive at the general formula

$$_{1t-z-1}a_z = {}_{t-z-1}a_z + \frac{\mathbf{b}}{\mathrm{P}'_x} \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If the share of the profits is allotted by way of a reversionary bonus payable at death, say **B**, the formula becomes

$$\frac{_{1t-z-1|}a_z}{A_z} = \frac{_{t-z-1}a_z}{A_z} + \frac{\mathbf{B}}{\mathrm{P}'_x}. \quad . \quad . \quad . \quad . \quad (44)$$

and Table **B** for reversions can be entered with $\dfrac{\mathbf{B}}{\mathrm{P}'_x}$. If the reciprocals of P'_x be tabulated, the formation of a table of $\mathbf{B}.\mathrm{P}'^{-1}_x$ to take the place of my Table **A** should not present any very great difficulties.

If we use $\mathrm{T}[z{:}t]$ to denote the tabular value of $_{t-z-1}a_z$, and $\mathbb{T}[z{:}t]$ the tabular value of $\dfrac{_{t-z-1}a_z}{A_z}$, the above formulas will be concisely represented by

$$\mathrm{T}[z{:}t] + \frac{\mathbf{b}}{\mathrm{P}'_x} = \mathrm{T}[z{:}^1t] \quad . \quad . \quad . \quad . \quad . \quad (45)$$

and

$$\mathbb{T}[z{:}t] + \frac{\mathbf{B}}{\mathrm{P}'_x} = \mathbb{T}[z{:}^1t]* \quad . \quad . \quad . \quad . \quad . \quad (46)$$

which expressions denote more clearly how the tables are to be used.

The question arises again in this problem : Whether, when the last renewal date arrives, the whole annual premium should be exacted, or only the proportion up to the date when it is made to cease by the application of the bonus ; and I again consider that only the proportion should be paid, for the formula distinctly assumes that the premium due from the date fixed has been commuted by the surrender of the bonuses.

---

* In like manner formulas 24 and 26 in my paper will be represented by

$$\mathrm{T}[z{:}t] + \mathbf{b}(1 + a_x) = \mathrm{T}[z{:}^1t] \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

$$\text{and} \quad \mathbb{T}[z{:}t] + \mathbf{B}(1 + a_x) = \mathbb{T}[z{:}^1t] \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

DISCUSSION.

The PRESIDENT (Mr. William Sutton, M.A.) having invited discussion,

Mr. GEORGE KING said that when Mr. Manly read his former paper on this subject he (Mr. King) had expressed a certain amount of dissatisfaction. It was not with the paper itself, but he was rather afraid of the effect it might have upon the course of business. Mr. Manly had now produced a second paper, which, he confessed, went a long way to reconcile him to the first. The paper showed how simple the process might be. He had no idea it was so simple; and certainly the statement was startling that 400 of these bonuses could be calculated in an hour. He had not found any other bonus option work so fast as that. The paper was a reply to various points raised in the discussion upon the first paper, and it had the result of elucidating many interesting points. He asked how Mr. Manly would treat a valuation made on the combined H^M and $H^{M(5)}$ basis, his tables being on the H^M Table. He should like to know, also, something of the effects of different rates of interest upon the dates when the endowment assurances would become payable. He quite agreed that the valuation data, at least under certain circumstances, were the proper ones to be used for the option, but it would be very interesting to know how changes in the tables and rates of interest affected the dates when policies would mature. For instance, if a company had been valuing at $3\frac{1}{2}$ per-cent and reduced the rate to 3 per-cent, to what extent would the dates of maturity be affected? If the valuation data were adopted no option should be allowed to the policyholders when they had once accepted the endowment assurance plan as the method of taking profit. If, at a subsequent date, they were allowed to take reversionary bonus instead of continuing to apply the bonus to reduce the term of the policy, then he did not think the valuation data should be those employed, but the policyholder should be fined for the benefit of the option allowed him. There was one very small point in which he thought the author was not quite accurate as regards the practice of companies. He spoke of companies giving in a quinquennium six years' bonuses. He never heard of that being done, and a policy effected just at the close of a quinquennium would take, if it took a bonus at all for the first premium, one year's bonus in respect of that quinquennium, and not six for the succeeding one. With regard to the advantages of the author's method over others, he would ask whether he had looked into Mr. Colenso's method, and ascertained if that was really shorter than his own. Speaking of methods of valuation, the author took care to say that he would confine himself as closely as possible to this one method of distribution, and not discuss methods of valuation generally; but further on he seemed to have gone away from the rule so laid down, and to speak of what he thought was *the* proper method. There might be other methods besides this particular one, and very good reasons might be urged for a method upon which the author threw a little ridicule. He (Mr. King) did not agree that they had only to consider the sum assured in dealing with their policy-

holders.   Although that might be the only liability according to the written contract, they were under other moral contracts which should be fulfilled, and as long as they were doing "with-profit" business they must consider these and adapt their valuations accordingly. They held out certain inducements to the public which practically were to give an increasing assurance, and, that being the case, if they were to be fair to subsequent policyholders they must value accordingly.   It would be found in a case of that kind that if they valued their policies as increasing assurances at the rate of bonus that was fair and reasonable in respect of a with-profit premium, and if they valued the corresponding net premiums for such increasing assurances, say, at 4 per-cent. they would produce similar results in their reserves to those produced when they valued the policies as ordinary assurances at 3 per-cent.   If they held out the inducement of a bonus percentage on the sum assured they were bound to make the valuation accordingly.   It was a matter of convenience only whether they adopted the method of valuing as increasing assurances at a higher rate of interest, or whether they adopted a valuation at a lower rate, treating the policies as ordinary ones.   For these reasons he strongly urged that the method spoken of as the proper one was only the proper one under certain circumstances.

Mr. T. B. SPRAGUE disclaimed any credit for being a practical authority on the subject under discussion.   It so happened that it was one that he dealt with in the earliest papers he submitted to the Institute, more than 30 years ago (*J.I.A.*, vi, 290, 344), and since then he had never had occasion to consider the point.   This was an option that had never come under his practical consideration, and he was not familiar with Mr. Manly's first paper.   Looking at the paper on its merits, he could not say that he altogether agreed with some of the principles that had been laid down.   For instance, when the author said the premium used in this option should be the pure premium according to the data used in the valuation, he was not convinced by his arguments.   There seemed to be a good deal more to be said in favour of taking the office premium.   Of course, if the bonus was applied to redeem the premium for a limited number of years, or the whole of life, or for a term of years after a certain age, it was clearly admitted that they had to make the office premium the basis of the calculation.   That of itself suggested that they should also, in this case, take the office premium, but was not conclusive. Mr. Manly said, "As the policy runs off the books at, say, age t, as " completely as any other endowment assurance, it would be unjust " to charge the policyholder for profits he will never receive, and to " debit him with expenses he will never occasion."   That also was not clear.   Endowment assurance premiums were sometimes loaded in such a way that the total contribution to expenses and profits came to much the same thing as in the case of a whole-life policy, and it was clear that when they converted a whole-life policy into an endowment assurance, they should do so on such terms that the contribution to profits of the altered policy would be just the same as before.   Of course, it might be said that the policyholder should not be charged for profits he would never receive, but that again was very much a matter of contract.   The author said it was the office

making the contract that altered the term of the policy, but in
that case the office had a right to say that it would only alter
the contract upon certain terms, and the office might perhaps
consider that it was just that the office premium should be used
in the calculation. With regard to the valuation the author
had connected with his (Mr. Sprague's) name, in which, instead
of valuing the pure premium as at entry, they valued the pure
premium for one year older, he would like to say, having reconsidered
at intervals the opinion he formerly expressed on that subject, that
he still adhered to it. If the first year's premiums were practically
absorbed by the initial expenses of the policy and the current
risk—and he believed in practice this was very much the case, if
the expenses were fairly apportioned between the new and the old
business—then the method of valuation, of applying the net
premium, P_{x+1}, was strictly correct, and did justice between the
different classes of members. Of course, if an office was sufficiently
strong to make a net-premium valuation, taking the net premium P_x,
so much the better, but this method took out of the profits of the
past the difference necessary to make up the net-premium valuation
reserve. But when they came to deal with the policies afterwards,
they were entitled to disregard that reserve, and to calculate the
surrender-value as if they had only made the smaller reserve. Then,
with regard to the rate of interest to be used in calculating the option,
he was not satisfied with Mr. Manly's arguments, although he was
not prepared to speak against them very decidedly. The way he
would look at the question was from a point of view that did
not seem to have occurred to the author, namely, what was just
between the members who took their bonuses in one way and those
who took them in the other way. If they valued at 3 per-cent
interest, and the office was making $4\frac{1}{4}$, they calculated their surplus
at 3 per-cent and turned it into reversion at 3 per-cent, so that if
they made a new valuation after the declaration of the bonus, they
would have just the same result as before. Then, if they considered
on what terms they were to allow a member to surrender his bonus,
he would reason in this way: Here was a member who had a
reversionary bonus; if he left that in their hands they were able to
make $4\frac{1}{4}$ per-cent upon it and realize a certain profit. If a man
surrendered his bonus, he deprived the office of the future profit,
between 3 and $4\frac{1}{4}$ per-cent, and in that way he lessened their profits
as compared with the man who left his bonus with them. In
calculating the cash value of the bonus, it was, therefore, just and
right that they should calculate the cash value at the higher rate of
interest they were actually making. That was a point he would like
the author to consider. With regard to the conversion of the
reversionary bonus into cash, there was another point to be considered,
that on the average, those who surrendered their bonuses for cash were
in better health than those who did not, and for that reason the office
was entitled to use a different table in calculating the cash bonus.
For instance, if the H$^{M(5)}$ Table was used for calculating the amount
of the reversionary bonus, in surrendering that bonus they were
entitled to adopt the H^M Table.

Mr. F. BELL said that those who selected this option might be

considered to take the cash bonus, which they afterwards employed to
their own advantage. All who had watched applications for surrender
would agree that in the majority of cases it would be found that the
bonuses, or at any rate some of them, where there had been an
option at each division, had been taken in cash. He had not investi-
gated the subject, and had no figures to give, but he had a strong
impression that this was so, and it was an accepted fact that
discontinuants were, as a class, select lives. That conclusion agreed
with the results of Mr. Hardy's inquiries into the mortality among
the bonus policies of the British Empire Mutual Life Assurance
Company (*J.I.A.*, xxiii, 1). It, therefore, seemed to him that the
office should in some way protect itself from the selection which the
assured here exercised, and this might be done by making a_z in
formula (39) "select." By a slight re-arrangement of that formula,
the effect of that alteration could be made apparent. Transferring
the temporary annuity to the right-hand side of the equation, the
formula became:

$$(\mathrm{P}_x + d)a_z = (\mathrm{P}_x + d)_{t-z-1}a_z + \mathrm{b}^1$$

Here they had again an equation of benefits. The office was to give
up the annuity of $\mathrm{P}_x + d$ for the whole of life, the insured giving in
exchange an annuity of $\mathrm{P}_x + d$ for the term of $t-z-1$ years, and in
addition the amount of bonus. It seemed that the annuity which
the office gave up might be made "select," so as to compensate for the
selection which the assured was able to exercise.

Mr. F. E. Colenso regretted that he could not add anything of
interest with regard to the use of Mr. Chisholm's tables in the place
of Mr. Manly's. He had had no opportunity of testing the relative
merits of the two methods in the matter of speed, but it was quite
certain that the example of rapid work given in the paper could not
be surpassed. At the same time, if they had the policy reserves in
each case, as well as the cash bonuses, Mr. Chisholm's tables afforded
a suitable means of writing down the desired short-term annuities
without any reference to rates of interest or mortality. The *pure*
premium must, however, be purchased—in other words, the formula
employed must be that upon which the tables are based.

Mr. Manly, in reply, said that Mr. King asked what he would
do when the valuation was made upon a combined H^M and $\mathrm{H}^{\mathrm{M}(5)}$ Table.
If they used the H^M pure premium in the valuation they should use
P_x or a_x by the H^M Table, and the other annuities should be
calculated by the $\mathrm{H}^{\mathrm{M}(5)}$ Table. That, of course, would involve special
tables being formed upon the basis of the $\mathrm{H}^{\mathrm{M}(5)}$ Table. He had not
calculated the difference in the ages at maturity when different rates of
interest were used, but they were given by Mr. Sprague in his paper
30 years ago. The maturing ages were calculated by the Carlisle
Table at 3, $3\frac{1}{2}$, and 4 per-cent. If he remembered rightly the
difference in the final maturing ages were small, not amounting to
more than a year or a year and a half. He was surprised to hear
Mr. King say he had never heard of a bonus given for six years. He
thought it was a rule that policies which had only been in force for
one year did not participate in the profits of the quinquennium, but

when the next valuation was made a six years' bonus was given. He had known cases where an office had not given a bonus for two years at one quinquennium, and at the following quinquennium the policy had received a seven years' bonus. He thought that was a general custom where bonuses were not given until two or three years' premiums had been paid. Mr. King had certainly caught him tripping with regard to "the proper way of making a valuation", and he would alter the language there. Mr. Sprague suggested that the loading of endowment assurance premiums might be equal to that of whole-life assurances. That might possibly be the case, but in these days of competition for endowment assurances he had some doubt on the subject. As to the method of calculating the cash value of reversionary bonuses, he agreed with Mr. Sprague that the value should not be calculated upon the valuation basis; that the policyholder was really taking out of the office sums which could be immediately invested to yield a higher rate of interest, and probably, being a select life, that he was also exercising an option against the office. As to the rapidity of the working which he mentioned, it was a fact that 8,000 cases were calculated in 20 hours, but they were all calculated upon the valuation sheets, where, of course, all the policies were classified according to the present ages, which, of course, made the work much more simple. He had completed his tables and printed them on a card, on the one side tables for applying cash bonuses, and the other side tables for applying the reversionary bonus, and now presented two copies to the Institute.

On the Premiums for Assurances against Issue to Widowers, Bachelors, and Married Men, or on the Calculation of the Values of Benefits depending on Marriage and on Birth of Issue. By T. B. SPRAGUE, *M.A., Manager of the Scottish Equitable Life Assurance Society.*

[Read before the Institute, 31 March 1890.]

I BELIEVE that the first publisht paper on the subject of premiums for insurance against issue was that of Mr. Archibald Day, which was read before the Institute on 31 January 1859 (*J.I.A.*, viii, 127). In this paper he gave from the Census Statistics the numbers of bachelors and widowers living in 1851, and the numbers of marriages of bachelors and widowers in the same year; and from these he deduced the probabilities of a bachelor and of a widower of any age marrying in a year. Then by means of the last-mentioned probabilities, combined with the probabilities of life given by the English Table No. 1, he calculated the value, at 3 per-cent interest, of £1 payable at the end of the year in which a husband shall contract a second

marriage. The formula he used for this purpose may be written
$_np_{\text{H}}(1 - _np_{\text{W}})\phi_{\text{H}+n-1} \times v^n$, where H and W represent the respective
ages of husband and wife, and ϕ is the probability of a widower
marrying in a year. It may be remarked in passing, that this
formula is only approximately correct, inasmuch as there is a
contingency that is not provided for in it, namely, that the
husband may be alive and his wife dead, but that he may already
have married a second time.

In this paper Mr. Day stated (p. 137) that he had little doubt
that the probabilities of marriage among the higher classes would
be found to be greater than those obtained from the general
population; and three years later, namely, on 24 March 1862, he
submitted to the *Institute* a second paper on the same subject
(*J.I.A.*, x, 181). In this he gave new marriage statistics, which
he had deduced from an examination of the marriages among
bachelors and widowers belonging to the peerage families; and
combining these with the mortality shown by the same table as
he used in his first paper, namely, the English Table No. 1, he
calculated a new set of single premiums for the same contingency
as in his first paper. These new premiums were found to be very
much larger than the former ones; and there can be no doubt
that they are much more suitable for adoption in the transactions
which come before life insurance offices. He also gave a short
table showing the single premium for the insurance of £1 payable
on the second marriage of a man who is already a widower. It is
not stated by what formula the values in this table were calculated,
but we may safely assume that it was the formula obtained from
the one above quoted, by assuming that the wife is dead—
$_np_{\text{H}}\phi_{\text{H}+n-1}.v^n$. The same remark applies to this formula as to
the former one, namely, that it does not take into account the
contingency of the man being alive but already remarried; and
this will have a tendency to make the values given by the formula
too large. The exact probability that a widower of any age will
be alive and unmarried after n years, is very simply found by
means of a combined marriage and mortality table, such as the
Tables C and E appended to this paper.

The premiums calculated by Mr. Day are, as he points out,
much more than sufficient to cover the risk of an insurance
against leaving issue, for the two reasons (1) that they assume the
sum assured will be payable immediately on the second marriage
taking place, whereas in practice the payment is almost always
deferred until the death of the husband; and (2) they also make

no allowance for the probability that there will be no issue of the marriage.

It seems to have been Mr. Day's intention to carry on his investigations, so as to obtain closer approximations to the true premiums for insurance against issue risk. In a third paper, read before the Institute on 27 March 1865 (*J.I.A.*, xii, 185), he dealt with the statistics of second marriages among the families of the peerage and gave a variety of curious and interesting statistics, which he had obtained by means of a careful study of the experience of the peerage families. One of the most important tables given in this paper relates to the probability that a marriage entered into by a man of any age will be fruitful. In his first paper he had said (p. 134): " There is at present hardly " sufficient data from which to calculate the probability of issue " from second marriages, and that part of the question must still " be solved by conjecture "; but the figures he now gives lead him to the conclusion that " the chances of issue to a widower " remarried at the age of 50, seem to be about evenly balanced ; " and to a marriage at 70, the chance of having issue appears to " be about 25 per-cent "; also, that " of all remarriages con- " tracted above the age of 50, scarcely more than one-third will " prove fruitful." It is obvious what an important bearing these conclusions have on the calculation of issue premiums. Here the question rested, so far as publisht results go, until I took it up.

In May 1879 I read to the Royal Society of Edinburgh a paper (reprinted *J.I.A.*, xxii, 359) " On the Probability that a Marriage entered into by a man above 40 will be Fruitful." I shall presently show how far the conclusions I there arrived at agree with those of Mr. Day, and I will now only say that, as I intended to apply my results to the actual calculation of issue premiums, it was necessary to adjust the probabilities, and I did this by the graphic method. On 31 March 1879, I submitted to the Institute my paper " On the Construction of a combined Marriage and Mortality Table " (*J.I.A.*, xxi, 406). In this paper, after dealing fully with the theory of the construction of such a table, and deducing the proper formulas to employ when we have the probabilities of marriage and death separately given, I calcu-lated those probabilities for the bachelors of the peerage families by means of Mr. Day's statistics adjusted by the graphic method. Having then constructed a combined marriage and mortality table for bachelors, I next introduced the element of interest, and

calculated by means of a table analogous to the familiar D-and-N Table, the values of the following benefits at 3 per-cent interest:

$(ba)_x$ = the value of an annuity of 1 payable so long as a bachelor of age x remains unmarried;

$(bA)_x$ = the value of an assurance of 1 payable at the end of the year in which a bachelor of age x dies unmarried;

$(bmE)_x$ = the value of an endowment of 1 payable at the end of the year in which a bachelor of the age x marries.

The values of these benefits are connected, as I showed, by the following equation:

$$(bA)_x + (bmE)_x = \frac{1 - i(ba)_x}{1 + i} .$$

In order to obtain the value of an assurance payable on the death of a bachelor after marrying, I thought it desirable to form a new mortality table applicable to married men, inasmuch as it is well establisht that the mortality among bachelors is heavier than among married men of the same age. I obtained the necessary table by comparing the results given by Mr. Day for bachelors with those given by Messrs. Bailey and Day (*J.I.A.*, ix, 305) for the whole of the peerage males; and from this table I calculated at 3 per-cent the values of annuities and assurances on the lives of married men. These values are given in the above-mentioned paper (*J.I.A.*, xxi, 440) in the columns headed with the symbols $(ma)_x$ and $(mA)_x$, and the mortality table itself is now given in the appended Table L. By means of these assurances, and by the process described on p. 441 of my paper, I then calculated

$(bmA)_x$ = the value of an assurance payable on the death of a bachelor after marrying.

Lastly, by means of the above-mentioned graduated probabilities of a marriage being fruitful, I calculated

$(bfmA)_x$ = the value of an assurance payable on the death of a bachelor of 40 or upwards, after contracting a fruitful marriage.

My next paper bearing on the subject was the one I read to the Institute on 24 November 1879, "On the Rate of Remarriage among Widowers" (*J.I.A.*, xxii, 77). In this I showed that, as might have been anticipated, the marriage rate depends not only on the age of the widower, but also on the length of time which has elapst since the death of his wife. For the purpose of showing to what extent the marriage rate depends upon this

length of time, I have compiled the following table, which
contains the different central marriage rates for ages 26, 36, 46,
56, and 66.   The values as calculated from Mr. Day's probability
of marrying in a year (see *J.I.A.*, xxii, 93) are given in column (2);
those for widowers whose wives have been so long dead that the
rate of remarriage has reached its ultimate magnitude, are given
in column (3); and in the remaining columns the rates for men
who became widowers at the ages 25, 35, 45, 55, 65, respectively,
and have attained the above ages without marrying.   My reason
for taking the ages 26, 36, &c., is that the marriage rate attains
its greatest value, not in the year immediately following the death
of the wife, but in the next year.

Age	Day	Ultimate Rate	SPRAGUE				
			Age at Entry (Death of Wife)				
			25	35	45	55	65
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
26	·168	·231	·231	...	...	...	...
36	·115	·091	·091	·156	...	...	...
46	·085	·043	·043	·052	·148	...	...
56	·046	·023	·023	·023	·033	·146	...
66	·024	·011	·011	·011	·011	·011	·065

Soon after the publication of the last-named paper, I calcu-
lated, by means of the probabilities of remarriage given in it, the
values, at 3 per-cent interest, of (1) endowments payable on the
remarriage of a widower of any age; (2) assurances payable on the
death of a widower after remarrying; and (3) similar assurances
if there is issue of the subsequent marriage.   As, however, I was
not satisfied with the probabilities of issue which I had thus far
obtained, I have delayed until now the publication of these
results.

In July 1887 I read to the Royal Society of Edinburgh a
paper (reprinted *J.I.A.*, xxvii, 195) "On the Probability that a
Marriage entered into by a Man of any Age will be Fruitful."
In this I showed that the probability of a marriage being
fruitful was greater in the case of peers and heirs apparent than
in the case of other men belonging to the peerage families.   It
may be useful here to compare briefly the results obtained in this
paper and the former one on the same subject, with those
obtained previously by Mr. Day; and the more so, because I did

not make any such comparison in either of those papers, having in fact overlooked Mr. Day's investigations as to this point, until my own had been completed and publisht. The following table contains the number of marriages observed at each quinquennium of age, the number of these that were childless, and the percentage of the childless to the total marriages. It will be observed that Mr. Day's figures relate entirely to widowers. In my paper of 1879 I gave separately the figures for bachelors and widowers; but I subsequently combined them, as I saw no reason for thinking that there would be any difference in the probabilities of issue in the two classes of men.

Ages at Marriage	DAY (Widowers)			SPRAGUE (1879) (Bachelors and Widowers)			SPRAGUE (1887)					
							Peers and Heirs Apparent			Other Peerage Males		
	Marriages	Of which were Childless	Percentage	Marriages	Of which were Childless	Percentage	Marriages	Of which were Childless	Percentage	Marriages	Of which were Childless	Percentage
16–23	...	...	...	...	...	...	115	14	12·2	131	15	11·5
24–29	32	4	12·5	...	...	...	159	21	13·2	401	78	19·5
30–34	53	3	5·7	...	...	...	82	8	9·8	218	45	20·6
35–39	77	9	11·7	...	...	...	45	10	22·2	112	31	27·7
40–44	88	28	31·8	127	36	28·4	28	9	32·1	62	20	32·3
45–49	66	16	24·3	69	20	29·0	20	6	30·0	41	15	36·6
50–54	52	26	50·0	49	16	32·7	16	4	25·0	19	8	42·1
55–59	53	33	62·3	43	26	60·5	18	10	55·6	12	10	83·3
60–64	37	27	73·0	27	19	70·4	15	10	66·7	8	7	87·5
65–69	25	20	80·0	13	10	76·9	9	8	88·9	3	3	100·0
70–74	19	11	73·7	8	7	87·5	5	5	100·0	..	...	...
75 & upds.	5	5	100·0	3	3	100·0	3	3	100·0	...	...	...
All Ages	507	185	...	339	137	...	515	108	21·0	1,007	232	23·0
Under 50	316	60	19·0	...	...	...	449	68	15·1	965	204	21·1
50 & upds.	191	125	65·5	143	81	56·6	66	40	60·6	42	28	66·7

Comparing the three sets of figures, it will be seen that my figures of 1879 give the lowest probability that a marriage will be childless; but I am satisfied that, for the reason stated in my paper of 1887, this probability is too low, whereas, in consequence of the extreme care I took in that paper to sift the facts thoroughly, I feel great confidence in the results contained in it. Taking the ages over 50, which for business purposes are the most important, it will be seen that out of 108 marriages observed by me, 68 were childless, or 63 per-cent; while Mr. Day's corresponding percentage is 65·5; which is as close an agreement as could be expected from the nature of the case. Taken in the

aggregate, then, my later results may be said to be identical with
Mr. Day's; but the figures clearly show that the distinction I
have drawn between the men who married when they were either
in possession of the title, or the next in succession to it, and the
men who belonged to younger branches of the family, is a valid
distinction of practical importance.

Having now, by means of my new probabilities of issue,
calculated more correct values of the single premiums for issue
assurances on the lives of bachelors and widowers, that is to say,
for assurances payable only if they marry (or remarry) and have
issue, I submit the results to the Institute in the following tables.
Mr. James Chatham, F.I.A., F.F.A., a member of the actuarial
staff of the *Scottish Equitable* Life Assurance Society, has at my
suggestion written the full description of the construction and use
of the tables, which is appended to them (see page 384); and
it will therefore be unnecessary for me to say anything on that
part of the subject. The most important results are those
contained in Tables J and H, namely,

$(wmE)_{[x]} =$ the value of an endowment payable on the remarriage
of a man of the age x, who has just become a
widower. Such a man I call, for brevity, a
commencing widower.

$(wmE)_x =$ the value of an endowment payable on the remarriage
of a man who has been a widower so long that the
marriage rate has reached its ultimate value. Such
a man I call a *chronic* widower; and those who have
not been widowers so long, I call *recent* widowers.

$(wmA)_{[x]} =$ the value of an assurance payable on the death of a
commencing widower if he shall remarry.

$(wmA)_x =$ the value of a similar assurance payable on the death
of a chronic widower.

$(wfmA)_{[x]} =$ the value of an assurance payable on the death of a
commencing widower if he shall remarry and have
issue.

$(wfmA)_x =$ the value of a similar assurance payable on the death
of a chronic widower.

Consistently with the notation adopted in my paper on
"Select" Mortality Tables, *J.I.A.*, xxii, 391, I use the symbol
$[x]$ to denote that x was the widower's age when his wife died;
and I call this age, his *age at entry*.

The values of $(wmE)_{[x]}$ and $(wmA)_{[x]}$ do not run quite so

regularly as could be desired, but the fluctuations are not great. Practically speaking, the values of the latter are constant from ages 25 to 55, the greater probability of death at the higher ages being counterbalanced by the smaller probability of remarriage.

In the following table Mr. Day's single premiums for an assurance (or, more properly speaking, an endowment) payable on the second marriage of a widower, are compared with the figures now brought out:

Age	DAY	SPRAGUE					
	$(wmE)_x$	$(wmE)_{[x]}$	$(wmE)_x$	$(wmA)_{[x]}$	$(wmA)_x$	$(wfmA)_{[x]}$	$(wfmA)_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
50	·5163	·4716	·2315	·2829	·1412	·1545	·0696
55	·3666	·4709	·1597	·3062	·1072	·1337	·0401
60	·2397	·3067	·0999	·2154	·0749	·0637	·0181
65	·1471	·1740	·0529	·1313	·0405	·0208	·0057
70	·0879	·0954	·0241	·0762	·0173	·0067	·0013
75	·0384	·0658	·0045	·0554	·0038	·0021	·0001

It will be observed that Mr. Day's values, as given in the second column, are, except at age 50, intermediate to the corresponding values now obtained and given in the third and fourth columns. The other columns of the table require no explanation.

By means of the new issue probabilities, I have calculated fresh values of $(bfmA)_x$, or the single premium for an assurance payable on the death of a bachelor if he shall marry and have issue; and these are given in Table K, in which are also reprinted, for convenience of reference, the values of $(bmE)_x$ and $(bmA)_x$. I have also given in the same table the probabilities that a bachelor of any age will (1) marry, and (2) marry and have issue.

Thus far I have said nothing about the case which most commonly occurs in practice, namely, the assurance against issue to a married man by a future wife, the present wife being past child-bearing age. The foregoing results may be used to calculate the single premiums for such assurances, and the premiums in the following table have been calculated by Mr. Chatham by the process he describes in his paper.

Value at 3 per-cent interest of an Endowment payable at the end of the year in which a married man shall contract a second marriage.

(1) Mr. Sprague's Results.

Age of Present Wife	AGE OF HUSBAND										
	25	30	35	40	45	50	55	60	65	70	75
25	·11829	·09859	·08171	·06706	·05328	...	...	...	...	...	...
30	·13463	·11156	·09170	·07462	·05910	·04456	...	...	...	...	...
35	...	·12910	·10543	·08492	·06655	·05005	·03316	...	...	...	...
40	...	...	·12322	·09842	·07602	·05621	·03690	·01993	...	...	...
45	...	...	...	·11865	·09108	·06633	·04285	·02297	·01157	...	...
50	...	...	...	...	·11447	·08343	·05357	·02870	·01442	·00730	...
55	...	...	...	...	..	·10808	·06999	·03769	·01909	·00970	·00473
60	...	...	...	...	...	...	·09149	·04965	·02535	·01310	·00642
65	...	...	...	...	...	...	...	·06502	·03354	·01758	·00897
70	...	...	...	...	...	...	...	...	·04241	·02256	·01175
75	...	...	...	...	...	...	...	...	...	·02851	·01533
80	...	...	...	...	...	...	...	...	...	...	·01978

(2) Mr. Day's Results, *J.I.A.*, x, 195.

Age of Present Wife	AGE OF HUSBAND					
	50	55	60	65	70	75
45	·0637	·0375	·0208	...	...	...
50	·0782	·0458	·0251	·0128	...	...
55	·1012	·0618	·0342	·0172	·0077	...
60	...	·0842	·0473	·0244	·0111	·0039
65	...	...	·0643	·0338	·0156	·0055
70	...	...	...	·0456	·0215	·0077
75	...	...	...	...	·0289	·0106
80	...	...	...	...	...	·0144

For the purpose of comparison, I give also Mr. Day's similar but smaller table (*J.I.A.*, x, 195). It will be noticed that, although my values are in many cases very nearly equal to Mr. Day's, they are throughout larger, with the exception of the three cases where the husband is 70 and the wife 75, and the husband 65 and the wife 65 or 70, the difference amounting to as much as 67 per-cent when the husband is 75 and the wife 60.

It will be useful to give here a list of the following tables.

Table A.—Death and Issue Rates used in the construction of the Tables.

Table B.—Chronic Widowers—ages 45 onwards—Central Marriage Rates, $(wm\mu)_{x+\frac{1}{2}}$.

Table C.—Chronic Widowers—ages 45 onwards—Combined Marriage and Mortality Tables; also probabilities, (1) of marrying, and (2) of marrying and having issue.

Table D.—Recent Widowers—Quinquennial ages at entry (death of wife)—Central Marriage Rates for all durations of widowerhood, $(wm\mu)_{[x]+n+\frac{1}{2}}$.

Table E.—Recent Widowers—Quinquennial ages at entry (death of wife)—Combined Marriage and Mortality Tables; also probabilities, (1) of marrying, and (2) of marrying and having issue.

Table F.—Chronic Widowers—ages 45 onwards—3 per-cent Commutation Table.

Table G.—Recent Widowers—Quinquennial ages at entry—3 per-cent Commutation Tables.

Table H.—Chronic Widowers—Values at 3 per-cent interest of Marriage Endowments, $(wmE)_x$, and of Assurances depending on Marriage, $(wmA)_x$, and on birth of issue, $(wfmA)_x$.

Table I.—Recent Widowers—Quinquennial ages at entry—Values at 3 per-cent interest of Marriage Endowments, $(wmE)_{[x]+n}$, and of Assurances depending on Marriage, $(wmA)_{[x]+n}$, and on birth of issue, $(wfmA)_{[x]+n}$.

Table J.—Commencing Widowers—all ages at entry $(n=0)$—Values at 3 per-cent interest of Marriage Endowments, $(wmE)_{[x]}$, and of Assurances depending on Marriage, $(wmA)_{[x]}$, and on birth of issue, $(wfmA)_{[x]}$.

Table K.—Bachelors—Values at 3 per-cent interest of Marriage Endowments, $(bmE)_x$, and of Assurances depending on Marriage, $(bmA)_x$, and on birth of issue, $(bfmA)_x$; also probabilities, (1) of marrying, and (2) of marrying and having issue.

Table L.—Mortality Table for Married Men, as deduced by Mr. Sprague from the Peerage Experience.

Table M.—Mortality Tables for Females, deduced from the observations on the Peerage Females by Messrs. Bailey and Day, and adjusted by Dr. T. M. Thiele.

TABLE A.—*Death and Issue Rates used in the construction of the following tables.*

The Central Death Rate has been calculated from the probability of a married man dying in a year, *J.I.A.*, xxi, 132, col. (6). The Probabilities of a marriage being fruitful (or more briefly the probabilities of issue) are those for Peers and Heirs Apparent, *J.I.A.*, xxvii, 212.

Age x	Central Death Rate $(wd\mu)_{x+\frac{1}{2}}$	Probability of Issue $(pfm)_x$	Age x	Central Death Rate $(wd\mu)_{x+\frac{1}{2}}$	Probability of Issue $(pfm)_x$	Age x	Central Death Rate $(wd\mu)_{x+\frac{1}{2}}$	Probability of Issue $(pfm)_x$
15	·0050	...	42	·0106	·755	69	·0544	·117
16	·0057	...	43	·0112	·740	70	·0592	·102
17	·0065	...	44	·0118	·725	71	·0645	·088
18	·0077	...	45	·0124	·709	72	·0708	·075
19	·0089	·879	46	·0130	·693	73	·0780	·063
20	·0104	·878	47	·0135	·677	74	·0863	·052
21	·0109	·877	48	·0141	·661	75	·0954	·042
22	·0075	·876	49	·0147	·645	76	·1057	·033
23	·0067	·875	50	·0152	·628	77	·1169	·025
24	·0045	·874	51	·0157	·611	78	·1289	·018
25	·0039	·873	52	·0162	·593	79	·1419	·012
26	·0042	·872	53	·0168	·574	80	·1559	·007
27	·0048	·871	54	·0176	·553	81	·1716	·003
28	·0056	·870	55	·0184	·530	82	·1875	...
29	·0062	·869	56	·0193	·505	83	·2050	...
30	·0065	·867	57	·0203	·478	84	·2209	...
31	·0066	·865	58	·0213	·449	85	·2345	...
32	·0067	·862	59	·0224	·418	86	·2468	...
33	·0068	·858	60	·0238	·385	87	·2614	...
34	·0072	·852	61	·0256	·350	88	·2718	...
35	·0074	·844	62	·0286	·313	89	·2899	...
36	·0077	·834	63	·0308	·275	90	·3249	...
37	·0080	·823	64	·0341	·239	91	·3706	...
38	·0084	·811	65	·0377	·206	92	·4262	...
39	·0089	·798	66	·0415	·178	93	·5249	...
40	·0094	·784	67	·0456	·154	94	·6797	...
41	·0099	·770	68	·0499	·134	95	·9347	...

TABLE B.—*Chronic Widowers—Ages 45 onwards—Central Marriage Rates*, $(wm\mu)_{x+\frac{1}{2}}$.

Reprinted from *J.I.A.*, xxii, 93.

Age x	$(wm\mu)_{x+\frac{1}{2}}$	Age x	$(wm\mu)_{x+\frac{1}{2}}$	Age x	$(wm\mu)_{x+\frac{1}{2}}$
45	·0163	57	·0214	69	·0074
46	·0131	58	·0201	70	·0064
47	·0402	59	·0188	71	·0055
48	·0376	60	·0175	72	·0046
49	·0354	61	·0163	73	·0038
50	·0333	62	·0151	74	·0030
51	·0313	63	·0139	75	·0023
52	·0294	64	·0127	76	·0016
53	·0276	65	·0116	77	·0010
54	·0259	66	·0105	78	·0005
55	·0243	67	·0094	79	·0002
56	·0228	68	·0084	...	...

TABLE C.—*Chronic Widowers—Ages 45 onwards—Combined Marriage and Mortality Table; also probabilities* (1) *of marrying, and* (2) *of marrying and having issue.*

Age x	Widowers living	Decrements caused by		Widowers marrying who have issue	Probability that a Widower will marry	Probability that a Widower will marry and have issue
		Widowers dying	Widowers marrying			
	$(wl)_x$	$(wd)_x$	$(wm)_x$	$(wfm)_x$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
45	100,000	1,204	4,498	3,189	·39554	·22399
46	94,298	1,188	3,953	2,740	·37176	·20372
47	89,157	1,175	3,490	2,363	·34885	·18474
48	84,492	1,164	3,097	2,047	·32680	·16697
49	80,231	1,153	2,770	1,787	·30556	·15032
50	76,308	1,134	2,481	1,558	·28496	·13463
51	72,693	1,117	2,223	1,358	·26500	·11989
52	69,353	1,100	1,994	1,182	·24571	·10608
53	66,259	1,091	1,789	1,027	·22710	·09319
54	63,379	1,089	1,606	888	·20919	·08123
55	60,684	1,092	1,414	765	·19201	·07019
56	58,148	1,098	1,298	656	·17555	·06009
57	55,752	1,109	1,169	559	·15981	·05092
58	53,174	1,117	1,053	473	·14176	·04264
59	51,304	1,129	945	395	·13036	·03523
60	49,230	1,117	844	325	·11665	·02869
61	47,239	1,185	754	264	·10370	·02302
62	45,300	1,211	670	210	·09419	·01817
63	43,389	1,306	590	162	·08009	·01414
64	41,493	1,381	515	123	·06953	·01088
65	39,597	1,457	448	92	·05986	·00829
66	37,692	1,526	386	69	·05099	·00626
67	35,780	1,589	327	50	·04294	·00468
68	33,864	1,643	276	37	·03570	·00346
69	31,945	1,687	229	27	·02919	·00250
70	30,029	1,722	186	19	·02312	·00177
71	28,121	1,753	149	13	·01839	·00122
72	26,219	1,790	116	9	·01102	·00080
73	24,313	1,822	89	6	·01034	·00051
74	22,402	1,850	64	3	·00726	·00030
75	20,488	1,865	45	2	·00480	·00016
76	18,578	1,864	28	1	·00288	·00008
77	16,686	1,841	16	...	·00152	...
78	14,829	1,795	7	...	·00063	...
79	13,027	1,726	2	...	·00018	...
80	11,299	1,634	...	...	...	...
81	9,665	1,527	...	...	...	...
82	8,138	1,395	...	...	...	...
83	6,743	1,254	...	...	...	...
84	5,489	1,091	...	...	...	...
85	4,398	923	...	...	...	...
86	3,475	764	...	...	...	...
87	2,711	627	...	...	...	...
88	2,084	498	...	...	...	...
89	1,586	402	...	...	...	...
90	1,184	331	...	...	...	...
91	853	267	...	...	...	...
92	586	206	...	...	...	...
93	380	158	...	...	...	...
94	222	113	...	...	...	...
95	109	69	...	...	...	...
96	40	33	...	...	...	...
97	7	7	...	...	...	...

TABLE D.—*Recent Widowers—Quinquennial ages at entry (death of wife): Central Marriage Rates for all durations of widowerhood,* $(wm\mu)_{[x]+n+\frac{1}{2}}$.

Reprinted from *J.I.A.*, xxii, 96, 97.

Years elapst n	Age attained $[x]+n$	Central Marriage Rate	Years elapst n	Age attained $[x]+n$	Central Marriage Rate	Years elapst n	Age attained $[x]+n$	Central Marriage Rate	Years elapst n	Age attained $[x]+n$	Central Marriage Rate
Age at Entry 25			Age at Entry 35			Age at Entry 45			Age at Entry 60		
0	25	·116	0	35	·050	0	45	·032	0	60	·033
1	26	·231	1	36	·156	1	46	·148	1	61	·117
2	27	·223	2	37	·149	2	47	·140	2	62	·090
3	28	·192	3	38	·130	3	48	·122	3	63	·070
4	29	·165	4	39	·112	4	49	·099	4	64	·051
5	30	·119	5	40	·097	5	50	·080	5	65	·033
6	31	·136	6	41	·088	6	51	·069	6	66	·020
7	32	·125	7	42	·079	7	52	·061	7	67	·015
8	33	·115	8	43	·072	8	53	·056	8	68	·008
9	34	·109	9	44	·067	9	54	·048			
10	35	·099	10	45	·060	10	55	·040	Age at Entry 65		
11	36	·091	11	46	·052	11	56	·033			
12	37	·085	12	47	·046	12	57	·027			
13	38	·078	13	48	·041	13	58	·023	0	65	·035
14	39	·072	14	49	·036	14	59	·019	1	66	·065
15	40	·068	15	50	·033				2	67	·050
16	41	·063							3	68	·035
17	42	·058				Age at Entry 50			4	69	·021
18	43	·054	Age at Entry 40						5	70	·014
19	44	·050				0	50	·028	6	71	·006
20	45	·046	0	40	·038	1	51	·145			
			1	41	·151	2	52	·135			
			2	42	·134	3	53	·109	Age at Entry 70		
Age at Entry 30			3	43	·115	4	54	·090			
			4	44	·098	5	55	·070	0	70	·040
0	30	·073	5	45	·082	6	56	·055	1	71	·039
1	31	·188	6	46	·071	7	57	·043	2	72	·022
2	32	·170	7	47	·064	8	58	·034	3	73	·011
3	33	·148	8	48	·057	9	59	·029	4	74	·003
4	34	·134	9	49	·050	10	60	·025			
5	35	·120	10	50	·045	11	61	·020			
6	36	·109	11	51	·039	12	62	·015	Age at Entry 75		
7	37	·100	12	52	·033						
8	38	·094	13	53	·029				0	75	·047
9	39	·089	14	54	·026	Age at Entry 55			1	76	·029
10	40	·080	15	55	·024				2	77	·001
11	41	·072	...	...	...	0	55	·031			
12	42	·064	...	...	...	1	56	·146	Age at Entry 80		
13	43	·058	...	...	...	2	57	·137			
14	44	·052	...	...	...	3	58	·121			
15	45	·046	...	...	...	4	59	·104	0	80	·055
...	...	...	...	...	...	5	60	·085	...	...	...
...	...	...	...	...	...	6	61	·069	...	...	...
...	...	...	...	...	...	7	62	·051	...	...	...
...	...	...	...	...	...	8	63	·034	...	...	...
...	...	...	...	...	...	9	64	·022	...	...	...
...	...	...	...	...	...	10	65	·012	...	...	...

TABLE E.—*Recent Widowers—Quinquennial Ages at entry (death of wife): Combined Marriage and Mortality Tables: also probabilities (1) of marrying, and (2) of marrying and having issue.*

Years elapst u	Age attained $[x] + u$	Widowers living	Widowers dying	Widowers marrying	Widowers marrying who have issue	Probability that a Widower will marry	Probability that a Widower will marry and have issue
		$(wl)_{[x]=u}$	$(wd)_{[x]=u}$	$(wm)_{[x]=u}$	$(wfm)_{[x]=u}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Age at Entry 25				
0	25	1,142,664	4,215	125,052	109,171	·91059	·77173
1	26	1,013,397	3,818	200,160	182,649	·90334	·76582
2	27	800,119	3,455	160,184	139,518	·88235	·74167
3	28	636,483	3,255	111,214	96,756	·85753	·71316
4	29	522,014	2,994	79,340	68,946	·83253	·68419
5	30	439,683	2,659	60,787	52,702	·80797	·65549
6	31	376,237	2,325	47,763	41,315	·78265	·62595
7	32	326,149	2,056	38,250	32,971	·75640	·59541
8	33	285,843	1,838	30,985	26,585	·72925	·56402
9	34	253,020	1,729	26,064	22,207	·70439	·53211
10	35	225,227	1,589	21,171	17,868	·67222	·49918
11	36	202,167	1,491	17,558	14,643	·64322	·46704
12	37	183,448	1,408	14,897	12,261	·61129	·43571
13	38	167,113	1,352	12,495	10,133	·58508	·40485
14	39	153,266	1,317	10,606	8,464	·55612	·37531
15	40	141,343	1,284	9,253	7,255	·52832	·34709
16	41	130,806	1,256	7,951	6,122	·50014	·31959
17	42	121,599	1,241	6,819	5,148	·47262	·29344
18	43	113,539	1,227	5,938	4,394	·44611	·26893
19	44	106,374	1,215	5,159	3,740	·42034	·24573
20	45	100,000	1,204	4,498	3,189	·39554	·22399
			Age at Entry 30				
0	30	536,978	3,367	37,700	32,686	·83707	·68534
1	31	495,911	2,992	84,963	73,493	·83037	·67618
2	32	407,956	2,519	63,722	54,928	·80113	·64182
3	33	341,715	2,163	46,939	40,274	·76995	·60549
4	34	292,613	1,976	36,624	31,203	·73874	·56947
5	35	254,013	1,774	28,656	24,185	·70681	·53316
6	36	223,583	1,633	23,026	19,204	·67485	·49755
7	37	198,924	1,515	18,873	15,532	·64275	·46269
8	38	178,536	1,433	15,965	12,917	·61044	·42853
9	39	161,138	1,374	13,672	10,910	·57727	·39445
10	40	146,092	1,320	11,187	8,771	·54314	·36039
11	41	133,585	1,277	9,239	7,114	·51025	·32847
12	42	123,069	1,253	7,593	5,733	·47877	·29874
13	43	114,223	1,232	6,404	4,739	·44938	·27168
14	44	106,587	1,216	5,371	3,894	·42149	·24668
15	45	100,000	1,204	4,498	3,189	·39554	·22399

TABLE E—(*continued*).

Years elapsed n	Age attained $[x]+n$	Widowers living	Widowers dying	Widowers marrying	Widowers marrying who have issue	Probability that a Widower will marry	Probability that a Widower will marry and have issue
		$(wl)_{[x]+n}$	$(wd)_{[x]+n}$	$(wm)_{[x]+n}$	$(wfm)_{[x]+n}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Age at Entry 35

0	35	308,724	2,230	15,005	12,664	·75175	·57363
1	36	291,189	2,083	42,031	35,054	·74172	·56410
2	37	247,375	1,841	34,175	28,127	·70762	·52299
3	38	211,359	1,669	25,698	20,841	·66650	·47904
4	39	183,992	1,554	19,432	15,507	·62597	·43702
5	40	163,009	1,461	15,013	11,770	·58734	·39815
6	41	146,535	1,390	12,293	9,466	·55092	·36259
7	42	132,852	1,343	10,046	7,584	·51513	·32868
8	43	121,463	1,301	8,396	6,213	·48073	·29706
9	44	111,766	1,266	7,204	5,223	·44731	·26724
10	45	103,296	1,235	5,981	4,241	·41425	·23859
11	46	96,080	1,205	4,839	3,353	·38311	·21237
12	47	90,036	1,183	4,022	2,723	·35508	·18939
13	48	84,831	1,166	3,385	2,237	·32946	·16891
14	49	80,280	1,153	2,849	1,818	·30597	·15062
15	50	76,308	1,134	2,481	1,558	·28496	·13463

Age at Entry 40

0	40	208,721	1,925	7,748	6,074	·66771	·46294
1	41	199,048	1,833	27,817	21,420	·66123	·45492
2	42	169,398	1,668	21,169	15,983	·61275	·40810
3	43	146,561	1,539	15,854	11,732	·56380	·36264
4	44	129,168	1,442	12,000	8,700	·51697	·32065
5	45	115,726	1,369	9,061	6,425	·47332	·28271
6	46	105,296	1,309	7,175	4,972	·43415	·24970
7	47	96,812	1,261	5,965	4,038	·39808	·22022
8	48	89,586	1,222	4,931	3,259	·36361	·19291
9	49	83,433	1,191	4,041	2,606	·33132	·16807
10	50	78,201	1,155	3,416	2,145	·30182	·14598
11	51	73,630	1,127	2,795	1,708	·27416	·12591
12	52	69,708	1,104	2,245	1,331	·24949	·10849
13	53	66,359	1,093	1,881	1,080	·22824	·09390
14	54	63,385	1,088	1,613	892	·20927	·08127
15	55	60,684	1,092	1,444	765	·19201	·07019

TABLE E—(*continued*).

Years elapst n	Age attained $[x]+n$	Widowers living	Decrements caused by		Widowers marrying who have issue	Probability that a Widower will marry	Probability that a Widower will marry and have issue
			Widowers dying	Widowers marrying			
		$(wl)_{[x]+n}$	$(wd)_{[x]+n}$	$(wm)_{[x]+n}$	$(wfm)_{[x]+n}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Age at Entry 45				
0	45	171,673	2,081	5,374	3,810	·62664	·39004
1	46	164,218	1,968	22,494	15,588	·62236	·38454
2	47	139,756	1,756	18,171	12,302	·57034	·34031
3	48	119,829	1,585	13,688	9,048	·51354	·29425
4	49	104,556	1,457	9,794	6,317	·45765	·25069
5	50	93,305	1,356	7,125	4,475	·40787	·21322
6	51	84,824	1,279	5,615	3,431	·36465	·18179
7	52	77,930	1,218	4,577	2,714	·32485	·15384
8	53	72,135	1,172	3,898	2,237	·28750	·12858
9	54	67,065	1,139	3,117	1,724	·25112	·10494
10	55	62,809	1,121	2,441	1,294	·21851	·08461
11	56	59,247	1,113	1,906	962	·19044	·06786
12	57	56,228	1,115	1,483	709	·16678	·05439
13	58	53,630	1,119	1,207	542	·14721	·04380
14	59	51,304	1,129	945	395	·13036	·03523
			Age at Entry 50				
0	50	125,219	1,866	3,432	2,155	·54329	·29395
1	51	119,921	1,745	16,095	9,834	·53867	·28896
2	52	102,081	1,510	12,812	7,598	·47514	·24312
3	53	87,729	1,390	8,997	5,164	·40683	·19629
4	54	77,342	1,288	6,605	3,653	·34514	·15589
5	55	69,449	1,222	4,656	2,467	·28925	·12101
6	56	63,571	1,182	3,371	1,702	·21276	·09338
7	57	59,018	1,161	2,460	1,176	·20437	·07174
8	58	55,397	1,150	1,833	823	·17332	·05520
9	59	52,414	1,147	1,482	619	·14822	·04264
10	60	49,785	1,155	1,215	468	·12628	·03245
11	61	47,415	1,188	927	324	·10697	·02421
12	62	45,300	1,241	670	210	·09149	·01817
			Age at Entry 55				
0	55	112,628	2,019	3,407	1,806	·53396	·23141
1	56	107,202	1,909	14,457	7,301	·52920	·22628
2	57	90,836	1,710	11,537	5,515	·46540	·18667
3	58	77,589	1,545	8,764	3,935	·39616	·14747
4	59	67,280	1,421	6,581	2,751	·32659	·11157
5	60	59,278	1,337	4,779	1,840	·25966	·08023
6	61	53,162	1,300	3,503	1,226	·19964	·05485
7	62	48,359	1,302	2,372	743	·14704	·03495
8	63	44,685	1,332	1,472	405	·10603	·02120
9	64	41,881	1,388	896	214	·07799	·01296
10	65	39,597	1,457	448	92	·05986	·00829

TABLE E—*(continued)*.

Years elapst n	Age attained $[x]+n$	Widowers living	Widowers dying	Widowers marrying	Widowers marrying who have issue	Probability that a Widower will marry	Probability that a Widower will marry and have issue
		$(wl)_{[x]+n}$	$(wd)_{[x]+n}$	$(wm)_{[x]+n}$	$(wfm)_{[x]+n}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Age at Entry 60							
0	60	67,969	1,572	2,181	840	·34124	·10011
1	61	64,216	1,536	7,013	2,455	·32722	·09288
2	62	55,667	1,471	4,731	1,481	·25148	·06305
3	63	49,465	1,449	3,297	907	·18737	·04102
4	64	44,719	1,461	2,188	523	·13354	·02510
5	65	41,070	1,495	1,309	270	·09214	·01460
6	66	38,266	1,542	743	132	·06468	·00862
7	67	35,981	1,593	524	81	·04816	·00549
8	68	33,864	1,643	276	37	·03570	·00346
Age at Entry 65							
0	65	46,772	1,701	1,580	325	·19045	·03000
1	66	43,491	1,716	2,684	478	·16850	·02479
2	67	39,091	1,702	1,865	287	·11880	·01535
3	68	35,524	1,701	1,193	160	·07822	·00881
4	69	32,630	1,712	660	77	·04861	·00469
5	70	30,258	1,728	409	42	·03060	·00251
6	71	28,121	1,753	149	13	·01839	·00122
Age at Entry 70							
0	70	32,918	1,857	1,254	128	·10163	·00883
1	71	29,807	1,829	1,105	97	·07015	·00546
2	72	26,873	1,819	565	42	·03668	·00243
3	73	24,489	1,829	258	16	·01718	·00094
4	74	22,402	1,850	64	3	·00726	·00030
Age at Entry 75							
0	75	22,025	1,963	966	41	·06858	·00265
1	76	19,096	1,891	519	17	·02850	·00093
2	77	16,686	1,841	16	...	·00152	...
Age at Entry 80							
0	80	11,943	1,684	594	4	·00498	·00000
1	81	9,665	1,527	...	...	...	...

TABLE F.—*Chronic Widowers—ages 45 onwards—3 per-cent Commutation Table.*

Age x	$(wD)_x$	$(wmEN)_x$	$(wmM)_x$	$(wfmM)_x$	Age x
(1)	(2)	(3)	(4)	(5)	(6)
45	26,444	8,373	4,812·4	2,780·7	45
46	24,210	7,219	4,218·6	2,359·7	46
47	22,223	6,233	3,702·9	2,002·3	47
48	20,447	5,388	3,252·9	1,697·6	48
49	18,850	4,661	2,858·4	1,436·9	49
50	17,406	4,029	2,509·6	1,211·9	50
51	16,099	3,479	2,201·0	1,018·1	51
52	14,912	3,001	1,927·8	851·1	52
53	13,832	2,585	1,685·6	707·5	53
54	12,845	2,223	1,470·7	584·2	54
55	11,941	1,907	1,280·0	478·7	55
56	11,108	1,631	1,110·5	388·9	56
57	10,340	1,390	959·9	312·9	57
58	9,629	1,179	825·9	248·8	58
59	8,969	995	706·5	195·2	59
60	8,356	835	600·7	150·9	60
61	7,785	696	507·2	115·0	61
62	7,248	575	424·6	86·1	62
63	6,740	471	352·2	63·4	63
64	6,257	382	289·1	46·0	64
65	5,798	307	234·8	33·1	65
66	5,358	243	188·2	23·5	66
67	4,938	190	148·6	16·4	67
68	4,537	146	115·5	11·3	68
69	4,156	110	88·0	7·6	69
70	3,793	81	65·5	5·0	70
71	3,448	58	47·6	3·2	71
72	3,121	41	33·4	1·9	72
73	2,810	27	22·5	1·1	73
74	2,514	17	14·4	·6	74
75	2,232	10	8·6	·3	75
76	1,965	5	4·6	·1	76
77	1,714	2	2·1	...	77
78	1,478	1	·8	...	78
79	1,261	...	·2	...	79

Table G.—*Recent Widowers—Quinquennial Ages at Entry:*
3 per-cent Commutation Tables.

Years elapst n	Age attained $[x]+n$	$(wD)_{[x]+n}$	$(wmEN)_{[x]+n}$	$(wmM)_{[x]+n}$	$(wfmM)_{[x]+n}$
(1)	(2)	(3)	(4)	(5)	(6)
		Age at Entry 25			
0	25	545,741	423,610	160,909·2	137,470·7
1	26	469,907	365,624	140,706·6	119,833·8
2	27	360,205	271,327	107,117·3	90,543·9
3	28	278,192	201,315	81,633·9	68,347·9
4	29	221,514	154,122	64,099·3	53,092·8
5	30	181,144	121,435	51,714·2	42,330·1
6	31	150,490	97,121	42,322·9	34,187·8
7	32	126,656	78,573	35,018·6	27,869·6
8	33	107,770	64,152	29,227·8	22,877·9
9	34	92,617	52,810	24,582·8	18,892·5
10	35	80,042	43,547	20,714·1	15,596·4
11	36	69,858	36,243	17,603·0	12,970·6
12	37	61,442	30,361	15,048·5	10,840·2
13	38	54,349	25,516	12,902·9	9,074·4
14	39	48,394	21,571	11,121·5	7,629·7
15	40	43,330	18,319	9,625·0	6,435·5
16	41	38,932	15,565	8,333·1	5,422·6
17	42	35,137	13,268	7,235·0	4,577·0
18	43	31,852	11,355	6,303·4	3,873·7
19	44	28,973	9,738	5,501·4	3,280·2
20	45	26,444	8,373	4,812·4	2,780·7
		Age at Entry 30			
0	30	221,228	154,934	65,692·1	54,175·4
1	31	198,358	139,855	59,867·6	49,125·6
2	32	158,425	106,861	46,874·3	37,886·4
3	33	128,836	82,836	37,227·2	29,570·6
4	34	107,109	65,654	30,190·5	23,533·1
5	35	90,272	52,638	24,754·5	18,901·6
6	36	77,143	42,751	20,543·5	15,347·5
7	37	66,636	35,037	17,193·4	12,553·5
8	38	58,064	28,900	14,475·2	10,316·5
9	39	50,880	23,859	12,199·2	8,470·6
10	40	44,786	19,667	10,270·1	6,931·2
11	41	39,759	16,338	8,708·1	5,706·6
12	42	35,562	13,668	7,432·0	4,723·9
13	43	32,044	11,538	6,394·7	3,940·8
14	44	29,031	9,794	5,529·7	3,300·7
15	45	26,444	8,373	4,812·4	2,780·7

TABLE G—*(continued)*.

Years elapsd n (1)	Age attained $[x] + n$ (2)	$(wD)_{[x]+n}$ (3)	$(wmEN)_{[x]+n}$ (4)	$(wmM)_{[x]+n}$ (5)	$(wfmM)_{[x]+n}$ (6)
			Age at Entry 35		
0	35	109,715	68,656	32,145·3	24,768·8
1	36	100,573	63,178	29,940·2	22,907·7
2	37	82,867	49,399	23,825·2	17,807·8
3	38	68,739	38,284	18,903·0	13,756·8
4	39	58,096	30,170	15,239·3	10,785·6
5	40	49,972	24,213	12,497·4	8,597·6
6	41	43,613	19,714	10,404·4	6,954·3
7	42	38,389	16,192	8,703·5	5,646·9
8	43	34,076	13,374	7,331·2	4,610·8
9	44	30,442	11,087	6,197·0	3,774·5
10	45	27,315	9,182	5,234·9	3,071·0
11	46	24,667	7,646	4,445·4	2,514·3
12	47	22,442	6,440	3,814·1	2,076·8
13	48	20,529	5,467	3,295·6	1,725·8
14	49	18,862	4,672	2,864·4	1,440·8
15	50	17,406	4,029	2,509·6	1,211·9
			Age at Entry 40		
0	40	63,985	35,845	18,336·6	12,850·1
1	41	59,242	33,539	17,254·9	12,002·0
2	42	48,949	25,500	13,412·7	9,043·5
3	43	41,116	19,562	10,520·8	6,860·1
4	44	35,182	15,243	8,379·2	5,275·3
5	45	30,603	12,070	6,776·7	4,113·5
6	46	27,033	9,744	5,580·5	3,265·4
7	47	24,131	7,955	4,644·5	2,616·8
8	48	21,680	6,512	3,875·5	2,096·2
9	49	19,603	5,353	3,247·3	1,680·9
10	50	17,838	4,431	2,738·7	1,352·8
11	51	16,306	3,675	2,313·7	1,086·0
12	52	14,988	3,074	1,970·2	876·0
13	53	13,852	2,605	1,697·4	714·3
14	54	12,816	2,224	1,471·4	584·6
15	55	11,941	1,907	1,280·0	478·7

Table G—*(continued).*

Years elapst n (1)	Age attained $[x'] + n$ (2)	$(w\mathrm{D})_{[x]+n}$ (3)	$(wm\mathrm{EN})_{[x]+n}$ (4)	$(wm\mathrm{M})_{[x]+n}$ (5)	$(wfm\mathrm{M})_{[x]+n}$ (6)
		Age at Entry 45			
0	45	45,397	24,271	13,454·7	8,453·1
1	46	42,161	22,891	12,745·3	7,950·1
2	47	34,835	17,284	9,810·8	5,916·5
3	48	28,998	12,887	7,468·2	4,330·6
4	49	24,565	9,671	5,724·5	3,178·0
5	50	21,283	7,437	4,491·6	2,382·8
6	51	18,785	5,859	3,605·3	1,826·2
7	52	16,756	4,652	2,915·1	1,404·4
8	53	15,058	3,696	2,359·0	1,074·7
9	54	13,592	2,906	1,890·9	806·0
10	55	12,359	2,293	1,520·9	601·4
11	56	11,318	1,827	1,234·4	449·5
12	57	10,429	1,473	1,013·4	337·9
13	58	9,657	1,206	843·3	256·6
14	59	8,969	995	706·5	195·2
		Age at Entry 50			
0	50	28,563	13,472	8,080·7	4,411·8
1	51	26,558	12,712	7,653·8	4,143·7
2	52	21,949	9,251	5,675·2	2,934·9
3	53	18,313	6,576	4,118·6	2,011·8
4	54	15,675	4,753	3,038·2	1,391·6
5	55	13,665	3,453	2,254·0	958·0
6	56	12,144	2,564	1,707·6	668·4
7	57	10,946	1,939	1,316·6	470·9
8	58	9,975	1,496	1,034·5	336·1
9	59	9,163	1,175	826·7	242·8
10	60	8,450	924	660·7	173·4
11	61	7,813	724	526·1	121·6
12	62	7,248	575	424·6	86·1
		Age at Entry 55			
0	55	22,162	10,435	6,786·8	2,962·8
1	56	20,479	9,784	6,386·9	2,750·8
2	57	16,847	7,103	4,709·9	1,903·9
3	58	13,971	5,026	3,386·9	1,271·5
4	59	11,762	3,493	2,393·4	825·5
5	60	10,061	2,376	1,656·0	517·2
6	61	8,761	1,589	1,126·7	313·4
7	62	7,737	1,029	743·2	179·2
8	63	6,941	660	486·6	98·9
9	64	6,316	438	329·4	55·7
10	65	5,798	307	234·8	33·1

Table G—*(continued)*.

Years elapsed n	Age attained $[x]+n$	$(wD)_{[x]+n}$	$(wmEN)_{[x]+n}$	$(wmM)_{[x]+n}$	$(wfmM)_{[x]+n}$
(1)	(2)	(3)	(4)	(5)	(6)
Age at Entry 60					
0	60	11,537	3,539	2,484·9	735·0
1	61	10,582	3,179	2,243·3	642·0
2	62	8,906	2,057	1,475·5	373·3
3	63	7,683	1,322	963·7	213·1
4	64	6,744	825	611·6	116·2
5	65	6,013	505	380·9	61·1
6	66	5,439	319	244·7	33·0
7	67	4,966	216	168·5	19·5
8	68	4,537	146	115·5	11·3
Age at Entry 65					
0	65	6,818	1,192	898·8	142·6
1	66	6,182	967	734·5	108·7
2	67	5,395	597	459·1	59·7
3	68	4,760	347	270·5	30·7
4	69	4,245	192	151·8	14·8
5	70	3,822	108	87·0	7·2
6	71	3,448	58	47·6	3·2
Age at Entry 70					
0	70	4,157	397	316·9	27·6
1	71	3,655	243	195·8	15·3
2	72	3,199	111	90·8	6·0
3	73	2,830	46	38·1	2·1
4	74	2,514	17	14·4	·6
Age at Entry 75					
0	75	2,400	158	132·9	5·1
1	76	2,020	56	47·2	1·5
2	77	1,714	2	2·1	...
Age at Entry 80					
0	80	1,122	54	47·6	·3

TABLE H.—*Chronic Widowers—Values at 3 per-cent interest of Marriage Endowments, $(wmE)_x$, and of Assurances depending on marriage, $(wmA)_x$, and on birth of issue, $(wfmA)_x$.*

Age x	$(wmE)_x$	$(wmA)_x$	$(wfmA)_x$	Age x	$(wmE)_x$	$(wmA)_x$	$(wfmA)_x$
45	·31665	·18198	·10515	63	·06992	·05225	·00940
46	·29817	·17425	·09747	64	·06109	·04621	·00736
47	·28048	·16662	·09010	65	·05293	·04051	·00570
48	·26353	·15909	·08303	66	·04538	·03513	·00438
49	·24725	·15161	·07623	67	·03846	·03010	·00333
50	·23146	·14418	·06962	68	·03219	·02546	·00250
51	·21613	·13672	·06324	69	·02649	·02118	·00184
52	·20128	·12928	·05708	70	·02139	·01728	·00132
53	·18691	·12186	·05115	71	·01691	·01379	·00092
54	·17304	·11450	·04548	72	·01299	·01069	·00062
55	·15967	·10720	·04009	73	·00965	·00802	·00040
56	·14680	·09997	·03501	74	·00682	·00572	·00024
57	·13442	·09283	·03026	75	·00454	·00384	·00013
58	·12249	·08577	·02584	76	·00274	·00234	·00007
59	·11098	·07877	·02176	77	·00145	·00125	·00000
60	·09993	·07188	·01806	78	·00061	·00053	...
61	·08939	·06515	·01477	79	·00018	·00016	...
62	·07937	·05859	·01187	80	·00000	·00000	...

TABLE I.—*Recent Widowers—Quinquennial ages at entry: Values at 3 per-cent interest of Marriage Endowments,* $(wmE)_{[x]+n}$, *and of Assurances depending on marriage,* $(wmA)_{[x]+n}$, *and on birth of issue,* $(wfmA)_{[x]+n}$.

Column headings (both halves):

Years elapst n	Age attained $[x]+n$	$(wmE)_{[x]+n}$	$(wmA)_{[x]+n}$	$(wfmA)_{[x]+n}$
(1)	(2)	(3)	(4)	(5)

Age at Entry 25

n	$[x]+n$	(wmE)	(wmA)	$(wfmA)$
0	25	·77621	·29485	·25190
1	26	·77808	·29914	·25502
2	27	·75326	·29738	·25137
3	28	·72366	·29314	·24569
4	29	·69577	·28937	·23968
5	30	·67038	·28549	·23368
6	31	·64587	·28123	·22718
7	32	·62037	·27649	·22004
8	33	·59527	·27120	·21228
9	34	·57020	·26543	·20399
10	35	·54406	·25879	·19485
11	36	·51881	·25198	·18567
12	37	·49114	·24492	·17643
13	38	·46918	·23741	·16696
14	39	·44573	·22981	·15766
15	40	·42279	·22214	·14852
16	41	·39951	·21405	·13929
17	42	·37760	·20591	·13026
18	43	·35648	·19790	·12161
19	44	·33609	·18988	·11321
20	45	·31665	·18198	·10515

Age at Entry 30

n	$[x]+n$	(wmE)	(wmA)	$(wfmA)$
0	30	·70034	·29694	·24489
1	31	·70506	·30182	·24766
2	32	·67452	·29588	·23914
3	33	·64295	·28895	·22952
4	34	·61296	·28187	·21971
5	35	·58311	·27422	·20939
6	36	·55418	·26630	·19895
7	37	·52580	·25802	·18839
8	38	·49772	·24930	·17767
9	39	·46892	·23976	·16648
10	40	·43915	·22932	·15476
11	41	·41093	·21902	·14353
12	42	·38435	·20899	·13284
13	43	·36006	·19956	·12298
14	44	·33735	·19017	·11369
15	45	·31665	·18198	·10515

Age at Entry 35

n	$[x]+n$	(wmE)	(wmA)	$(wfmA)$
0	35	·62576	·29299	·22576
1	36	·63116	·29770	·22777
2	37	·59612	·28751	·21190
3	38	·55694	·27500	·20013
4	39	·51931	·26231	·18565
5	40	·48453	·25009	·17205
6	41	·45272	·23849	·15946
7	42	·42179	·22672	·14710
8	43	·39248	·21514	·13531
9	44	·36421	·20357	·12389
10	45	·33615	·19165	·11254
11	46	·30998	·18022	·10193
12	47	·28697	·16995	·09254
13	48	·26631	·16054	·08107
14	49	·24768	·15187	·07639
15	50	·23146	·14418	·06962

Age at Entry 40

n	$[x]+n$	(wmE)	(wmA)	$(wfmA)$
0	40	·56020	·28658	·20083
1	41	·56613	·29126	·20259
2	42	·52096	·27401	·18475
3	43	·47576	·25588	·16685
4	44	·43328	·23817	·14995
5	45	·39412	·22144	·13412
6	46	·36043	·20643	·12079
7	47	·32966	·19247	·10844
8	48	·30036	·17876	·09669
9	49	·27308	·16566	·08575
10	50	·24842	·15353	·07584
11	51	·22536	·14489	·06660
12	52	·20500	·13145	·05845
13	53	·18807	·12253	·05156
14	54	·17312	·11454	·04551
15	55	·15967	·10720	·04009

TABLE I—(continued).

Years elapsed n (1)	Age attained $[x]+n$ (2)	$(wmE)_{[x]+n}$ (3)	$(wm\Lambda)_{[x]+n}$ (4)	$(wfm\Lambda)_{[x]+n}$ (5)	Years elapsed n (1)	Age attained $[x]+n$ (2)	$(wmE)_{[x]+n}$ (3)	$(wm\Lambda)_{[x]+n}$ (4)	$(wfm\Lambda)_{[x]+n}$ (5)
Age at Entry 45					Age at Entry 60				
0	45	·53464	·29638	·18620	0	60	·30672	·21539	·06371
1	46	·54295	·30230	·18857	1	61	·30012	·21199	·06067
2	47	·49617	·28164	·16984	2	62	·23097	·16567	·04191
3	48	·44441	·25754	·14934	3	63	·17209	·12543	·02773
4	49	·39369	·23303	·12937	4	64	·12235	·09069	·01724
5	50	·34943	·21104	·11195	5	65	·08395	·06334	·01016
6	51	·31190	·19192	·09721	6	66	·05859	·04498	·00607
7	52	·27762	·17397	·08382	7	67	·04355	·03393	·00392
8	53	·24547	·15666	·07137	8	68	·03219	·02546	·00250
9	54	·21383	·13912	·05930					
10	55	·18555	·12306	·04866	Age at Entry 65				
11	56	·16140	·10907	·03972					
12	57	·14128	·09717	·03240	0	65	·17404	·13125	·02082
13	58	·12492	·08733	·02657	1	66	·15617	·11880	·01759
14	59	·11098	·07877	·02176	2	67	·11065	·08511	·01107
					3	68	·07290	·05684	·00645
					4	69	·04519	·03575	·00348
Age at Entry 50					5	70	·02837	·02277	·00188
					6	71	·01691	·01379	·00092
0	50	·47164	·28290	·15446					
1	51	·47863	·28819	·15603	Age at Entry 70				
2	52	·42148	·25857	·13371					
3	53	·35910	·22190	·10985	0	70	·09544	·07622	·00665
4	54	·30323	·19382	·08878	1	71	·06648	·05357	·00418
5	55	·25271	·16494	·07010	2	72	·03482	·02839	·00189
6	56	·21112	·14061	·05504	3	73	·01628	·01344	·00074
7	57	·17711	·12028	·04302	4	74	·00682	·00572	·00024
8	58	·14994	·10370	·03369					
9	59	·12826	·09022	·02650	Age at Entry 75				
10	60	·10932	·07818	·02052					
11	61	·09260	·06733	·01556	0	75	·06584	·05537	·00214
12	62	·07937	·05859	·01187	1	76	·02761	·02339	·00076
					2	77	·00145	·00125	·00003
Age at Entry 55									
					Age at Entry 80				
0	55	·47088	·30624	·13369					
1	56	·47777	·31187	·13432	0	80	·04831	·04242	·00030
2	57	·42162	·27956	·11301					
3	58	·35971	·24241	·09101					
4	59	·29700	·20348	·07018					
5	60	·23619	·16459	·05141					
6	61	·18137	·12861	·03578					
7	62	·13294	·09606	·02317					
8	63	·09509	·07010	·01425					
9	64	·06936	·05215	·00881					
10	65	·05293	·04051	·00570					

Table J.—*Commencing Widowers—All ages at entry, $(n=0)$: Values at 3 per-cent interest of Marriage Endowments, $(wmE)_{[x]}$, and of Assurances depending on marriage, $(wmA)_{[x]}$, and on birth of issue, $(wfmA)_{[x]}$.*

Age x	$(wmE)_{[x]}$	$(wmA)_{[x]}$	$(wfmA)_{[x]}$	Age x	$(wmE)_{[x]}$	$(wmA)_{[x]}$	$(wfmA)_{[x]}$
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
25	·77621	·29485	·25190	53	·48161	·30438	·14579
26	·76052	·29542	·25144	54	·48175	·30902	·14146
27	·74523	·29602	·25054	55	·47088	·30624	·13369
28	·73018	·29653	·24917	56	·44999	·29666	·12282
29	·71525	·29686	·24729	57	·41859	·27996	·10870
30	·70034	·29694	·24489	58	·38040	·25845	·09274
31	·68541	·29674	·24197	59	·34474	·23607	·07721
32	·67044	·29622	·23855	60	·30672	·21539	·06371
33	·65546	·29540	·23468	61	·27470	·19602	·05192
34	·64054	·29431	·23039	62	·24591	·17812	·04197
35	·62576	·29299	·22576	63	·22012	·16163	·03373
36	·61106	·29140	·22077	64	·19635	·14609	·02677
37	·59619	·28942	·21537	65	·17404	·13125	·02082
38	·58199	·28752	·20992	66	·15375	·11748	·01606
39	·56976	·28644	·20497	67	·13574	·10496	·01257
40	·56020	·28658	·20083	68	·12004	·09382	·01010
41	·55327	·28792	·19749	69	·10663	·08422	·00822
42	·54937	·29072	·19510	70	·09544	·07622	·00665
43	·54671	·29401	·19302	71	·08632	·06970	·00537
44	·54236	·29620	·19026	72	·07907	·06455	·00435
45	·53464	·29638	·18620	73	·07345	·06059	·00350
46	·52359	·29456	·18089	74	·06915	·05761	·00278
47	·50830	·29016	·17408	75	·06584	·05537	·00214
48	·49169	·28500	·16659	76	·06311	·05354	·00157
49	·47857	·28203	·15979	77	·06052	·05177	·00107
50	·47164	·28290	·15446	78	·05756	·04964	·00066
51	·47024	·28719	·15044	79	·05369	·04670	·00038
52	·47518	·29541	·14802	80	·04831	·04242	·00030

Table K.—*Bachelors—Values at 3 per-cent interest of Marriage Endowments, $(bmE)_x$, and of Assurances depending on marriage, $(bmA)_x$, and on birth of issue, $(bfmA)_x$; also probabilities (1) of marrying, and (2) of marrying and having issue.*

The values of $(bmE)_x$ and $(bmA)_x$ are reprinted from *J.I.A.*, xxi, 440.

Age x (1)	$(bmE)_x$ (2)	$(bmA)_x$ (3)	$(bfmA)_x$ (4)	Probability that a Bachelor will marry (5)	Probability that a Bachelor will marry and have issue (6)
15	·45613	·17066	...	·70463	...
16	·47213	·17665	...	·70814	...
17	·48887	·18294	...	·71208	...
18	·50638	·18955	...	·71649	...
19	·52466	·19651	·16721	·72148	·61064
20	·54293	·20363	·17322	·72656	·61478
21	·56040	·21071	·17915	·73115	·61833
22	·57039	·21603	·18340	·73085	·61710
23	·57267	·21951	·18588	·72539	·61081
24	·57092	·22200	·18738	·71696	·60158
25	·56453	·22317	·18759	·70502	·58890
26	·55455	·22314	·18662	·69010	·57329
27	·54240	·22218	·18173	·67304	·55561
28	·52864	·22042	·18205	·65424	·53623
29	·51302	·21774	·17845	·63345	·51492
30	·49566	·21116	·17396	·61074	·49179
31	·47644	·20962	·16854	·58606	·46686
32	·45554	·20413	·16219	·55963	·44031
33	·43324	·19776	·15503	·53174	·41250
34	·40970	·19054	·14709	·50260	·38372
35	·38545	·18266	·13861	·47274	·35460
36	·36008	·17429	·12982	·44265	·32572
37	·33652	·16552	·12086	·41261	·29745
38	·31231	·15648	·11185	·38292	·27006
39	·28857	·14727	·10291	·35382	·24380
40	·26543	·13797	·09412	·32551	·21884
41	·24313	·12871	·08561	·29825	·19538
42	·22183	·11960	·07744	·27223	·17350
43	·20179	·11078	·06974	·24772	·15339
44	·18316	·10238	·06256	·22488	·13510
45	·16608	·09449	·05598	·20387	·11867
46	·15073	·08724	·05006	·18484	·10415
47	·13698	·08061	·04475	·16766	·09135
48	·12453	·07449	·03996	·15205	·08000
49	·11317	·06878	·03558	·13775	·06986
50	·10271	·06344	·03156	·12459	·06075
51	·09285	·05823	·02779	·11225	·05244
52	·08355	·05323	·02425	·10067	·04487
53	·07485	·04845	·02097	·08991	·03804
54	·06683	·04394	·01797	·08002	·03196
55	·05942	·03969	·01523	·07094	·02658
56	·05268	·03575	·01279	·06270	·02189
57	·04656	·03209	·01063	·05524	·01784
58	·04101	·02870	·00873	·04851	·01438
59	·03596	·02555	·00707	·04240	·01142
60	·03144	·02267	·00565	·03696	·00896
61	·02748	·02011	·00447	·03220	·00697
62	·02393	·01776	·00349	·02795	·00535
63	·02072	·01559	·00268	·02412	·00404

TABLE K—(*continued*).

Age x (1)	$(bmE)_x$ (2)	$(bmA)_x$ (3)	$(bfmA)_x$ (4)	Probability that a Bachelor will marry (5)	Probability that a Bachelor will marry and have issue (6)
64	·01795	·01369	·00205	·02084	·00304
65	·01557	·01202	·00157	·01800	·00229
66	·01358	·01061	·00121	·01563	·00174
67	·01190	·00941	·00094	·01362	·00133
68	·01044	·00834	·00074	·01187	·00103
69	·00912	·00736	·00057	·01030	·00079
70	·00793	·00646	·00044	·00888	·00060
71	·00677	·00557	·00033	·00753	·00044
72	·00574	·00476	·00024	·00633	·00032
73	·00474	·00397	·00017	·00520	·00022
74	·00388	·00327	·00012	·00422	·00015
75	·00305	·00260	·00008	·00329	·00010
76	·00236	·00202	·00005	·00253	·00006
77	·00170	·00147	·00003	·00181	·00003
78	·00118	·00103	·00001	·00125	·00002
79	·00070	·00062	·00001	·00074	·00001
80	·00037	·00033	·00000	·00039	·00000
81	·00010	·00009	...	·00010	...

TABLE L.—*Mortality Table for Married Men, as deduced by Mr. Sprague from the Peerage Experience. See J.I.A.*, xxi, 428–435.

Age x	l_x	d_x	Age x	l_x	d_x	Age x	l_x	d_x
15	1,000,000	5,000	43	816,452	9,062	71	403,548	25,222
16	995,000	5,672	44	807,390	9,447	72	378,326	25,878
17	989,328	6,430	45	797,943	9,821	73	352,448	26,469
18	982,898	7,570	46	788,122	10,144	74	325,979	26,958
19	975,328	8,680	47	777,978	10,156	75	299,021	27,244
20	966,648	9,668	48	767,522	10,767	76	271,780	27,287
21	956,980	10,335	49	756,755	11,063	77	244,493	26,992
22	946,645	7,099	50	745,692	11,260	78	217,501	26,340
23	939,546	6,296	51	734,432	11,157	79	191,161	25,329
24	933,250	4,200	52	722,975	11,640	80	165,832	23,979
25	929,050	3,624	53	711,335	11,880	81	141,853	22,413
26	925,426	3,886	54	699,455	12,170	82	119,440	20,472
27	921,540	4,424	55	687,285	12,508	83	98,968	18,398
28	917,116	5,136	56	674,777	12,888	84	80,570	16,025
29	911,980	5,655	57	661,889	13,304	85	64,545	13,548
30	906,325	5,891	58	648,585	13,685	86	50,997	11,204
31	900,434	5,912	59	634,900	14,096	87	39,793	9,200
32	894,492	5,994	60	620,804	14,588	88	30,593	7,321
33	888,498	6,042	61	606,216	15,337	89	23,272	5,893
34	882,456	6,354	62	590,879	16,309	90	17,379	4,857
35	876,102	6,484	63	571,570	17,410	91	12,522	3,916
36	869,618	6,696	64	557,160	18,665	92	8,606	3,023
37	862,922	6,902	65	538,495	19,924	93	5,583	2,321
38	856,020	7,190	66	518,571	21,107	94	3,262	1,655
39	848,830	7,555	67	497,464	22,187	95	1,607	1,024
40	841,275	7,909	68	475,277	23,146	96	583	476
41	833,366	8,250	69	452,131	23,963	97	107	107
42	825,116	8,664	70	428,168	24,620			

TABLE M.—*Mortality Table for Females, deduced from the observations on the Peerage Females by Messrs. Bailey & Day, and adjusted by Dr. T. M. Thiele.*

[The values of l_x and d_x are reprinted from *J.I.A.*, xvi, 43.]

Age x	l_x	d_x	μ_x	Age x	l_x	d_x	μ_x
20	8,294	67	·00808	56	5,542	108	·01913
21	8,227	67	·00814	57	5,434	114	·02043
22	8,160	66	·00815	58	5,320	118	·02180
23	8,094	67	·00822	59	5,202	124	·02326
24	8,027	66	·00828	60	5,078	128	·02481
25	7,961	66	·00829	61	4,950	132	·02626
26	7,895	66	·00836	62	4,818	138	·02802
27	7,829	66	·00843	63	4,680	144	·03013
28	7,763	67	·00857	64	4,536	150	·03241
29	7,696	67	·00871	65	4,386	157	·03500
30	7,629	68	·00885	66	4,229	163	·03783
31	7,561	69	·00906	67	4,066	169	·04083
32	7,492	69	·00921	68	3,897	173	·04388
33	7,423	70	·00936	69	3,724	175	·04672
34	7,353	72	·00966	70	3,549	177	·04959
35	7,281	74	·01003	71	3,372	177	·05249
36	7,207	74	·01027	72	3,195	177	·05540
37	7,133	77	·01058	73	3,018	176	·05848
38	7,056	78	·01098	74	2,842	176	·06193
39	6,978	78	·01118	75	2,666	175	·06583
40	6,900	78	·01130	76	2,491	176	·07045
41	6,822	78	·01143	77	2,315	174	·07559
42	6,744	78	·01157	78	2,141	173	·08104
43	6,666	78	·01170	79	1,968	169	·08689
44	6,588	78	·01184	80	1,799	164	·09255
45	6,510	78	·01198	81	1,635	158	·09847
46	6,432	80	·01228	82	1,477	150	·10427
47	6,352	80	·01259	83	1,327	143	·11040
48	6,272	83	·01299	84	1,184	136	·11782
49	6,189	83	·01341	85	1,048	128	·12599
50	6,106	86	·01384	86	920	121	·13533
51	6,020	88	·01445	87	799	114	·14706
52	5,932	92	·01517	88	685	107	·16131
53	5,840	95	·01601	89	578	98	·17734
54	5,745	99	·01688	90	480	90	·19583
55	5,646	104	·01798	...	...	...	...

DISCUSSION.

The PRESIDENT (Mr. William Sutton, M.A.), in inviting discussion, said that papers like that which had been read were not meant to show exactly what premiums should be charged for insurances against issue, but they were most valuable as giving actuaries a broad indication of the sort of risks they were about to undertake. The question of insurance against issue had come before him practically in the last few years. No doubt many members had heard of Mr. Samuel Brown's Reports on the Indian Civil and Military

Funds, which in their way formed a monument of actuarial labour and skill. In those reports Mr. Brown had to deal with a question very much of the nature of the one they were discussing, and it appeared under the name of "potential benefits." The question arose in making the valuation as to the reserve which should be made to meet the liability of present bachelors hereafter marrying and bringing liabilities upon the fund both in the form of widows and children. The whole question was a very complicated one, and Mr. Sprague in treating it had shown that remarkable command of actuarial resources with which his numerous contributions to the Institute had made them so familiar. A part of the subject which did not fall within the limits of the paper was the probabilities of widows and spinsters marrying. He did not know to what extent the experience of actuaries led them to think that information on that subject was wanted, but his own experience in actuarial questions had led him to consider the question. Only a fortnight previously he had to deal with a case in which two sisters, both widows and childless, took the life income from trust funds, the absolute reversions being in favour of someone else on their decease. Those reversions were the subject—in the event of either of those widows re-marrying— of postponement to the extent of the duration of the husband's life. Then, again, as to the facts upon which Mr. Sprague, and also Mr. Day, had based their tables. If they took the Institute of Actuaries' Table they knew all the lives had been assured, but in the facts of the peerage it did not necessarily follow that there had been any question as to issue risk. If the Institute or the offices saw their way to combine to bring out a new Institute of Actuaries' Table possibly the question of issue risks might find a place therein. There appeared to be, on the surface at all events, no reason why the issue insurances should not form part of the investigation, and if they did it was very likely that they would, by the use of Mr. Sprague's tables, get a definite idea as to the value of selection in this kind of risk. As Mr. Sprague and Mr. Day were present, perhaps they would say whether the widowers referred to as having been tabulated were childless widowers, or whether they were taken up quite irrespective of whether they were childless or not. He thought that was a material point. Then, as bearing upon the curious fact Mr. Sprague had alluded to as the probability of a marriage being fruitful being greater in the case of peers and direct heirs to peerages than in that of other male members of peerage families, it would be very desirable if they could get some information as to the ages of the wives. One possible explanation might be, that a peer or his direct heir might desire for various reasons to make sure of having issue, and marry a comparatively young woman. On the other hand, with regard to those who were not in the direct line, and possibly were in many cases not particularly well supplied as regards this world's goods, they might in many cases marry wives older than themselves. If the ages of the wives were given, some light would be thrown upon what seemed to be a very curious fact.

Mr. A. H. BAILEY said that he thought most of them would hesitate to take some of the issue risks for which Mr. Sprague had calculated premiums. These assurances were usually for the com-

pletion of securities for advances on or the purchase of reversionary interests. The most common case was that relating to the probability of a married man whose wife was past the child-bearing age having issue: in other words, of the wife dying and the widower marrying a second wife and having issue by her. He found, on referring to his own experience of such assurances, that, having had a fair amount of these issue risks, there had been only one case, and that was 25 years ago, in which a claim arose, and as far as he had been able to hear that was the only case of claim that had ever occurred. It therefore seemed that, at present, the business of assurance against issue was not unprofitable. It was not, however, likely that there would be a sufficient number of these risks to enable the law of average to apply. They were included with other special risks. He was afraid that in these as in other risks, premiums were going down, so they would not do perhaps as well in the future as they had done in the past. He doubted whether any company had dared to take a case in which the wife was within the child-bearing period. He was very much struck with the pains and ingenuity that had been bestowed upon these tables, showing as they did that there was a law of marriage as well as a law of mortality, and that general actuarial principles were applicable to other subjects besides laws of mortality. But when they attempted to apply these principles to practice, other considerations would have to be taken into account.

Mr. ARCHIBALD DAY said that Mr. Sprague had first stated, and compared what had been done in the past; the second part of the paper consisted of a series of tables, some of which had appeared before, while others were very valuable additions to what had been already published; the third part referred to the construction of the tables, of which more was to be heard on a future occasion. In Mr. Sprague's references to what he (Mr. Day) had undertaken 29 years ago, he had dealt very kindly with him, but it must be remembered that he had absolutely no one to follow when he broke ground upon this question. The credit, indeed, was not all due to himself, for Mr. Bailey and he, in those days as now, were most intimate friends, and what was then done was no doubt the result of conversations between them. He was extremely gratified to find that with all the new light which Mr. Sprague had been enabled to throw upon this subject, the results he had brought out were singularly confirmatory of much that had been done in times past. Whether they would ever be able to make use of this form of insurance to any very great extent seemed problematical, because all that these assurances had hitherto been used for had been as collateral aids to make contingent securities complete. Independent assurances of this nature would, he supposed, be very few, and the selection against the offices would be so great that companies would hardly be disposed to undertake such risks at all freely. The question of loading the premiums to which Mr. Sprague referred was a matter almost more important than the premium itself.

Mr. A. HENDRIKS agreed with Mr. Bailey, that however valuable Mr. Sprague's tables might be for statistical purposes as a means of comparison, yet for practical purposes actuaries would certainly have to be guided much more by the surroundings of the case itself than

by any tables. When they had to assess a risk they must first decide whether there was any risk at all. Many cases came before insurance companies in which there was little or no risk. There were some also in which there was no risk, but according to the legal condition the policy had to be produced. He remembered a case in which the company of which he was actuary had taken a case of £15,000 at a premium of one guinea per-cent, the sum assured being payable if a lady aged 59 produced issue. As a remarkable fact, that lady's Christian name was Sarah, but she did not produce issue. Following upon the view stated by Mr. Bailey, he could say that, as regards issue risks where there had been a living husband and a living wife past the child-bearing period, including with that risk the chances of survivorship and other insurance considerations, his company had so far not had to pay a single claim. They had even had cases in which the risk was much greater, and within the last week he had been called upon to assess the premium for the chances of issue of a living husband of 62 with a living wife of 42 who had been married eight years. Even in cases in which the wife had passed the child-bearing period a great deal depended upon the idiosyncrasies of both the husband and wife. Some men were looked upon as undoubted "marrying men," who would marry again if left widowers, while in other instances this was not so. Further, as was the case in fire insurance, the "moral hazard" had sometimes to be considered. Whilst entertaining a profound admiration for the way in which Mr. Sprague brought his statistics together, he still thought that very much more depended upon selection, apart from the facts of the actual ages, than was the case in any other description of life assurance which came before them.

Mr. F. E. COLENSO observed that whatever might be said as to the insufficiency of Mr. Sprague's rates as the basis for the calculation of actual premiums, it seemed that they might very well have formed the basis of the rates charged by the offices during the century. Looking at what had been reported in the *Journal of the Institute* as to the total amounts at risk, and to what was known of the experience of the offices under issue policies, they had evidently kept well on the safe side by paying regard to the special circumstances of each case. Mr. Sprague had provided a very full set of symbols for issue risks, but there did not appear to be one given for the cases referred to in the paper as those which most commonly presented themselves in practice—namely, where married men were concerned. In a very common case, moreover, the risk was much less than that of payment at the death of the tenant-for-life. Where a reversioner was entitled to a share of a fund, and there was a provision in the settlement entitling children by any subsequent marriage to shares, what they had to provide against was a diminution in the share, and this would be met by a graduated assurance: so much for the first child, so much for the second, and so on. But the further shares would not vest unless the sons attained 21, or the daughters attained that age or married. Thus the total risk was much less in such a case than that provided for in the formula.

Mr. GEORGE KING said that Mr. Sprague had spoken of "chronic widowers", but there was another description of chronic widowers,

which in his (Mr. King's) experience he had come across. He had to value a large fund not long ago, where the question of marriage occurred, and he found that one of the persons had been a widower four times. The question of re-marriage might therefore sometimes be a very important one. No light, so far as he could see, had been thrown upon it in the paper, but he presumed that, in speaking of widowers, no account was taken of whether they had been married more than once.

Mr. G. H. RYAN said that certain criticisms had been passed upon the paper, having as their object the inculcation of the necessity for prudence in dealing with issue statistics practically, and there did unquestionably exist every cause for extreme circumspection in dealing with insurances against issue. It was on record that this branch of insurance business was increasing with some rapidity. Within a space of ten years, the amount of issue insurances in force had been doubled, and in 1887 (according to a letter published by Mr. Sprague in *J.I.A.*, xxvi, 391), policies to the extent of £1,696,747 were in force on risks of that description, the amount of premiums received thereon having been £116,704. In view of this fact, it was surely desirable that they should have the fullest and most accurate statistics available to guide them in calculating issue premiums, and allowing to the fullest that other considerations must be borne in mind, he still could not think that such limitations in their use should make them stint the praise that should be accorded to Mr. Sprague for his comprehensive and invaluable tables. He had been about to ask a question which Mr. Bailey in his speech had answered in anticipation, namely, whether there was any recorded case of a claim having arisen under an issue policy. It now appeared that there was one. To have a considerable business upon which, over a long series of years, but a single claim had arisen did not appear to him to betoken that actuaries had been exactly enterprising in this matter in the past. The probability was that, with their information on these points becoming more exact, more claims in the future would occur, and this outlook need, he thought, cause them no alarm. So long as they could get adequate premiums which, after paying claims, would yield a fair margin of profit, he did not think anyone would object to the payment of a fair amount of claims.

Mr. M. N. ADLER asked whether the statistics collected in Scotland by Dr. Matthews Duncan, to which Mr. Sprague had alluded in a former paper, would in any way confirm those statistics which were now quoted as derived from the peerage?

Mr. SPRAGUE, in reply, said that there was not much in the paper to discuss, inasmuch as it was to a very great extent an historic record of what had been done in the past, and a comparison of the present results with those formerly obtained. He had not given on the present occasion any description of the statistics the tables were founded upon or even a description of the process by which they were formed, that part of the subject being contained in a paper written by Mr. Chatham. [See page 384]. Of course the premiums that he had brought out were not to be looked at as the premiums that were to be charged. The question of loading was very important, perhaps quite as important as the calculation of the premiums them-

selves, but it was a very important thing to be able to say that however much they were disposed to reduce the premiums, they must not go below these net premiums which are required to cover the risk.   Of course there were many questions that he had not dealt with, one, for instance, mentioned by the Chairman, where a reversion was liable to be postponed until the death of the future husband whom a widow might marry.   Every now and then a fund was settled upon a lady for her life, and after her death to any husband she might leave surviving her, and in buying a reversion of that sort they reflected, " Here is a lady " 60 or so; she has a comfortable annuity, £300 or £400 a year; is " it not very possible that some young man may marry her for " the sake of getting the annuity after her death?   We should then " have a reversion payable on the death of a healthy man of 30 " instead of a widow of 60."   In reply to an inquiry that had been made, he stated that he had included in his statistics all the widowers, whether they had children by the first marriage or not.   He had drawn a few conclusions, which he had not yet published, as to the effect of there being issue of the first marriage upon the probability of issue of a second marriage, but these conclusions were very uncertain in consequence of the small number of facts at his disposal, and he had never put them into exact shape for publication.   A wish had been expressed that they might have the ages of the wives as being an important element in considering these assurances against issue, but this would not be of much use practically, if any, and it was very difficult to get information as to the lady's age.   Lodge's Peerage was the one which gave the fullest information, and if a member of the peerage happened to marry a lady also a member of a peerage family one could generally get at her age ; but probably in more cases than not the wife did not belong to one of the peerage families, and in that case they had no evidence of her age whatever. Why peers and heirs-apparent should have a larger probability of issue than other members was a question he had touched upon in one of his papers.   He thought that the man in possession of the title and estates or the next in succession was more at liberty to please himself and to marry for love, and might therefore be expected to marry a young healthy lady; whereas the younger brothers frequently preferred to marry money, and heiresses were sometimes elderly ladies, whose fathers had made money in business, the fact being that until the death of the father it could not be ascertained what fortune a lady would have.   In other cases the heiress was an only child, and so was less likely to have children.   Of course, the paper had no bearing whatever upon the risk of issue while the wife was within child-bearing age, but he had good reason for saying that insurances of that kind were in practice sometimes granted, insurances which formerly would not have been entertained on any terms.   It was quite possible now to effect an insurance against issue to a marriage where the lady was well within the child-bearing age, say 35 or 40, supposing she had been married for a considerable number of years.   That question he had also considered in a former paper. As to the widowers who had married three or four times, he thought it would be more appropriate to call them chronic marrying men.   Insurances against further issue to a married man or a

widower were, comparatively speaking, common. It often happened that a fund was to be divisible on the death of a man among all his children by any wife he might have, and when a reversionary interest in such a fund was bought it was proper to insure against the risk of the share being reduced by the birth of further children. Every office that bought reversions must have several such cases on its books. With regard to Dr. Duncan's statistics, they were very interesting, but for this purpose they were useless, because they related only to fruitful marriages. With regard to the total amount of issue risks in force, he inferred from certain extracts he had made from the blue-books, that in the three years that had elapsed since the date of the tables quoted by Mr. Ryan the amount insured had increased by about 30 per-cent, which would correspond in five years to an increase of about 50 per-cent, so that there would probably be considerably more than two millions assured when the time came for a new edition of those tables. He was very glad to find his paper had been received with so much interest, and had given rise to so much discussion, though he could not claim that the discussion had arisen directly out of the paper.

On the Construction and Use of the combined Marriage and Mortality Tables for Widowers. By JAMES CHATHAM, *F.I.A., F.F.A., of the Scottish Equitable Life Assurance Society.*

AS the marriage rate among widowers depends not only on their age, but also upon the length of time for which they have been widowers, it was necessary to form the combined marriage and mortality table for widowers on principles analogous to those adopted by Mr. Sprague in the construction of his Select Life Tables, where the rate of mortality among assured lives for each age at entry was, after the lapse of five or a less number of years, joined on to the $H^{M(5)}$ rate of mortality. The first step, therefore, was to form a combined marriage and mortality table for chronic widowers—that is to say, those among whom the marriage rate has reached its ultimate magnitude; in other words, to find $(wl)_x$, the number of widowers alive and still unmarried at any age, and number marrying, $(wm)_x$, and dying, $(wd)_x$, in the following year. These were found by the formulas

$$(wm)_x = (wl)_x \cdot \frac{\mu_1}{1 + \frac{1}{2}(\mu_1 + \mu_2)}$$

$$(wd)_x = (wl)_x \cdot \frac{\mu_2}{1 + \frac{1}{2}(\mu_1 + \mu_2)},$$

and checked by the formula

$$(wl)_{x+1} = (wl)_x \cdot \frac{1 - \frac{1}{2}(\mu_1 + \mu_2)}{1 + \frac{1}{2}(\mu_1 + \mu_2)};$$

where $\mu_1 = (wm\mu)_{x+\frac{1}{2}} = $ force of marriage in the middle of the year of age x to $x+1$, or the central marriage rate;

$\mu_2 = (wd\mu)_{x+\frac{1}{2}} = $ force of mortality or central death rate at the same age.

The above formulas were given by Mr. Sprague in his paper, *J.I.A.*, xxi, 406, from which the following demonstration is taken.

Let y and z denote the numbers of marriages and deaths. Then assuming, as usual, the deaths and marriages to be uniformly distributed over the year, the number of widowers remaining alive and unmarried in the middle of the year will be $(wl)_x - \frac{1}{2}(y+z)$. Hence, the force of marriage in the middle of the year will be $y \div \{(wl)_x - \frac{1}{2}(y+z)\}$; but this is given $= \mu_1$, so that we get the equation

$$y = \mu_1 \left\{ (wl)_x - \frac{y+z}{2} \right\}.$$

Similarly we get the equation

$$z = \mu_2 \left\{ (wl)_x - \frac{y+z}{2} \right\};$$

and solving these equations we get

$$y = (wl)_x \cdot \frac{\mu_1}{1 + \frac{1}{2}(\mu_1 + \mu_2)}$$

$$z = (wl)_x \cdot \frac{\mu_2}{1 + \frac{1}{2}(\mu_1 + \mu_2)},$$

and consequently

$$(wl)_{x+1} = (wl)_x \cdot \frac{1 - \frac{1}{2}(\mu_1 + \mu_2)}{1 + \frac{1}{2}(\mu_1 + \mu_2)}.$$

The values of $(wm\mu)_{x+\frac{1}{2}}$ for the ultimate marriage rate were given by Mr. Sprague in his paper on the remarriage of widowers, and are reprinted here for convenience of reference (see Table B).

As regards the death rate, it was assumed that the same rate would prevail among the widowers as among married men of the same age, and the values of the central death rate, $(wd\mu)_{x+\frac{1}{2}}$, are given in Table A. These were deduced from Mr. Sprague's probabilities of married men dying in a year by means of the fixed relation he has shown to subsist between the annual death

rate among widowers, for instance, who do not marry in the year, $(wdr)_x = r_2$ say, and the force of mortality in the middle of the year of age x to $x+1$, $(wd\mu)_{x+\frac{1}{2}} = \mu_2$.

We have

$$r_2 = \frac{(wd)_x}{(wl)_x - \frac{1}{2}(wm)_x} = \frac{q_2}{1 - \frac{1}{2}q_1} \quad \text{where } q_1 = (wm)_x \div (wl)_x$$

$$q_2 = (wd)_x \div (wl)_x.$$

Similarly $$\mu_2 = \frac{q_2}{1 - \frac{1}{2}(q_1 + q_2)}.$$

Hence, $$\frac{q_2}{r_2} = 1 - \frac{1}{2}q_1, \qquad \frac{q_2}{\mu_2} = 1 - \frac{1}{2}(q_1 + q_2);$$

and, therefore, $$\frac{q_2}{r_2} - \frac{q_2}{\mu_2} = \frac{1}{2}q_2,$$

or $$\frac{1}{r_2} - \frac{1}{\mu_2} = \frac{1}{2},$$

whence $$\mu_2 = \left(\frac{1}{r_2} - \frac{1}{2}\right)^{-1}.$$

Starting now with a radix of 100,000 at age 45, we have

$$(wm)_{45} = 100,000 \times \frac{\cdot0463}{1 + \frac{1}{2}(\cdot0463 + \cdot0124)} = 4,498;$$

also $$(wd)_{45} = 100,000 \times \frac{\cdot0124}{1 + \frac{1}{2}(\cdot0463 + \cdot0124)} = 1,204,$$

and $$(wl)_{46} = (wl)_{45} - (wm)_{45} - (wd)_{45}$$
$$= 100,000 - 4,498 - 1,204$$
$$= 94,298.$$

To check the last result, we have

$$(wl)_{46} = 100,000 \frac{1 - \frac{1}{2}(\cdot0463 + \cdot0124)}{1 + \frac{1}{2}(\cdot0463 + \cdot0124)} = 94,298 \text{ as before.}$$

Proceeding in this way we obtain the number of widowers who have remained such living, remarrying, or dying at any age, and these are given in columns (2), (3), and (4) of Table C. These columns will enable us to calculate the values of benefits that depend simply upon the remarriage of the widowers; but if we wish also to take into account the probability of issue, we require to know the number of widowers remarrying at each age whose marriages were fruitful. These figures are given in column (5), and are got from the figures in column (4) by multiplying them by the probabilities given in Table A that a

peer or heir apparent marrying at any age will have issue. For instance, the number of widowers marrying at age 45 being 4,498, and the probability that a peer or heir apparent marrying at that age will have issue being ·709, we get $4,498 \times ·709 = 3,189$ as the number of widowers marrying at that age who have issue. It is sometimes useful to know the probability that a widower will marry, $\dfrac{\Sigma (wm)_x}{(wl)_x}$, where $\Sigma (wm)_x$ denotes the sum of $(wm)_x + (wm)_{x+1} + \ldots \ldots$ to the end of life, and the probability that he will marry and have issue, $\dfrac{\Sigma (wfm)_x}{(wl)_x}$, and these are given in columns (6) and (7) respectively.

In order next to form the combined marriage and mortality table suitable for recent widowers, it was necessary to work backwards from the age at which the marriage rate among them became equal to the ultimate rate. The ages at which this takes place for widowers who became such at the quinquennial ages 25, 30, &c., are shown in Table D, which gives for those widowers the force of marriage in each year of widowerhood until the rate joins on to the ultimate rate. It will be seen that the last rate under each age at entry is the same as given in Table B as the ultimate rate for the same age attained. For instance, among widowers who became such at the age of 35, the marriage rate attains its ultimate value after 15 years, so that the rate of remarriage among widowers of the age of 50, is the same whether they became such at the age of 35 or at a younger age. Then the number of widowers at the age of 50 in Table C being 76,308, this was taken as the number at the same age for widowers who became such at age 35.

In order to get the numbers of widowers who remained such living, marrying, or dying at age 49, it will be necessary to make a slight transformation upon the working formulas already given. We have

$$(wm)_x = (wl)_x \cdot \frac{\mu_1}{1 + \frac{1}{2}(\mu_1 + \mu_2)} = (wl)_{x+1} \cdot \frac{\mu_1}{1 - \frac{1}{2}(\mu_1 + \mu_2)}$$

since
$$(wl)_x = (wl)_{x+1} \cdot \frac{1 + \frac{1}{2}(\mu_1 + \mu_2)}{1 - \frac{1}{2}(\mu_1 + \mu_2)}.$$

Similarly $$(wd)_x = (wl)_{x+1} \cdot \frac{\mu_2}{1 - \frac{1}{2}(\mu_1 + \mu_2)}$$

and proceeding as before, we get

$$(wm)_{49} = 2,819 \qquad (wd)_{49} = 1,153$$

and
$$(wl)_{49} = (wm)_{49} + (wd)_{49} + (wl)_{50}$$
$$= 2,819 + 1,153 + 76,308$$
$$= 80,280.$$

Working backwards in this way, we find that the number of widowers at the age of 35 was 308,724. Similar calculations being made for each age at entry gave the figures in columns (3), (4), and (5) of Table E. The process of finding the number of fruitful marriages, and the probability of a widower marrying, and of marrying and having issue, is similar to that employed in the ultimate rate, and it is not therefore necessary to give examples.

By means of these tables have been calculated (1) the value of an endowment payable on a widower marrying; (2) the value of an assurance payable on the death of a widower if he should marry; and (3) the value of an assurance payable on the death of a widower if he should marry and have issue. The formula for obtaining the value of an endowment payable at the end of the year of marriage of chronic widowers is

$$(wmE)_x = \frac{(wm)_x r + (wm)_{x+1} r^2 + \ \ldots \ldots}{(wl)_x}$$

$$= \frac{(wm)_x v^{x+1} + (wm)_{x+1} v^{x+2} + \ \ldots \ldots}{(wl)_x v^x}$$

$$= \frac{\Sigma\{(wm)_x r^{x+1}\}}{(wl)_x r^x}.$$

The value of an assurance payable on the death of a chronic widower if he should marry, which is denoted by $(wmA)_x$, is obtained as follows. Out of $(wl)_x$ widowers now of the age x, $(wm)_{x+n}$ marry in the $(n+1)$th year from the present time, or (on the average) at the age of $x + n + \frac{1}{2}$; and assuming them all to marry in the middle of the year, the sum to be then paid in order to provide an assurance of 1 payable on the death of each will be $(wm)_{x+n}(mA)_{x+n+\frac{1}{2}}$, where $(mA)_{x+n+\frac{1}{2}}$ denotes the value of an assurance payable on the death of a married man aged $x + n + \frac{1}{2}$, and the present value of this sum is found by multiplying by $r^{n+\frac{1}{2}}$. The total present sum, therefore, that is necessary to provide for assurances on the death of all the widowers who marry out of the $(wl)_x$ is

$$(wm)_x v^{\frac{1}{2}}(mA)_{x+\frac{1}{2}} + (wm)_{x+1} r^{\frac{3}{2}}(mA)_{x+\frac{3}{2}} + \ \ldots$$

and, therefore,

$$(wm\Lambda)_x = \frac{(wm)_x r'^{\frac{1}{2}}(m\Lambda)_{x+\frac{1}{2}} + (wm)_{x+1} r^{\frac{3}{2}}(m\Lambda)_{x+\frac{3}{2}} + \ldots}{(wl)_x}$$

$$= \frac{(wm)_x r^{x+\frac{1}{2}}(m\Lambda)_{x+\frac{1}{2}} + (wm)_{x+1} r^{x+\frac{3}{2}}(m\Lambda)_{x+\frac{3}{2}} + \ldots}{(wl)_x r^x}$$

$$= \frac{\Sigma\{(wm)_x r^{x+\frac{1}{2}}(m\Lambda)_{x+\frac{1}{2}}\}}{(wl)_x r^x}.$$

In order to find the value of an assurance payable on the death of a chronic widower if he should marry and have issue, we have only in the formula for $(wm\Lambda)_x$ to substitute $(wfm)_x$ instead of $(wm)_x$, that is,

$$(wfm\Lambda)_x = \frac{\Sigma\{(wfm)_x r^{x+\frac{1}{2}}(m\Lambda)_{x+\frac{1}{2}}\}}{(wl)_x r^x}.$$

These formulas are analogous to those for bachelors given by Mr. Sprague in his paper, *J.I.A.*, xxi, 106, from which the above demonstrations have been taken.

It will be observed that if columns of $(wl)_x r^x$, $\Sigma\{(wm)_x v^{x+1}\}$, $\Sigma\{(wm)_x v^{x+\frac{1}{2}}(m\Lambda)_{x+\frac{1}{2}}\}$, and $\Sigma\{(wfm)_x r^{x+\frac{1}{2}}(m\Lambda)_{x+\frac{1}{2}}\}$, are formed, the calculation of the above-mentioned benefits can be rapidly proceeded with; and this was accordingly done, the rate of interest used being 3 per-cent. These are similar to the ordinary D and N columns, and may be conveniently denoted by $(wD)_x$, $(wmEN)_x$, $(wmM)_x$, and $(wfmM)_x$ respectively.

The changes in the above formulas that must be made in order to obtain the values of the corresponding functions for recent widowers are obvious. The values of these four functions for chronic and recent widowers respectively are given in Tables F and G. The benefits described above were then calculated by means of them, and the values for chronic and recent widowers respectively are given in Tables H and I. Having thus got the values of $(wmE)_{[x]}$, $(wm\Lambda)_{[x]}$, and $(wfm\Lambda)_{[x]}$, for quinquennial ages at entry, the next step was to calculate the values for each age at entry. This was done by interpolation, the formula used being that given and fully explained with examples by Mr. Sprague in his paper, *J.I.A.*, xxii, 270, and the results are given in Table J. These are for commencing widowers only, and should a value be required for a recent widower, it will be necessary, unless his age at entry be a multiple of five, to interpolate in order to get the required value, for which purpose first differences may be used.

For instance, if we wish to obtain the value of an assurance payable on the death of a man now aged 63 who has been a widower for six years, if he marries and has issue, we proceed as follows:

$$(wfmA)_{[55]+6} = \cdot 03578 \qquad\qquad (wfmA)_{[55]+6} = \cdot 03578$$
$$(wfmA)_{[60]+6} = \cdot 00607 \qquad\qquad \text{less } \tfrac{2}{5} \text{ of } \cdot 02971 = \cdot 01188$$
$$\text{Difference} \quad \cdot 02971 \qquad\qquad (wfmA)_{[57]+6} = \cdot 02390$$

In order to obtain from these results the values of benefits depending on the remarriage of married men, namely, (1) an endowment payable on their remarriage, (2) an assurance payable on their death if they have remarried, (3) an assurance payable on their death if they have remarried and had issue, we proceed as follows. Let x be the age of the husband, and y the age of his wife. If the wife die in the nth year leaving her husband surviving, the then value of an endowment payable on his remarriage will be $(wmE)_{[x+n-\frac{1}{2}]}$, as the wife will, on the average, die in the middle of the year. The probability of the wife dying in the nth year and the husband surviving, is

$$\frac{d_{y+n-1}}{l_y} \cdot \frac{l_{x+n-\frac{1}{2}}}{l_x} \, ;$$

and the present value of the sum required to provide for this benefit if her death occur in the nth year, is therefore

$$v^{n-\frac{1}{2}} \frac{d_{y+n-1}}{l_y} \cdot \frac{l_{x+n-\frac{1}{2}}}{l_x} \cdot wmE_{[x+n-\frac{1}{2}]}.$$

The total present value of the insurance is therefore

$$v^{\frac{1}{2}} \frac{d_y}{l_y} \cdot \frac{l_{x+\frac{1}{2}}}{l_x} \cdot wmE_{[x+\frac{1}{2}]} + v^{\frac{3}{2}} \frac{d_{y+1}}{l_y} \cdot \frac{l_{x+\frac{3}{2}}}{l_x} \cdot (wmE)_{[x+\frac{3}{2}]} + \ldots$$
$$= \Sigma v^{\frac{1}{2}} \frac{d_y}{l_y} \cdot \frac{l_{x+\frac{1}{2}}}{l_x} \cdot (wmE)_{[x+\frac{1}{2}]}.$$

For the other benefits we have only to substitute $(wmA)_{[x+\frac{1}{2}]}$ and $(wfmA)_{[x+\frac{1}{2}]}$ respectively, for $(wmE)_{[x+\frac{1}{2}]}$ in the last expression. The mortality table used for the husband was that of Mr. Sprague for married men given in Table L, the number living at the middle of a year being taken as the mean of two consecutive ages. For the wife, Dr. Thiele's table of mortality for British Peerage Females (*J.I.A.*, xvi, 43) was used, the l_x and d_x columns of which from ages 20 to 90 are given in Table M, and a column has been added giving the force of mortality, μ_x, at each age,

calculated by the well-known approximate formula $\dfrac{l_{x-1}-l_{x+1}}{2l_x}$.
The values of $(wm\mathrm{E})_{[x+n-\frac{1}{2}]}$ were taken as the mean of $(wm\mathrm{E})_{[x+n-1]}$ and $(wm\mathrm{E})_{[x+n]}$. The values of an endowment payable on remarriage were calculated for each year for various ages of the husband and wife; and the summation gave the required single premiums.

For the calculation of isolated values we might use the more exact formula

$$\frac{1}{l_x l_y}\int r^n l_{x+n}.l_{y+n}.\mu_{y+n}.(wm\mathrm{E})_{[x+n]}dn,$$

obtaining the value by means of an approximate formula of summation, for instance, the approximate formula given by Mr. Woolhouse at the end of his paper, *J.I.A.*, xxvii, 122. Taking the husband's age as 55 and the wife's as 50, we have only five values of the function to calculate, and obtain ·05364 as the result, against ·05357 by the other formula.

KEY TO THE NOTATION.

I. Widowers subject to the ultimate rate of remarriage, called *chronic* widowers:

$(wm\mu)_{x+\frac{1}{2}}$ = force of marriage in the middle of the year of age x to $x+1$, or the central marriage rate.

$(wd\mu)_{x+\frac{1}{2}}$ = force of mortality at the same age, or central death rate.

$(wl)_x$ = number of widowers living at age x in a combined marriage and mortality table.

$(wd)_x$ = number of widowers dying between x and $x+1$.

$(wm)_x$ = number of widowers marrying between x and $x+1$.

$(wfm)_x$ = number of widowers who marry between x and $x+1$ and have issue.

$\dfrac{\Sigma(wm)_x}{(wl)_x}$ = probability that a widower will marry.

$\dfrac{\Sigma(wfm)_x}{(wl)_x}$ = probability that a widower will marry and have issue.

II. Commencing and recent widowers:

$[x]$ denotes that x was the widower's age at his wife's death, or his "age at entry"; and

$[x]+n$ that n years have elapsed since then.

For example, calling a widower whose wife is just dead, a *commencing* widower,

$(wl)_{[x]}$ = number of commencing widowers of the age x.

$(wl)_{[x]+n}$ = number of widowers of the age $x+n$, who became such at age x.

$= (wl)_{x+n}$, when the marriage rate has reached its ultimate value.

Widowers among whom the marriage rate has not reached its ultimate value are called *recent* widowers.

III. Commutation columns:

$$(wD)_x = (wl)_x v^x.$$
$$(wmEN)_x = \Sigma\left[(wm)_x v^{x+1}\right].$$
$$(wmM)_x = \Sigma\left[(wm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}}\right].$$
$$(wfmM)_x = \Sigma\left[(wfm)_x v^{x+\frac{1}{2}}(mA)_{x+\frac{1}{2}}\right].$$

We have similar columns for quinquennial ages at entry.

IV. Benefits—Widowers:

$(wmE)_x = (wmEN)_x \div (wD)_x =$ value of an endowment of 1 payable at the end of the year in which a chronic widower shall remarry.

$(wmA)_x = (wmM)_x \div (wD)_x =$ value of an assurance of 1 payable on the death of a chronic widower if he marry.

$(wfmA)_x = (wfmM)_x \div (wD)_x =$ value of an assurance of 1 payable on the death of a chronic widower if he marry and have issue.

We have similar benefits for recent widowers for quinquennial ages at entry, and for commencing widowers of all ages.

V. Bachelors:

$(bmE)_x =$ value of an endowment of 1 payable at the end of the year in which a bachelor shall marry.

$(bmA)_x =$ value of an assurance of 1 payable on the death of a bachelor if he marry.

$(bfmA)_x =$ value of an assurance of 1 payable on the death of a bachelor if he marry and have issue.

$\dfrac{\Sigma(bm)_x}{(bl)_x} =$ probability that a bachelor will marry.

$\dfrac{\Sigma(bfm)_x}{(bl)_x} =$ probability that a bachelor will marry and have issue.

ACTUARIAL NOTE.

Demonstration of a Formula relating to the Theory of Errors.

IN Dr. Thiele's article on a proposed "mathematical formula" for expressing the law of mortality (*J.I.A.*, xvi, 313), we have a valuable example of a strictly scientific test, proposed by a practical astronomer, who doubtless must have had considerable experience in the application of the theory of errors of observations. Among much other very interesting matter, Dr. Thiele gives (but without demonstrating) an approximate formula for determining the weight of an observation of the logarithm of the probability of surviving the year. The following very simple demonstration of Dr. Thiele's formula may perhaps not be without interest to the readers of the *Journal*.

Dr. Thiele's "mean error", it will be recollected, denotes the function more accurately defined as "the square root of the mean square error." Let l denote the number of individuals comprised in the observation, s, s', s'', &c., the numbers of the observed survivors in the several trials (n in number); then the actual mean square error of the Naperian logarithm of the probability of surviving

$$= \frac{\left(\log_\epsilon \frac{s}{l} - \log_\epsilon p\right)^2 + \left(\log_\epsilon \frac{s'}{l} - \log_\epsilon p\right)^2 + \left(\log_\epsilon \frac{s''}{l} - \log_\epsilon p\right)^2 + \ldots}{n}$$

$$= \frac{1}{n} \left\{ \left(\log_\epsilon \frac{s}{lp}\right)^2 + \left(\log_\epsilon \frac{s'}{lp}\right)^2 + \left(\log_\epsilon \frac{s''}{lp}\right)^2 + \ldots \right\}.$$

Now $\log_\epsilon \frac{s}{lp} = \log_\epsilon \left(1 + \frac{s - lp}{lp}\right) = \frac{s - lp}{lp}$ (approximately), and so on, with the other terms. Hence the approximate mean square error of the given function becomes

$$\frac{(s - lp)^2 + (s' - lp)^2 + (s'' - lp)^2 + \ldots}{n(lp)^2} = \frac{m \cdot e^2}{(lp)^2},$$

that is to say, = the mean square error of the *observed survivors*, divided by $(lp)^2$. But, as I showed in the *Journal* for April last, $m \cdot e^2 = lpq$. Therefore the mean square error of the *Naperian* logarithm of the probability of surviving the year $= \dfrac{q}{lp}$, and the weight of the observation (which is inversely proportional to the

mean square error) $= \dfrac{lp}{q}$. If, in the above process, we substitute

$\log \epsilon \times \log_\epsilon \cdot \dfrac{s}{lp}$ for $\log_\epsilon \cdot \dfrac{s}{lp}$, &c., we get for the weight of the

common logarithm, $\dfrac{lp}{q(\log \epsilon)^2}$, which is Dr. Thiele's result.

W. M. MAKEHAM.

London, 30 May 1890.

CORRESPONDENCE.

MR. MANLY'S PAPER.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—Referring to the subject discussed at the Institute on Monday last, permit me to bring to the notice of your readers the following modification of the processes advocated by Mr. Manly.

In place of Mr. Sprague's substitution of P_{x+1} for P_x, we can, where practicable, and with, perhaps, greater equity as between entrants at different ages, assess our new business expenditure by way of fixed charge (say r) per £1 assured. This will be represented in the annual loading by $\dfrac{r}{1+a_x}$, and at age t the amount of initial expenditure not repaid will be $r\dfrac{1+a_t}{1+a_x}$, or $r(1-V_{x|t})$.

Now we know that the net formula will resolve itself into

$$b^1 = \frac{D_t}{D_z}(1-V_{x|t});$$

and if we add, as a further endowment to be purchased, the above outstanding debt, we have

$$b^1 = \frac{D_t}{D_z}(1-V_{x|t})(1+r).$$

At a second valuation we shall have

$$b^2 + b^1 \frac{D_z}{D_{z'}} = \frac{D_{t'}}{D_{z'}}(1-V_{x'|t})(1+r)$$

in the same way, since the reserve value at any subsequent date of the endowment purchased by b^1 is irrespective altogether of the amount or term of the endowment so purchased, and the introduction of $r(1-V_{x'|t})$ represents the amount of expenditure outstanding at the new date of maturity.

The same reasoning will apply to all future valuations, and the formula is reduced to the ordinary net one by dividing out by $(1+r)$,

i.e., that all provision can be made, and Mr. Manly's processes and tables still employed by the simple expedient of first discounting the amount of the cash bonus under treatment.

> I am Sir,
>   Your obedient Servant,

London, E.C.,   H. P. CALDERON.
    3 *March* 1890.

THE INSTITUTE OF ACTUARIES.

REVISED RULES RELATING TO THE EXAMINATIONS OF THE INSTITUTE.

It will be noticed that the following rules differ from those hitherto adopted, in that the subjects formerly optional (Elementary Plane Geometry in Part I, Elementary Differential and Integral Calculus in Part II, and the application of the higher mathematics to the Theory of Life Contingencies in Part III) are now included in the ordinary subjects of the syllabus.

Attention may also be drawn to the fact that, in future, the names of the successful candidates, in each Part of the Examinations, will be arranged alphabetically in three classes or orders of merit. The new rules will come into force at the Examinations to be held in April 1891.—Ed. *J.I.A.*

RULES prescribed by the Council of the Institute to regulate the Examinations qualifying for admission to the Classes of Students, Associates, and Fellows respectively.

GENERAL REGULATIONS.

1. Except in the case of Examination for admission as a Student, at least one month's public notice will be given of the days and hours when, and of the place or places where, the Examinations will take place.

2. Candidates for any Examination must give fourteen days' notice in writing, addressed to the Honorary Secretaries, of their intention to present themselves for Examination, specifying the particular Examination for which they intend to present themselves.

3. The Examiners shall place successful Candidates in three Classes, and the names in each class will be arranged in alphabetical order.

4. The Examinations shall be conducted partly in writing and partly *vivâ voce*, or entirely in writing, as the Examination Committee may think fit, and under such other conditions as the

Examination Committee may prescribe, subject to the approval of the Council.

Rules relating to Examinations for Admission to the Class of Students.

1. Examinations will be held as often as the Examination Committee may prescribe.

2. Candidates whose applications for admission have been approved by the Council shall, unless exempted, be examined in the following subjects :—

    (1) Writing from Dictation.
    (2) English Composition.
    (3) Arithmetic.
    (4) Elementary Algebra (not beyond Equations of the first degree).

3. In the event of a Candidate passing the Examination, he will be admitted a Student, after he has signed the Form of Obligation prescribed by the Council, and paid his Subscription for the current year.

4. In the event of a Candidate failing to pass the Examination, he shall be permitted to offer himself for Examination once more within a period not exceeding six months from the date of the First Examination. If he fail to pass on the second occasion, or do not present himself a second time for Examination within such prescribed time, the fee paid upon application for admission shall be forfeited.

Rules relating to Examinations for Admission to the Class of Associates.

1. The Examination to qualify for admission as an Associate shall consist of two parts, and no Student shall present himself for Examination in the second part, unless he has previously succeeded in passing the first part of the said Examination.

2. Examinations in the first part of the Examination will be held in April and October of each year ; and Examinations in the second part will be held in April of each year, or oftener if the Council shall think fit.

3. Candidates for any Examination in either the first or second part must, at the time of giving notice of their intention to present themselves for Examination, pay a fee of One Guinea.

4. In the first part of the Examination, Students shall be examined in the following subjects :—

    (1) Arithmetic and Algebra, including theory and practice of logarithms, and the elements of the theory of probabilities.
    (2) Euclid—Books I, II, III, IV, and VI, and Definitions of Book V.

(3) Elements of the Theory of Finite Differences, including its practical application, Interpolation.

(4) Elementary Plane Co-ordinate Geometry, as applied to the Straight Line and the Conic Sections.

(5) Principles of Book-keeping.

5. In the second part of the Examination, Students who have passed the first Examination shall be examined in the following subjects:—

(1) The Theory of Compound Interest, including Annuities-Certain.

(2) The application of the Theory of Probabilities to Life Contingencies.

(3) The Theory of Life Contingencies, including Annuities and Assurances on Lives and Survivorships.

(4) The use of Mortality Tables, and the construction and use of other Tables relating to the Theory of Life Contingencies.

(5) Practical examples in all the foregoing subjects.

(6) Elementary Differential and Integral Calculus.

Rules relating to Examinations for Admission to the Class of Fellows.

1. Examinations will be held in April of each year, and oftener if the Council shall think fit.

2. Subject to the exceptions hereinafter mentioned, the Examination to qualify an Associate or Student for admission as a Fellow shall consist of three parts. The first and second parts of the Examination shall be in the same subjects and subject to the same rules as those prescribed in the Rules relating to Examinations for Admission of Associates; and in the third part, for which they shall not present themselves until an interval of at least twelve months after passing the second part, Candidates shall be examined in the following subjects:—

(1) The methods of compilation and treatment of the materials for the formation of Mortality Tables and other Tables relating to the Theory of Life Contingencies, and the Methods of graduation of such tables, with special reference to existing well-known tables, and with practical examples.

(2) The methods of valuation of the liabilities and assets of Life Assurance Companies and analogous institutions, and of dealing with the results shown by such valuations, with practical examples.

(3) Elements of the laws of real and personal property, and of the laws relating to life assurance contracts and to joint-stock companies.

(4) The principles of banking and finance, including the constitution and operations of the Bank of

England, the National Debt of this country, and
investments generally.

(5) Miscellaneous subjects connected with the foregoing,
such as practical valuation of Life and Rever-
sionary Interests, preparation of Statements and
Reports, &c.

(6) The application of the higher mathematics to the
Theory of Life Contingencies.

3. Candidates for Examination in the third part must, at
the time of giving notice of their intention to present them-
selves for Examination, pay a fee of One Guinea.

4. Associates who have prior to the commencement of
Bye-Laws passed the First or Second Year's Examination
prescribed by the General Regulations issued by the Council
prior to the commencement of Bye-Laws, shall be required to
pass only the second and third parts, or only the third part,
respectively, of the Examination, to qualify for admission as
a Fellow.

5. Associates admitted after the commencement of Bye-
Laws shall be required to pass only the third part of the
Examination to qualify for admission as a Fellow.

May, 1890.

EXAMINATIONS OF THE INSTITUTE, April 1890.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).

Examiners—B. A. BERRY, Esq.; F. E. COLENSO, Esq., M.A.;
J. E. FAULKS, Esq.; G. KING, Esq.

First Paper.

1. State and prove the rule for reducing a recurring decimal to
a vulgar fraction, and reduce the following expression to a single
decimal fraction :

$$\frac{919\frac{2}{17}}{7\cdot95\dot{4}} \times \frac{4\cdot1\dot{0}\dot{0}}{442\frac{30}{37}} \times \tfrac{7}{22} \text{ of } \cdot073\dot{4}\dot{4} ;$$

and prove that $\sqrt[3]{\cdot03\dot{7}} = \cdot\dot{3}$.

2. 12,000 sums of money, consisting of (besides pounds and
shillings) equal numbers of all possible sums of pence from 0*d.* to
11*d.* inclusive, are each expressed in pounds and decimals of a pound,
3 places of decimals being retained, and the usual rule being observed
by which the figure in the last place is increased by unity if the first
of the figures rejected be not less than 5. What will be the error in
the amount obtained by adding these decimals together?

3. There are a sopranos, b contraltos, c tenors, and d basses. How many choirs of eight voices, two of each kind, can be formed?

4. The assumed value of a quantity exceeds its real value, 12, by ·01. Find the amount of the error in the common logarithm of this assumed value, expressing your result to 7 places of decimals, having given that the modulus of common logarithms $= ·43429$ approximately.

5. What is the most probable number of aces that will be thrown in 35 throws with 1 die? Give an algebraical proof of the correctness of your answer.

6. A bag contains 4 balls, each of which is either white or red: a ball is drawn at random and is found to be red; what is the probability that the original contents of the bag were 3 red balls and 1 white ball?

7. Three men, A, B, and C, throw dice upon the condition that he who makes the lowest throw shall give each of the others the sum he has already. Each loses in turn in the order named, and at the end of the third game all have the same sum, namely, £n. How much had each at first?

8. A farmer spends £200 on cows and £540 on horses. If the cost of each cow had been £22 more, and that of each horse £3 less, the cost of a cow and of a horse would have been the same, and the farmer would have spent £184 more. Find how many of each he purchased.

9. Answer *one only* of the following questions:

  (a) Describe an isosceles triangle having each of the angles at the base double of the third angle; or

  (b) Show how to trisect a given straight line.

10. Show how to interpolate an unknown value of a function, by means of known values which are not consecutive.

By means of the formula, having given

$$\log 280 = 2·4472$$
$$,,\ \ 281 = 2·4487$$
$$,,\ \ 283 = 2·4518$$
$$,,\ \ 286 = 2·4564$$

deduce $\log 282$.

11. In keeping a set of accounts by double entry, would it be possible to dispense with the use of a journal?

Give reasons for your answer.

It is optional on the part of the Candidate to answer the following question; but due weight will be given to answers sent in.

12. Investigate the general equation to a straight line referred to rectangular axes, and show by diagrams the consequences of making different assumptions as to the values of the constants.

Second Paper.

13. Find the Least Common Multiple of 24, 15, 9, 36, and 54, and explain concisely the reason for each step in the process.

14. Find the greatest term in the expansion of $(x+a)^n$, and expand $\left(\dfrac{a+x}{a-x}\right)^{\frac{1}{2}}$ in a series of ascending powers of x.

15. The gross income of a man was £30 more in the second of two particular years than in the first, but in consequence of the income tax rising from 5*d.* in the pound in the first year to 8*d.* in the pound in the second, his net income was unaltered. Find the income.

16. The price of wine having been raised 6*s.* per dozen, 6 bottles less than before can be bought for 8 guineas. What is the present price?

17. How many florins and half-crowns, whose diameters are $1\frac{1}{6}$ and $1\frac{1}{4}$ inches respectively, can be laid with their centres in a straight line, the row of coins being exactly a yard long?

18. In the quadratic equation $ax^2+bx+c=0$, what relations between the roots are known, without solving the equation?

The product of the roots of this equation is found to be equal to one-third of the sum of their squares: prove that $b^2=5ac$.

19. Given the expression $\log_\epsilon(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\&c.$, deduce a more convenient expression for finding the natural logarithms of a series of numbers, and from it calculate $\log_\epsilon 2$, and show how to proceed to calculate common logarithms.

20. The figures 1, 2, 4, 5, 7, 8, are arranged at random as the period of a circulating decimal, which is then reduced to a vulgar fraction in its lowest terms. What are the odds against the denominator being 7?

21. A bag contains 6 balls, 4 black and 2 white. A and B draw in succession, the first to draw a white ball being entitled to receive £1. Find the value of the expectation of each man (i) if the balls are replaced after each draw, (ii) if they are not so replaced.

22. Answer *one only* of the following questions:

(*a*) Explain, in few words, the principles and advantages of the method of "Central Differences"; or

(*b*) What is meant by the "Differences of Nothing"? How may they be calculated?

23. Deduce an expression for the $(n+1)$th difference of u_x, a function of x, in terms of u_x and its successive values, and prove that this difference will vanish if u_x be a rational integral function of x of the nth degree.

24. Prove that a straight line drawn parallel to one of the sides of a triangle cuts the other sides, or those sides produced, proportionally.

It is optional on the part of the Candidate to answer the following question; but due weight will be given to answers sent in.

25. Trace the curve having the following equation to rectangular co-ordinates:

$$(x-2c)^2 + (y-c)^2 = c^2.$$

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE, OR TO THE CLASS OF FELLOW (PART II).

Examiners—A. F. BURRIDGE, Esq.; J. H. DUNCAN, Esq.; W. J. H. WHITTALL, Esq.

First Paper.

1. Given the time in which a debt bearing interest is discharged by annual instalments; show how to find, in how much less time the debt will be discharged if the instalments are payable half-yearly.

2. The purchaser of an annuity-certain for n years wishes to obtain interest at rate i' from his investment, and to provide for the accumulation of the sinking fund annually at rate i. Find the annuity required by him if the sum invested be S.

Given $S = £1,000$, $n = 20$, $i' = \cdot05$, and $i = \cdot035$; find the annuity.

Ascertain the formula for the amount of the original capital which may be considered to have been repaid by the end of t years.

3. Define the term *Force of Discount*.

Derive a formula for its value, and calculate to six places of decimals the force of discount corresponding to 3 per-cent interest.

Prove that

$$\delta = \frac{i+d}{2} - \frac{i^2-d^2}{4} + \frac{i^3+d^3}{6} - \&c.$$

4. Prove that the sum of the first n terms of a table of amounts of annuities-certain $(S_{n|})$ is equal to the $(n+1)$th term diminished by $n+1$ and divided by the rate of interest.

Find the sum of the first 20 values of S_n at 4 per-cent, having given $v^{21} = \cdot 456387$.

5. Find the probabilities of the following events:

(a) that one at least of (x) and (y) will die within n years;
(b) that (x) will be alive at the end of the nth year succeeding that in which (y) dies;
(c) that (x) will die after (y) within n years.

6. What does $e^1_{n|x}$ represent? Determine the formula expressing its value.

7. Obtain the value of an annuity for the remainder of a term of n years, after the failure of the joint existence of (x) and (y), provided such failure take place within t years. Suggest a symbol for this value.

8. Given (L_x) a census of a population in the middle of a calendar year, and the deaths at each age during that calendar year, show how the l_x and d_x columns of a life table may be ascertained. If the death registers alone were at your disposal for a series of years, and it were known that the births had increased annually by 2 per-cent, how would you proceed to construct the column l_x?

9. Give an expression in terms of e_x for the mean annual death-rate of persons aged x and upwards.

Given a table of e_x, how would you proceed to construct the columns l_x and d_x? What methods of verification would you adopt?

10. Discuss a formula for finding e_x without any assumption as to distribution of deaths:

And find an approximate value of e_{30}, having given $e_{30} = 34 \cdot 226$, $l_{30} = 89,685$, $d_{30} = 691$, $d_{31} = 700$.

11. Obtain an expression for the *Force of Mortality*, and show that, approximately,

$$p_x = 1 - \mu_{x+\frac{1}{2}} + \tfrac{1}{2}\mu^2_{x+\frac{1}{2}} - \tfrac{1}{4}\mu^3_{x+\frac{1}{2}} + \&c.$$

12. Investigate the change in the values of q_x produced by assuming an increase in the rate of interest to represent an increased mortality. Take the case where a_x is extracted from the 4 per-cent Table, and assumed to be a 3 per-cent value.

Second Paper.

13. How would you estimate the amount of claims expected to occur in the ordinary whole-term business of an office in a year?

14. State the formula which you regard as most satisfactory for

obtaining the value of a life annuity, payable m times a year, from the ordinary value, and show how it is derived.

What modification in your result is necessary if the annuity be made complete?

Given, at 4 per-cent interest, $a_{24} = 18.520$, find an approximate value of $a_{24}^{(2)}$.

15. Find in terms of commutation columns the reserve for the following policies, which have been n years in force:

(a) an endowment assurance payable at age $(x+z)$, the premiums being returnable along with the sum assured;

(b) a deferred annuity, payable half-yearly, maintained by annual premiums, the annuity to be entered on at age $(x+z)$: examine the three cases of $n =, <,$ or $>, z$.

16. Prove that

$$1 - (P_y + d)(1 + a_{xy}) = A_{xy}^1 (1 - V_y)$$

where V_y is the value of a policy on (y) at the end of the year in which (x) dies.

Give a verbal interpretation of the above equation, and also of the following:

$$(a) \qquad A_x = \frac{a_x - a_x'}{1 + a_x};$$

$$(b) \qquad P_{x.\,\overline{n}|}^1 = \frac{D_x - D_{x+n}}{N_{x-1} - N_{x+n-1}} - d.$$

17. Deduce a general formula, in terms of D, M, R, for the value of $(vA)_x$, a varying assurance under which u_0 would be paid in the first year, u_1 in the second, u_2 in the third, and so on.

Hence, find the uniform annual premium for an assurance under which the sum assured in successive years would be 1, 6, 15, 28, &c.

18. Find a convenient expression for the value of the total additions to a whole-life policy which will be entitled to bonuses, declared at decennial intervals, at the rate of £1 per-cent per annum in respect of each premium paid from the outset; an interim bonus of £1 per cent. per annum being allowed after the first five years.

Take the cases of policies effected in the first and last years of a decennial period.

19. If P_x be increased by a constant, how will $_nV_x$ be affected? What should be the nature of the addition to P_x so that the policy-values may be unaltered?

20. Given a complete table of temporary annuities and also of single premiums for whole-life assurances, show how to form a table of single premiums for deferred assurances.

21. Describe the method adopted by Mr. Peter Gray in the construction of the tables of joint-life annuities based on the H^M Table.

22. Find an expression for the value of a life interest in terms of the annual premium to assure the life and the rate of interest.

Show how to construct a table of the values of life interests corresponding to various rates of annual premium increasing by differences of ·025.

23. Deduce formulas for tabulating by a continued process the following functions, and state how you would check the results:

(a) P_{n1}^{1} .   (b) $_nV_{s,\overline{m}|}$.

The following questions are optional on the part of the Candidate.

24. State Leibnitz's theorem for finding the nth differential coefficient of the product of two functions.

Find the nth differential coefficient of $x^{n-1}.\log x$.

25. Find, by making use of the differential and integral calculus, the value of a complete annuity payable annually, assuming a uniform distribution of deaths.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

Examiners—M. N. ADLER, Esq., M.A.; J. CHISHOLM, Esq.;
G. H. RYAN, Esq.

First Paper.

1. Discuss the question whether tables of experience, and the usual monetary values deduced therefrom, should be based on observations relating to lives or amounts assured.

2. Describe the construction of Select Mortality Tables. To what uses can such tables be appropriately put?

3. Explain Woolhouse's Method of Adjustment. Would it be correct to say that this method, when applied to certain perfectly smooth series, introduces irregularities?

4. How would you estimate the Expected Claims in the case of a Company transacting a large and increasing new business?

5. You are asked to compute tables of premiums for the common modes of assurance from the unadjusted mortality experience of a Life Office. Discuss the advantages of Makeham's Method of Graduation and the Graphic Method for this special purpose.

6. State on what principles you would apply (1) Cash Bonuses and (2) Reversionary Bonuses, to make the sum assured under a whole-life policy payable during lifetime:

> (a) When the option has to be exercised on effecting the policy with regard to all bonuses.
>
> (b) When the option may be exercised at any time during the currency of the policy.

7. A Life Assurance Company had a surplus, on 31 December 1880, of £200,000, whereof £180,000 was distributed among the Policyholders, and the balance carried forward. On 31 December 1885, the surplus, on the same basis of valuation, is found to be £240,000. How would you account for the latter surplus by tracing it approximately to its several sources?

8. What adjustments should, in your opinion, be applied to provide for the immediate payment of claims in the case of:—

> (1) An ordinary net-premium valuation at 3 per-cent interest;
>
> (2) A gross-premium valuation at 3 per-cent interest, 25 per-cent of the gross premiums having to be reserved for future expenses and profits?

9. State your opinion of the relative merits and stringency of the following tables for the valuation of single-life annuities:—

> (1) Carlisle.
>
> (2) Davies' Equitable.
>
> (3) Government Annuitants' (1883).

10. Does a net-premium valuation by the combined H^M and $H^{M(5)}$ Tables provide an adequate reserve for policies of less than five years' duration? If not, can you suggest any approximate method of making a suitable adjustment to meet the requirements of the case?

11. It is desired to make provision for an assurance to be granted at the end of three years on a healthy life, now aged x, without further evidence of health. How would you assess the fine to be charged at the present time for such deferred assurance, assuming the annual premium payable when the risk commences to be the normal rate for age $x+3$?

12. On what methods and bases of valuation would you estimate the liability of a large Industrial Company under its various contracts?

The following question is optional on the part of the Candidate.

13. Prove that, according to Makeham's Law of Mortality, it would be possible to dispense with tables of annuities by forming instead a complete set of tables of the average duration of life for equal ages, and adopting Woolhouse's suggestion of lateral interpolation.

Second Paper.

14. What are the provisions of "The Customs and Inland Revenue Act, 1888" with regard to the stamp duty on assignments of life policies? What are the *ad valorem* Stamp Duties on Mortgages, Re-assignments, Conveyances and Bonds securing Annuities, immediate and reversionary?

15. Mention the leading suggestions that have been made from time to time for the modification of the Life Assurance Companies Acts, 1870 to 1872.

16. What is an Advowson? Would you recommend the Directors of a Life Office to make an advance on such a security? State your reasons.

17. Give as full an account as you can of Mr. Goschen's re-adjustment of the National Debt.

18. What are the provisions of the Bank Act of 1844 respecting the issue of Notes by the Bank of England? What limitation is there to the issue of Notes in Great Britain by Banks other than the Bank of England?

19. A Company divides its quinquennial surplus between Proprietors and Policyholders, the former taking cash and the latter reversionary additions to their policies. The valuation is made at 3 per-cent interest. On what basis would you convert the Policyholders' share into reversion, so as to make an equitable division of the surplus between the two bodies?

20. State briefly what you consider the chief objections to a compulsory National System of Insurance?

21. What considerations would influence you in deciding what maximum sum a Life Assurance Company should retain at its own risk upon a single life?

22. Give a formula for the value of a reversionary annuity to x after y, the vendor to retain the option of redemption, exercisable at any time during 5 years, on payment of the purchase-money and the premiums for the necessary assurances, accumulated at 6 per-cent interest.

23. A, aged 30 (an only child), is entitled in reversion to a fund of £10,000, payable on the death of his father and mother, both aged 60 and in good health. The reversion is subject to proportionate diminution for each further child born to the father and surviving him, or attaining 21 in his lifetime, by his present or any future wife. Indicate fully the method in which you would determine the value of A's expectancy.

24. Give a list of the securities in which trustees are now authorized to invest.

Draft a Report discussing the merits of such investments from the point of view of Life Assurance Companies.

The following question is optional on the part of the Candidate.

25. Enunciate and prove the law of the composition of decremental forces. Show how it may be applied to the determination of a question like the following:

> What would be the probable effect upon the increase of population of the extinction of small-pox?

PROCEEDINGS OF THE INSTITUTE.—Session 1889–90.

First Ordinary Meeting, 25 November 1889.

The President (Mr. W. Sutton) in the Chair.

The President delivered an inaugural address.

Second Ordinary Meeting 23 December 1889.

The President (Mr. W. Sutton) in the Chair.

The following gentlemen were elected Associates of the Institute:
Fox, Charles Edward. | Lugton, Hugh.

The following gentleman was elected a Corresponding Member of the Institute:

Gottfried, Schaertlin.

Mr. Sunderland read a paper entitled " Bonuses on Endowment Assurance Policies."

The following gentlemen took part in the discussion:—Messrs. King, Schooling, Tarn, and Manly.

Third Ordinary Meeting, 27 January 1890.

The President (Mr. W. Sutton) in the Chair.

The subject for discussion, " Income Tax ", was introduced at length by Mr. Newbatt, who was followed by Messrs. Bailey, Ackland, F. Bell, A. Hendriks, Tucker, and McKenzie.

Fourth Ordinary Meeting, 24 February 1890.

The President (Mr. W. Sutton) in the Chair.

Mr. Manly read a paper entitled " On some of the Practical Questions " raised during the Discussion that followed the reading of my Paper printed " in the *Journal*, vol. xxvii, p. 362; and on a Further Use for my Tables in " applying Profits to make Annual Premiums cease at an Earlier Age than " fixed in the Policy."

The following gentlemen took part in the discussion:—Messrs. George King, Sprague, F. Bell, and Colenso.

Fifth Ordinary Meeting, 31 *March* 1890.

The President (Mr. W. SUTTON) in the Chair.

Mr. T. B. Sprague read a paper entitled "On the Premiums for "Assurances against Issue to Widowers, Bachelors, and Married Men, or on "the Calculation of the Values of Benefits depending on Marriage and on "Birth of Issue."

The following gentlemen took part in the discussion:—The President, Messrs. Bailey, A. Day, A. Hendriks, Colenso, Ackland, G. King, Ryan, and Adler.

Sixth Ordinary Meeting, 28 *April* 1890.

The President (Mr. W. SUTTON) in the Chair.

Mr. James Chatham read a paper entitled "On the Construction and Use of the Combined Marriage and Mortality Tables for Widowers."

The President read the first part of a paper upon the subject of the assessment system of life assurance lately introduced in America.

The Forty-Third Annual General Meeting, 2 *June* 1890.

The President (Mr. W. SUTTON) in the Chair.

The proceedings at the Annual General Meeting will be found on page 413.

REPORT 1889-90.

"The Council have pleasure in reporting to the members upon the progress of the Institute during the session of 1889-90, the forty-second year of its existence.

"The appended schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

"The increase in the number of members was 38, as compared with 42 in the preceding year. Since the date of the Charter the number of members has been as follows:

1884–85	.	.	434		1887–88	.	.	521
1885–86	.	.	441		1888–89	.	.	563
1886–87	.	.	484		1889–90	.	.	601

"These figures give very satisfactory proof that the younger members of the actuarial profession value and appreciate the facilities which the Institute offers for securing efficiency in their professional attainments.

Schedule of Membership, 31 March 1890.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1889 .	2	165	218	168	10	563
ii. Withdrawals by						
(1) Death . . .	...	1	...	1	...	
(2) Resignations. .	...	...	5	13	...	25
(3) Default in payment of Subscriptions. .	...	1	3	1	...	
	2	163	210	153	10	538
iii. Additions to Membership . . .	...	1	4	57	1	63
	2	164	214	210	11	601
iv. Transfers						
(1) By Examination:						
from Associates	...	...	9	...	...	...
to Fellows .	...	9	...	...	...	. .
	2	173	205	210	11	601
(2) By Examination:						
from Students	...	...	...	2	...	...
to Associates .	...	...	2	...	...	...
	2	173	207	208	11	601
(3) By Election:						
from Associates	...	...	...	...	...	...
to Fellows .	...	...	...	...	...	...
v. Number of Members in each class on 31 March 1890 .	2	173	207	208	11	601

"The accounts for the year are very satisfactory. Before the date of the Charter the total funds amounted to £3,255. 13s., and since that date some special items of expenditure have been incurred.—such as expenses of the Charter, removal to Staple Inn and furnishing the Hall and offices, and the *Text-Book*, Part II. Taking these into consideration, it is satisfactory to notice that the amount of funds, nevertheless, has been maintained, and an addition made by the sum of £291. 19s., constituting the Library Funds. The total amount of funds now being £3,543. 2s. 4d.

"The income and expenditure account and balance sheet are given herewith.

Dr. Income and Expenditure for Year ending 31 March 1890. **Cr.**

		£ s. d.	£ s. d.
1889-90.			
Amount of Funds, 31 March 1889—			
Messenger Legacy Fund		254 6 9	
Brown Prize Fund		264 8 3	
Library Fund		286 14 0	
General Fund		2,570 12 10	3,096 1 10
Annual Subscriptions—			
Town Fellows	£297 15 0		
Country "	75 12 0		
Fellows admitted since 1884:			
(1) By Examination	89 5 0		
(2) By Ballot	31 10 0		
Town Associates	£25 15 0		
Country "	21 18 0	464 2 0	
Associates admitted since 1884:			
(1) By Examination	8 8 0		
(2) By Ballot	29		
Students		348 12 0	
		217 7	1,030 1 0
Entrance Fees—			
Fellow		5 5 0	
Associates		10 10 0	
Students		67 4 0	82 19 0
Examination Fees			105 0 0
Sales of Publications—			
Text-Book, Part I.		29 4 3	
" " II.		8 5 5	
Log. Cards		7 15 11	
Mortality Experience		17 11 6	
Life Tables		18 13 6	
Messenger Prize Essay (Friendly Societies)		3 8 10	
Dividends		139 19 5	
Donations to Library Fund		64 5 0	
		5 5 0	
		£4,523 12 9	

		£ s. d.	£ s. d.
1889-90.			
Journal—			
Cost of Nos. 151-2-3-4		297 7 8	
Less Sales		91 10 6	205 17 2
Library—			
Purchases		13 4 0	
Binding		9 11 0	22 15 0
General Expenditure—			
Rent		250 0 0	
Salaries, including Income Tax		255 5 0	
Lecturers		52 10 0	
Examination Charges		17 13 11	
Meetings		24 16 0	
House Expenses		22 5 6	
Corporation Duty		3 5 0	
Fire Insurance		11 16 0	
Postage and Stationery		101 16 0	
Sundries		13 8 10	751 18 3
Funds 31 March 1890—			
Messenger Legacy Fund		280 8 8	
Brown Prize Fund		272 12 0	
Library Fund		291 19 0	
General Fund		2,698 5 10	3,543 2 1
			£4,523 12 9

Examined and found correct, 21 April 1890.
(Signed) ARTHUR B. WOODS, } Auditors.
W. G. TURPIN,
J. P. LONGSTAFF,

Balance Sheet, 31 March 1890.

LIABILITIES.

	£ s. d.	£ s. d.
Messenger Legacy Fund (£211. 1s. 10d. Consols), cost	203 17 8	
Accumulated Dividends	76 11 1	280 8 9
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent		
Debenture Stock), cost	200 0 0	
Accumulated Dividends	72 8 9	272 8 9
Library Fund		291 19 0
General Fund		2,698 5 10
Outstanding Account payable		71 8 3
		£3,614 1 7

ASSETS.

	£ s. d.	£ s. d.
Consols, 2¾ per-cent (£1,350) cost		1,246 4 6
Metropolitan Railway 4 per-cent Debenture Stock (£700), cost		715 1 0
On Deposit at the London and Westminster Bank		1,300 0 0
On Current Account at "		295 3 9
Cash in hand, including Subscriptions "		24 9 0
Subscriptions due and unpaid		33 12 0
		£3,614 1 7

Examined and found correct, 21 April 1890.
(Signed) ARTHUR B. WOODS, } Auditors.
W. G. TURPIN,
J. P. LONGSTAFF,

" The annual subscriptions, including admission and other fees, amount to £1,218, being £71 in excess of the previous year.

" The total expenditure for the year is £981, showing a considerable reduction.

" The stock in hand of the Institute publications is as follows:

No. of Copies					Description of Work
627	.	.	.	.	*Text-Book*, Part I.
811	.	.	.	.	,, ,, II.
202	.	.	.	.	Mortality Experience Tables.
38	.	.	.	.	Mortality Experience.
34	.	.	.	.	Logarithm Cards.
440	.	.	.	.	Messenger Prize Essay (Friendly Societies).
512	.	.	.	.	Index to 10 Vols.
138	.	.	.	.	,, 20 ,,
8,981	.	.	.	.	Parts of *Journal*.

" During the past year the Honorary Librarians, Mr. Henry Cockburn and Mr. W. J. H. Whittall, have devoted a great deal of time and attention to the Library, with the result that the arrangement of the books has been greatly improved. The Council again appeal to the members to aid it in its endeavours to make the Library as complete as possible from a historical point of view, by sending to the Honorary Librarians any old books and pamphlets relating to life assurance and cognate subjects which they can spare.

" The Library Fund amounts to £291. 19s., but the Council have not yet come to any decision as to its application, in the hope that it may be still further increased, so as to form a permanent endowment for the support of the Library.

" The Council are gratified to be able to announce that Mr. Sprague's renewed offer of a sum of £70, to be awarded to the writers of the best Essays on the subject of the effect of selection among assured lives, has resulted in three Essays being sent in, which are now being examined by the Adjudicators, and their award will be made known as soon as possible.

" At the first meeting of the session, held on 25 November 1889, the President (Mr. William Sutton) delivered an address; and the following papers have since been submitted:

" 23 *December* 1889—' Bonuses on Endowment Assurance Policies'—by Mr. A. W. Sunderland.

" 27 *January* 1890—A discussion on ' Income Tax'—opened by Mr. B. Newbatt.

" 21 *February* 1890—' On some of the Practical Questions raised during the Discussion that followed the reading of my Paper printed in the *Journal*, vol. xxvii, p. 362; and on a Further Use for my Tables in applying Profits to make Annual Premiums cease at an Earlier Age than fixed in the Policy '—by Mr. H. W. Manly.

" 31 *March* 1890—' On the Premiums for Assurances against Issue to Widowers, Bachelors, and Married Men, or on the Calculation of the Values of Benefits depending on Marriage and on Birth of Issue '—by Mr. T. B. Sprague.

" 28 *April* 1890—' On the Construction and Use of the combined Marriage and Mortality Tables for Widowers'—by Mr. J. Chatham.   Also a paper by the President—' On the Assessment System of Life Assurance,'

" The first Intermediate Examination in Part I was held on 26 October 1889, when eighteen candidates were examined, of whom eleven passed, namely :

Besant, A. D.	Lewis, J. W.
Carment, D. M.	Manwaring, H.
Clarke, A. H.	Taylor, F.
Docker, L.	Turnbull, A. D. L.
Hutton, W.	Worthington, W.

Vodden, H.

" For the examinations held on 25 and 26 April last, eighty-eight candidates, the largest number that has yet been recorded, entered their names.

" For the examination in the First Part, thirty-nine appeared, of whom eighteen passed, namely :

Archer, J. A.	Lawson, H. G. S.
Baker, H. J.	Legg, D.
Barlow, R. H.	Meikle, H. G. W.
Benthall, H. E.	Munro, D.
Coop, C. R.	Pagden, L. K.
Glover, H. W.	Parker, R. P.
Gooding, H. J.	Poston, R.
Gross, W.	Salmon, R. G.
Jackson, S.	Sanderson, F.

[Two candidates were absent owing to illness.]

" Thirty-three candidates were examined for the Second Part, of whom eleven passed, namely :

Alderoft, W. H.	McDonald, J.
Coote, C. E.	Phelps, W. P.
Davies, G. W. McC.	Powell, A.
Hart, J. R.	Straker, E. R.
Lidstone, G. J.	Thorne, A. C.

Watson, A. W.

" For the Third Part fourteen candidates were examined, and the following five passed, namely :

Bearman, H.	Hovil, L. F.
Griffin, R. W.	Marks, G.

Molyneux, A. E.

" The names in each case are alphabetically arranged.

" The Council have decided that in future the successful candidates in all the Examinations shall be arranged alphabetically in three classes.

" The Council have also decided that in future years the subjects which have hitherto been optional on the part of candidates for each Examination shall be included in the ordinary subjects of the syllabus.

" These will come into force at the Examination of next year.

" The classes in the subjects of Parts I and II of the Examinations have been satisfactorily attended.

" A desire having been expressed by some members to compound for payment of their future subscriptions, the Council, acting under the powers conferred upon them by No. 68 of the Bye-Laws, have given the matter careful consideration, and have decided as follows :

" That no member below the rank of Fellow be allowed to compound his subscription, and that the scale of fees as the basis of commutation be the fees paid by Town Fellows, and be the same for all Fellows alike, whether living in town or country.

" The following scale is proposed :

(*a*) That no commutation be allowed under age 40.

(*b*) That between ages 40-45 the fee be 36 Guineas.

		45-50		30	
,,	,,	50-55	,,	24	,,
,,	,,	55-60	,,	18	,,
,,	,,	60 & upwards		15	,,

(*c*) That on retirement from the profession, after age 60, the commutation be 10 Guineas.

" These are subject to alteration at any time if the Council think fit.

" The Council received in December last a requisition, under Bye-Law 88, to call a Special General Meeting of the Fellows and Associates for the purpose of making certain proposed alterations in the Bye-Law relating to the qualifications necessary for election to the Class of Fellows.

" Such Special General Meeting was duly convened, and held on 17 February 1890, and an amendment to the proposed alteration was moved by the President on behalf of the Council. The amendment was carried by a large majority, and after confirmation at a second Special General Meeting, has been duly submitted to the Lords of Her Majesty's Most Honourable Privy Council, and the amended Bye-Laws will come into force immediately they have been allowed by their Lordships.

" The Council cannot refrain from expressing an earnest hope that the alterations now made in the qualifications necessary for election to the Class of Fellows, and in the mode of election to the Classes of Fellows and Associates, will prove a lasting settlement of the important issues involved, and tend to the advancement of the Institute and the profession of Actuary.

" The Council have to repeat the request made last year that members of the Institute, particularly the younger members, will prepare papers for reading and for insertion in the *Journal*. It is only in this way that the high standard of interest hitherto attaching to the Sessional Meetings of the Institute can be preserved.

<div align="right">

" WILLIAM SUTTON,

"*President.*"
</div>

"13 *May* 1890."

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Monday, 2 June, the President, Mr. W. Sutton, M.A., in the chair.

The Report of the Council (given on p. 408) having been read,

The PRESIDENT, in moving its adoption, referred to several matters in the report upon which he thought it desirable to comment. In the first place the schedule of membership showed the numbers to be still on the increase. Life assurance business was not necessarily the only direction in which actuaries of the future might apply themselves, and he hoped that hereafter they would see " F.I.A.'s " figuring on the prospectuses of many

other companies than those given up solely to life assurance business. The schedule only showed one death for the year, but there had been two other deaths in the last three months. In addition to Mr. Charles Ansell's death, they had recently to chronicle with great regret that of two other very well-known men—Mr. Henry Ambrose Smith, and, within the last few days, Mr. William Vaughan. Every member who had occasion lately to make use of the library must unhesitatingly endorse the compliments which the report of the Council bestowed upon the Honorary Librarians. Speaking on behalf of himself and everyone who knew what the library was now and remembered what it was two or three years ago, they were all deeply indebted to Mr. Cockburn and Mr. Whittall. As regarded the library fund the Council hoped to see it made considerably larger in some way or other, and then would undertake, no doubt, after careful consideration, some well-regulated plan of spending it. With regard to that fund, it had occurred to him whether there was any other way of augmenting it except out of the pockets of the members. The Institute undoubtedly did good for its members, and it was right that the members should make some return; but there were a considerable number of very wealthy companies in this country who indirectly obtained a large amount of benefit from the Institute. He was referring to the large life offices of the country, and, perhaps, when the Institute got into hands that were identified more directly than his with life assurance business, perhaps the hint might be turned over and hereafter come to some account. The paragraph dealing with Mr. Sprague's prizes intimated that the award would shortly be made; and he had the pleasure on behalf of himself and the other adjudicators to read that award: "The "adjudicators appointed by the Council, namely, the President and Vice- "Presidents, beg leave to report, in conformity with the conditions laid down "by the Council, that three essays were received. After a careful examination "of the essays the adjudicators have to report that they have awarded the "first prize of £30 to the author of the essay bearing the motto 'Amicable'; "and two second prizes of £20 each to the respective authors of the essays "bearing the mottos 'Fide Fortitude' and 'Circumstances alter cases.'" The author of the essay to which they have awarded the first prize proved to be Mr. James Chatham. The authors of the essays to which they have awarded second prizes proved to be Mr. Philip Lewin Newman, of York, and Mr. Emory McClintock, of the United States. Everyone would be impressed with the immense labour the writers had bestowed upon their work. He took the opportunity of again mentioning for the consideration of the Council, the desirability of paying its Examiners. The Institute received £105 on account of examination fees, and the examination expenses were about £23 to £25—being £80 to the good. One was pleased to see the Institute making money, but he did not like to see £70 or £80 made out of examina- tions, and he thought it well worthy of the consideration of the Institute in the future whether, at all events, some portion of the surplus should not be paid in the form of an honorarium to the Examiners. It would be noticed that the Council had arranged that in future endeavours shall be made to meet what he understood to be the almost universal wish on the part of the Examiners, by adopting the plan which is common enough in the tripos examinations at the Universities, of having three classes of merit, each class arranged alphabetically. He announced that he had the formal allowance of the amendment of the bye-laws (adopted by the members a short time ago), granted by the Lords of the Privy Council on 28 May. He concluded by urging the younger Fellows to contribute papers for reading at the sessional meetings, and thanking the members for the kind and cordial way in which, for the past two years, his endeavours to act as President had been met.

Mr. A. J. FINLAISON seconded the adoption of the report, and the resolution was unanimously agreed to.

ELECTION OF OFFICERS.

A ballot having been taken, the Scrutineers, Mr. Y. R. ECCLES and Mr. W. J. LANCASTER reported that the following list had been unanimously adopted:

President.

BENJAMIN NEWBATT.

Vice-Presidents.

MARCUS NATHAN ADLER, M.A.	AUGUSTUS HENDRIKS.
GEORGE STEPHEN CRISFORD.	THOMAS EMLEY YOUNG, B.A.

Council.

THOMAS GANS ACKLAND.	AUGUSTUS HENDRIKS.
MARCUS NATHAN ADLER, M.A.	*CHARLES DANIEL HIGHAM.
ARTHUR HUTCHESON BAILEY.	GEORGE HUMPHREYS, M.A.
*GEO. WM. BERRIDGE.	GEORGE KING.
THOMAS G. C. BROWNE.	HENRY WILLIAM MANLY.
ARTHUR FRANCIS BURRIDGE.	THOMAS MARR.
*JAMES CHISHOLM.	BENJAMIN NEWBATT.
*FRANCIS ERNEST COLENSO.	ARTHUR PEARSON.
*ERNEST COLQUHOUN.	*HOWARD JAMES ROTHERY.
*THOS. HOMANS COOKE.	GERALD HEMMINGTON RYAN.
GEORGE STEPHEN CRISFORD.	THOMAS BOND SPRAGUE, M.A.
ARCHIBALD DAY.	WILLIAM SUTTON, M.A.
*THOMAS CHARLES DEWLY.	ANDREW HUGH TURNBULL.
ALEXANDER J. FINLAISON, C.B.	WILLIAM WALLIS.
RALPH PRICE HARDY.	THOMAS EMLEY YOUNG, B.A.

* New Members of Council.

Treasurer.

HENRY WILLIAM MANLY.

Honorary Secretaries.

THOMAS G. C. BROWNE.	THOMAS HOMANS COOKE.

Mr. B. NEWBATT said, that in having elected him President, they had done him the highest professional honour to which any member of the Institute could aspire, and paid him the greatest personal compliment for which he might hope. At that moment, above and beyond all feelings of natural pride and personal gratification, he felt a sense of the weight of the task imposed upon him, for it was not merely one's own record, but the accumulated achievements of other and more distinguished men that must form the high, and, it might well seem, the unattainable ideal at which one had to aim. In one important particular, indeed, he started with an advantage over his friend and immediate predecessor now in the chair. On the occasion of his election two years ago he indulged in a lament, that he (Mr. Sutton) was not a greybeard. He (Mr. Newbatt) was profoundly conscious that it was to his own maturity and comparative length of days—to their indulgent estimate of a long professional career, during which he had endeavoured with consistency, and he hoped some success, to carry to practical issues the sound doctrines which it was the privilege of the Institute to teach—that he owed chiefly, if not solely, the crowning dignity now conferred on him. The

motto to express the aim and be the guide of his career as President would be "Theory and practice combined. What truth and right join together let no man put asunder." By a wholly undesigned coincidence, the same or a very similar idea was uttered on a recent occasion, by the recently-elected President of the Faculty of Actuaries in Scotland. He hailed that pronouncement from him, because every point of identity was another link in the chain which was binding the Institute of Actuaries to the Faculty of Actuaries in Scotland. In endeavouring in this spirit to do his best to sustain the dignity of the Institute and the honour of the business of assurance of which the Institute was the scientific handmaid, he relied not only upon the tried and honoured friends by whom he was immediately surrounded, but upon those younger members of the Institute to whom the President had made an urgent appeal, because it was to them in their turn, to their zeal and devotion in the days to come, that the fortunes of both the Institute and of insurance business must alike be committed. Let him also urge, as Mr. Sutton had urged them, to remember that this was their sowing time, and to remember also that those who sowed well and faithfully now were bound in days to come to reap a fruitful harvest.

On the motion of Mr. J. LONGSTAFF, seconded by Mr. LEVEAUX, Messrs. Trew, Gamble, and A. B. Woods, were elected Auditors for the ensuing year.

Mr. S. C. THOMSON proposed, and Mr. Y. R. ECCLES seconded, a vote of thanks to the President, Vice-Presidents, Council, Officers, and Examiners of the Institute for their services during the past year.

The resolution was cordially adopted, and was replied to by Mr. T. E. YOUNG and Mr. G. HUMPHREYS.

On the motion of Mr. G. DAVIES, seconded by Mr. GEO. KING, a vote of thanks was accorded to the Auditors, for their services during the past year.

The proceedings then terminated.

Errata.

Page 318, line 10 (from bottom),

$$\text{for} \quad -\frac{d(q^x-1)}{\log_e q} \quad \text{read} \quad -\frac{a(q^x-1)}{\log_e q}.$$

Page 332, line 1 (from top),

$$\text{for} \quad \frac{1}{2c}\left(\int \epsilon^{-ck^2} - \epsilon^{-ck^2}.\,k\right) \quad \text{read} \quad \frac{1}{2c}\left(\int \epsilon^{-ck^2} dk - \epsilon^{-ck^2}.\,k\right).$$

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXVIII.—PART VI.

OCTOBER 1890.

CONTENTS OF NO. CLVIII.

NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

INSTITUTE OF ACTUARIES.

On the Assessment System of Life Insurance. By WILLIAM
SUTTON, *M.A., President of the Institute of Actuaries.*

PART I.

[Read before the Institute, 28 April 1890.]

IT will be within your recollection that in the course of the
discussion which followed the reading of Mr. Sunderland's paper
in December last (see p. 257 *ante*), I stated that while endeavouring
to obtain information with regard to American insurance business,
in connection with the subject of that paper, I found to my astonish-
ment that a large amount of business was apparently being carried
on in that country upon lines more or less resembling a fallacious
principle which had long ago been exploded in this country. The
desire I then expressed to have an opportunity of publicly
expressing my opinion upon the methods alluded to, has since
been increased by finding that these methods are being introduced
into this country; and I feel it my duty to take this opportunity,
being the last occasion on which I shall have the honour to
preside *ex officio* at an Ordinary General Meeting of the Institute
of Actuaries, to refer briefly to the methods in question. In
doing so, I wish it to be most distinctly understood that what I
am about to say is said solely on behalf of the true interests of

life insurance as a principle which, properly carried out, is one of the most beneficent results of civilization.

The need for insuring one's life must have often been felt when civilization was in its infancy, and it is probable that various attempts were made to meet it, of which nothing or little is now known. Among the forms of association to carry it out would probably be that still to some extent in use, where, in the event of a death, contributions became payable by those still surviving. These would either be fixed in amount, and the sum realized— which would be variable—handed over to the deceased's representatives, or would be varying in amount so as to realize a fixed sum payable to the deceased's representatives. It must soon, however, occur to some of the contributors that those who were old when they first became members would contribute in their lifetime much less than those who joined when young, and hence arose various methods of attempting to remedy matters—a common one would be to have a scale of entrance fees increasing with age on admission, or a sliding scale of assessment ratios increasing with age. In any case, the necessity of obtaining new members soon appeared—if the contribution per member was fixed, the sum receivable on death would otherwise tend to get less; and if the sum payable on death was fixed, the amount of contribution would increase per member as the number of contributories got reduced.

Where there is anything which *ensures* a steady influx of new members, of course the method may go on without any hitch, but is always liable to the risk of the continuity being broken, as it inevitably is in the long run with every voluntary institution of the kind. When this happens, the unfortunate residuum soon finds that the hand-to-mouth system which had been employed must, as far as it was concerned, end in failure and disappointment.

This description fairly represents the principal features of the levying system of life insurance where it has an independent existence. But in this country it is often directly connected with another kind of insurance—that of sick-pay insurance. This connection has an important effect in this way. Generally speaking, sick-pay insurance is provided by fixed contributions, bringing about generally an accumulation of funds in the early years of the institution, and this connection is found to tend to assist in prolonging the vitality of the levy system as regards the life insurance part. Let us now, for a time, look at another form in which the

levying system of life insurance is carried on, and that to a very large extent in this country. In the affiliated orders under the Friendly Societies Act, it is common for the lodges (or whatever their designation) to be constituted into districts, the lodges belonging to a particular district being generally in the same locality; and one of the reasons for constituting themselves into districts is in connection with this very question of levies. It is often considered desirable to reduce the chance of a particular lodge having exceptionally large claims upon its funds for payment of life insurances, and, accordingly, a district undertakes that responsibility, and to provide the means for payment, levies as much as is necessary upon the funds of all the lodges in the district. Sometimes districts are allowed to accumulate funds for life insurance in a similar way to that of life insurance companies, the contributions of members to their lodges in respect of life insurance being ear-marked and handed over to the district— but this is not the usual case. Again, other districts are allowed to provide out of the levies a reserve fund of fixed amount, the amount being generally small, and independent of the extent of the business.

In the past it may be said that, generally, no attempt was made to discriminate between one lodge and another as regards levies, so that a young lodge would pay the same amount of levy on a death as an old lodge with the same number of members; and it is hardly necessary for me to remind you of the financial mischief that would thus be caused, and which has of late led to all sorts of complicated but well-meant attempts on the part of the managing bodies to adjust the levies upon the lodges equitably.

I have mentioned this point at some length because I have an idea that the practice just described probably had some influence in the recent establishment, on the other side of the Atlantic, of the system of life insurance on the assessment plan. There is one specially-important feature of the levy system, as above described, which should never be lost sight of. The district levies are made on the accumulated funds of the lodges, not upon the pockets directly of the members, so that practically it stands thus:—The members of lodges pay fixed contributions (that is, I suppose I must say, "level premiums"), and out of the accumulated funds constituting the savings from these "level premiums" in the early years of membership, the hand-to-mouth levies for life insurance made by the district are met. I may add, incident-

ally, that I believe one of the reasons why the lodges do not, in
many cases, agree to hand over to the district intact the contribu-
tion for life insurance, is their unwillingness to let their funds go
out of their own control.

As far as I have been able to gather, the so-called assessment
system does not contemplate doing other than ordinary whole-life
insurance. In this respect I may be wrong, but, if so, I should much
like to see how short-term endowment assurances would be dealt
with, because it must not be overlooked that ordinary whole-life
insurance is only one form of endowment assurance. As a system,
there is nothing in it of importance which has not been in use in
this country among friendly societies for many years, and is, I
think I may say, growing into disuse among them. It may be
not unfairly described as a system which appeals directly to
human instincts and not to the understanding. We all of us
like to pay as little as possible for a given article, and provided
the article we get is what we really intended to buy, no blame
attaches to anyone. Persons insuring their lives are often giving
effect to the best instincts of humanity, by providing, when death
occurs, according to their means, for those left behind who
have claims upon them; but it is or should be incumbent upon
them, while effecting their insurances, to be quite certain they
have got good value for their money—in other words, that the
article they are buying is what they think they are paying for.
With most of us, I presume, there is no desire to pay a particular
company, say, a premium for insuring our lives, which *same*
insurance we can effect in another company for a premium of
half the amount. If the understanding is allowed to come into
play, it is not unnatural to assume that when considering where
to insure, something else should receive attention besides the
amount of the premium—in other words, the intending insurer
should ask himself whether the company which is willing to sell to
him an article at a price for which another would charge double
the amount, is really selling the same article. If the article which
a purchaser contemplates buying will stand pulling to pieces,
and is capable of close examination, and he is a judge of work-
manship and material, he will have little difficulty in settling the
question for himself; and even if he is not a judge of workmanship
and material, he will act wisely in obtaining the assistance of
someone who is. Luckily, in the case with which I am dealing,
that of the assessment system of insurance as compared with the

system prevalent in this country, as the uniform premium for the
whole of life, there is some possibility of examining the workman-
ship and material.

A gentleman connected with the assessment system a few
years back submitted tables to a committee of the Eleventh
Annual Convention of Mutual Benefit Associations of America.
There are given in the report two illustrations of the manner in
which his tables are arrived at.   Taking age 40 at entry, and
assuming the net premium (*i.e.*, apart from expenses of manage-
ment) to be £1. 6*s.* 1*d.* per-cent for the first year, he shows that
if the annual premium increase year by year for 18 years, so as to
stand at about £2. 17*s.* per-cent for the remainder of life, the
premium with the accumulations at $4\frac{1}{2}$ per-cent interest of the
amounts not required in the early years of insurance to pay
claims, will be sufficient to pay the sums assured on the following
further assumptions :

> (1) That to those under any circumstances ceasing to pay
>   their contribution no surrender-value is allowed.
>
> (2) That of every 1,000 joining at age 40 more than half
>   will have left at the end of 18 years, leaving *all*
>   their surplus contributions behind them.

Making use of the same assumption in his second illustration,
he shows that the net uniform premium for the whole of life
should be about £1. 18*s.* 1*d.* per-cent at age 40 of entry.

In another table he gives the results of similar calculations to
those of the first illustration for various ages at entry, and I have
given this table complete, and have put alongside it the rates for
insurance publicly advertized by one of the associations doing
this class of business.

TABLE I, *showing Rate per-cent for each Age, the Limit of Increase from each Age at Entry, and the Age when such Limit is attained.*

[This Table is taken from Eleventh Annual Convention Report, and contains no provision for Management Expenses.]

Age at Entry	Yearly Rate at each Age			Highest Rate, or Limit of Increase			Age when Limit of Increase is attained
	£	s.	d.	£	s.	d.	
25	1	1	6	1	16	0	49½
26	1	1	8	1	17	0	50
27	1	1	10	1	18	0	50½
28	1	2	0	1	19	2	51
29	1	2	3	2	0	3	52
30	1	2	5	2	1	4	52½
31	1	2	8	2	2	7	53
32	1	2	11	2	3	10	53
33	1	3	3	2	5	2	53½
34	1	3	7	2	6	7	54
35	1	3	11	2	8	3	54½
36	1	4	3	2	9	11	55
37	1	4	8	2	11	6	55½
38	1	5	1	2	13	4	56
39	1	5	7	2	15	4	56½
40	1	6	1	2	17	0	57
41	1	6	8	2	19	5	57½
42	1	7	4	3	1	9	58
43	1	8	0	3	4	1	58½
44	1	8	10	3	6	5	59
45	1	9	9	3	11	2	60
46	1	10	10	3	12	1	60½
47	1	12	0	3	15	0	61
48	1	13	4	3	18	6	61½
49	1	14	11	4	3	11	62
50	1	16	9	4	7	6	62½
51	1	18	9	4	9	7	63
52	2	1	0	4	13	9	63½
53	2	3	7	4	18	1	64
54	2	6	5	5	2	10	64½
55	2	9	5	5	8	2	65
56	2	13	0	5	13	7	65½
57	2	16	11	5	19	6	66
58	3	1	2	6	5	8	66½
59	3	5	11	6	11	1	67
60	3	11	2	6	18	4	68

(Odd half-pence are omitted.)

TABLE II, *giving Rates publicly advertized by one of the Associations doing business on the Assessment Plan.—Dues for Expenses.*

Amount of Insurance	Entrance Fee and Dues for First Year			Annual Dues for Second Year			Amount of Insurance	Entrance Fee and Dues for First Year			Annual Dues for Second Year		
£	£	s.	d.	£	s.	d.	£	£	s.	d.	£	s.	d.
100	1	18	0	0	8	0	900	10	2	0	3	12	0
200	2	16	0	0	16	0	1,000	11	0	0	4	0	0
300	3	14	0	1	4	0	1,500	15	10	0	6	0	0
400	4	12	0	1	12	0	2,000	20	0	0	8	0	0
500	6	10	0	2	0	0	2,500	25	10	0	10	0	0
600	7	8	0	2	8	0	3,000	30	0	0	12	0	0
700	8	6	0	2	16	0	4,000	40	0	0	16	0	0
800	9	4	0	3	4	0	5,000	50	0	0	20	0	0

The dues for first year includes the entrance fee, medical examination fees, stamp duty, &c.

Mortuary Calls.

On the 1st of February, April, June, August, October, and December, mortuary calls are made upon the living members for the sum required to meet the approved death claims.

Members desiring to pay annually in advance may do so at the maximum rate, as per following table, and the amount not required during the year will be returned as a dividend, and applied to reduce future payments.

Age	Maximum Amount which can be Collected Annually for Mortuary Fund			Annual Dues for Expenses Second and succeeding Years			Total Maximum Annual Cost Second Year			COMPARISON					
										Average Cost for same Insurance under the Old System			Saving to Members of this Association		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
25	1	1	7	0	8	0	1	9	7	2	4	3	0	14	8
26	1	1	8	0	8	0	1	9	8	2	5	5	0	15	9
27	1	1	11	0	8	0	1	9	11	2	6	6	0	16	7
28	1	2	0	0	8	0	1	10	0	2	7	8	0	17	8
29	1	2	3	0	8	0	1	10	3	2	8	9	0	18	6
30	1	2	5	0	8	0	1	10	5	2	9	11	0	19	6
31	1	2	8	0	8	0	1	10	8	2	11	3	1	0	7
32	1	3	0	0	8	0	1	11	0	2	12	7	1	1	7
33	1	3	3	0	8	0	1	11	3	2	14	0	1	2	8
34	1	3	7	0	8	0	1	11	7	2	15	4	1	3	9
35	1	3	11	0	8	0	1	11	11	2	16	8	1	4	9
36	1	4	3	0	8	0	1	12	3	2	18	2	1	5	11
37	1	4	8	0	8	0	1	12	8	2	19	9	1	7	1
38	1	5	1	0	8	0	1	13	1	3	1	4	1	8	3
39	1	5	7	0	8	0	1	13	7	3	2	10	1	9	3
40	1	6	5	0	8	0	1	14	5	3	5	5	1	11	0
41	1	7	0	0	8	0	1	15	0	3	7	7	1	12	7
42	1	7	9	0	8	0	1	15	9	3	9	9	1	14	0
43	1	8	6	0	8	0	1	16	6	3	11	11	1	15	5
44	1	9	3	0	8	0	1	17	3	3	14	1	1	16	10
45	1	9	11	0	8	0	1	17	11	3	16	3	1	18	4
46	1	10	11	0	8	0	1	18	11	3	19	3	2	0	4
47	1	12	0	0	8	0	2	0	0	4	2	3	2	2	3
48	1	13	4	0	8	0	2	1	4	4	5	3	2	3	11
49	1	15	0	0	8	0	2	3	0	4	8	3	2	5	3
50	1	16	9	0	8	0	2	4	9	4	11	3	2	6	6
51	2	0	11	0	8	0	2	8	11	4	15	9	2	6	10
52	2	5	7	0	8	0	2	13	7	5	0	3	2	6	7
53	2	9	11	0	8	0	2	17	11	5	4	8	2	6	9
54	2	14	5	0	8	0	3	2	5	5	9	2	2	6	9
55	2	18	11	0	8	0	3	6	11	5	13	8	2	6	9
56	3	3	5	0	8	0	3	11	5	5	19	9	2	8	4
57	3	7	11	0	8	0	3	15	11	6	5	10	2	9	11
58	3	12	5	0	8	0	4	0	5	6	11	10	2	11	5
59	3	16	11	0	8	0	4	4	11	6	17	11	2	13	0
60	4	1	5	0	8	0	4	9	5	7	4	0	2	14	7

The above maximum rate is based upon the mortality tables and the experience of the association for current ages, and includes 25 per-cent for the reserve fund, cost of collection, and payment of death claims and taxes.

The great care exercised in selecting risks, and the application of the interest on the reserve fund to reducing the mortuary calls, has enabled the association to make the mortuary calls at the rate at age of entry, in place of increase of cost by reason of increase of age.

A comparison of the figures here given will be found most instructive. It will be seen that for a considerable part of the Table I the minimum rates,—*i.e.*, those at entry, are to all intents and purposes the same as those of the association referred to given in Table II; but whereas the Table I contemplates assurers at every age paying an increasing premium at every age of insurance until the limit is reached, the rates of the association, given in Table II, do not specially indicate this. Instead of "age at entry" we have "age", with the following note: "The great care exercised in select-" ing risks, and the application of the interest on the reserve fund " to reducing the mortality calls, has enabled the association to " make the mortuary calls at the rate at age of entry, in place of " increase of cost by reason of increase of age." And I have not been able to find when the increase of rate of mortuary calls for any particular age at entry, *when it does begin*, is to stop.

Now the extraordinary difference between these two methods— that proposed in the tables laid before the convention, and that adopted apparently in the particular association's table of rates— calls for special notice. According to the former the increasing premium was to be charged after first year, and the accumulations with interest at $4\frac{1}{2}$ per-cent to go on, so as ultimately to be available to keep the increasing premium stationary at about £2. 17s. per-cent after 17 or 18 years. Whereas, according to the method adopted by the association, the increasing premium is apparently not at present charged, and the interest on accumulations is stated to be applied to enable this to be done, coupled with effect of selection. Under these circumstances, it would be most interesting to know how long the present state of things is contemplated to last.

It is my experience to see a great deal of the sanguine element introduced into the prosaic business of life insurance; but I do not remember anything ever having come before me more note-worthy than such a practice as that apparently here indicated.

Judging from the report of the convention, already alluded to, this sanguine spirit was not on that occasion universally shown. One of the speakers is reported to have said as follows. He held the opinion:

" (1) That man is mortal, and that the inevitable decline of " life cannot be neutralized by adding more lives.

" (2) That, in order to pay out a certain sum at the death of " every member, the aggregate amount must also be paid in by " the members.

" (3) That as some men die before they can have paid as
" much as their beneficiaries receive, others must make good
" that deficiency.

" (4) That this deficiency can be good only in two ways:
" *Either* by rates of assessment—increasing as the age of
" members and, naturally, their rate of mortality increases; *or*
" by paying in the early years a sum sufficient to accumulate
" a reserve, which, *increased* by interest, will balance the
" deficiency.

    *   *   *   *   *   *   *   *

" In 1883 I had the satisfaction of seeing your convention,
" by a four-fifths vote of all the delegates, adopt the resolution:
" ' That new members should be protected against bearing or
" ' sharing the cost of the increasing mortality of old members,
" ' either by accumulating a fund, or by increasing the rates with
" ' age.'

" Being in honour bound to place the institutions entrusted to
" our keeping on a sound and enduring basis, I would recommend
" the adoption of a table (already referred to) as the standard
" increasing rates, giving, however, the age when limit of increase
" is to be attained (col. 4) as a PROBABILITY only, and reserving
" the right to change the limit as future experience may dictate.

    *   *   *   *   *   *   *   *

" Considering that these rates are to serve the two-fold object
" of reducing the cost to members at an advanced age, and to
" protect the association against adverse selection, they are
" certainly *not too high*."

Statistics of Insanity in Scotland.

THE two following papers contain contributions to the statistics
of insanity in Scotland, which have been compiled with care and
accuracy.   They may therefore prove of service to actuaries who
may be called upon to deal with problems where the ordinary
vital contingencies are complicated by the element of insanity.
If the statistics in the second paper stood alone they might, as
will be observed, be made the basis of a mortality table for the
valuation of benefits depending upon the lives of the insane; but

the earlier paper demonstrates that the element of the duration of the malady has a much more important bearing upon the expectancy of life than the mere question of age; and that, if the first year or two years following the first attack be survived, the average after-lifetime is considerably higher than would be shown by any table constructed from the total experience embracing all stages of the malady. It is understood that no other investigation of a like kind has been made, and we are indebted to the author for permission to reproduce the papers here.—[Ed. *J.I.A.*]

Contribution to the Statistics of Insanity. By Sir Arthur Mitchell, M.D., LL.D., K.C.B , *Commissioner in Lunacy for Scotland.*

I. Nature of the Enquiry.

1. In this enquiry, all the asylums of Scotland are regarded as one asylum, and the different institutions merely as different wards of the asylum. A patient transferred from one institution to another is thus regarded as never leaving the asylum, but merely as passing from one ward to another. The words—*the asylum*—therefore, in this paper, mean an asylum made up of all the asylums of Scotland.

2. The enquiry does not deal with the whole population of the asylum. It deals only with the patients who were admitted into it during some single and remote year, and who had never been under asylum treatment before—in other words, who were *admitted* during the year in question for *the first time*.

3. The history of each of these patients is followed from year to year down to a certain fixed period. No cognizance is taken of the existence of any other patients. The wards of the asylum might thus have been empty when these patients went into them, so far as concerns this research, and in like manner no fresh admissions need have occurred during the time over which it extends. The enquiry is limited to the new cases which presented themselves in one remote year; and, at a fixed and comparatively recent period, it is asked, what has become of these patients?—how many of them are still in the asylum?—how many of them have died there?—how many have gone out and returned to it?—how many are out of it in life and sanity?—how many are out of it in life but in a state of insanity?—how many have died after leaving it, and what was their mental condition at the time of death? These and other such questions are asked, and as far as possible answered.

4. It is important to bear in mind that the whole history of each patient has been separately tabulated.

5. It is scarcely necessary to say that I am indebted to my official position for the means of making an enquiry of this kind.

II. General Results.

1. During the year 1858, there were 1,297 patients admitted for the first time into the asylum, and in the year 1870, or 12 years after, it is asked what has become of them. I am able to speak definitely regarding the condition of 1,096, or about $\frac{11}{13}$ths of the whole.

2. It is found that 412 had died in the asylum, and that 273 remained in it. We thus account for 685, or about 53 per-cent. There remain 612, or about 47 per-cent who had disappeared from the asylum—neither having died in it, nor being found in it at the end of the 12 years.

3. The enquiry has gone further, and an effort has been made to find out the history of the 612 patients who had disappeared from the asylum. With reference to them, it has been ascertained that at the end of the year 1869,

> 42 had died in a state of insanity,
> 78 had died in a state of sanity,
> 94 were living in a state of insanity,
> and 197 were living in a state of sanity.

This accounts for 411 of the 612, leaving 201, regarding whose condition at the end of the 12 years no trustworthy information could be obtained.*

---

* From what became known to me while making this enquiry, and from knowledge otherwise acquired, I think we may safely assume that what was found to be true of the 411, would have been found to be substantially true also of the 201, had we succeeded in getting the information regarding them which was desired. On this assumption, we should have the whole 612 thus accounted for:

> 62 as dead, being in a state of insanity at the time of death,
> 117 as dead, being in a state of sanity at the time of death,
> 139 as living, and in a state of insanity,
> and 294 as living, and in a state of sanity.

If we combine these figures with the figures in paragraph 2, we find that the 1,297 patients admitted for the first time into the asylum in 1858 are accounted for in 1870, or 12 years after, in the following manner:—

```
(1.) As insane—
        (a) Dead—while in the asylum    .    .  412
              ,,      after leaving the asylum   .   62
                                               ——  474
        (b) Alive—in the asylum    .    .    .  273
              ,,      out of the asylum    .    .  139
                                               ——  412
                                               ——— 886
(2.) As sane—
        (a) Dead—after leaving the asylum    .      117
        (b) Alive—out of the asylum    .    .      294
                                               ——  411
                           Total    .    .      1,297
```

Giving the figures in percentages, we have the 1,297 patients accounted for thus:

III. The Results considered in their bearings on Recoveries, Relapses, and Deaths.

1. The broader features of these results are given in the following table:

| Year | Admitted | Discharged | | | Readmitted | Remaining at 31 December of each Year | Mean Number Resident |
		Recovered	Not Recovered	Dead			
1858	1,297	320	88	103	47	833	416
1859	...	220	55	80	91	569	701
1860	...	75	36	56	65	467	518
1861	...	45	27	32	50	413	440
1862	...	39	10	33	42	373	393
1863	...	33	11	23	34	340	356
1864	...	20	11	14	34	329	334
1865	...	22	5	19	20	303	316
1866	...	28	4	13	35	293	298
1867	...	16	6	15	29	285	289
1868	...	20	5	11	28	277	281
1869	...	13	2	13	24	273	275
Totals . .		851	260	412	499	...	...

1. {
As dead in the asylum . . 31·7 per-cent.
As dead after leaving the asylum but insane at the time of death 4·9 per-cent.
————— 36·6 per-cent.

2. {
As still alive, and in the asylum 21·0 per-cent.
As still alive, but out of the asylum, and in a state of insanity . . . . 10·7 per-cent.
————— 31·7 per-cent.

Yielding a total, for all those who had either died in a state of insanity, or were still living in a state of insanity, of . 68·3 per-cent.

3. {
As having died out of the asylum, being in a state of sanity at the time of death . . . 9·0 per-cent.
As still alive, and in a state of sanity . . . . . 22·7 per-cent.
————— 31·7 per-cent.

100·0

More briefly stated, these figures stand thus:

36·6 per-cent are found to have died, being in a state of insanity at the time of death,

31·7 per-cent are found to be still alive and still insane,

and 31·7 per-cent are found to be either still alive and sane, or to have died, being in a state of sanity at the time of death.

These calculations are given in a foot note, because they involve the assumption (which I believe to be a safe one), that the 201 patients, regarding whose condition no trustworthy information was obtained, would present substantially the same results as the similarly situated patients, regarding whom satisfactory information was secured. In no other part of the paper is anything introduced which involves any assumption.

2. These figures, however, lose much of their interest if they are not examined in connection with the statement which gives the whole history from year to year of each patient separately.   In this enquiry, a patient, however often he may be discharged, either as recovered or as unrecovered, or however often he may be re-admitted, counts throughout as one and the same patient.   The effect of being able thus to deal individually with the 1,297 patients will be apparent when we discuss separately the discharges of the recovered and the unrecovered, the re-admissions, and the deaths.

(*a*) *Recoveries.*

1. The table shows that the 1,297 patients yielded during the 12 years, 851 discharges of recovered patients, or 65·6 per-cent.   If we add the re-admissions (499) to the admissions (1,297), we have 1,796, of which number the whole recoveries tabulated constitute 47·3 per-cent.

2. But these 851 discharges refer only to 538 patients, who are 41·5 per-cent of the patients admitted, and 30·0 per-cent of the patients admitted and re-admitted taken together.

3. Of the 538 patients who were at any time during the 12 years discharged as recovered, it is found that :

> 67 were re-admitted and were found in the asylum at the end of the 12 years,
>
> 33 were re-admitted and died in the asylum,
>
> and   9 were re-admitted and were ultimately discharged as unrecovered.

Total 109

If we deduct these (109) from the whole number of patients ever discharged as recovered (538), we have 429 as the number of patients who permanently disappeared from the asylum as recovered—that is, 33·0 per-cent of the original number admitted, and 23·8 per-cent of the numbers, both of the admitted and of the re-admitted taken together.

4. A large proportion of the recoveries occurred in the case of patients who had not been longer than from a year to a year and a half in the asylum.   Of the 538 patients who were at any time during the 12 years discharged as recovered, 505, or about 94 per-cent, were so discharged during the first two years.

5. The recoveries which occurred during these first two years appear to have been the most satisfactory of all the recoveries which took place.   We have evidence of this in the facts that 295, or 58·4 per-cent of the 505 patients discharged as recovered during these two years never entered the asylum again, and that 263 of these 295 patients are included in the total number of patients (411) who appear to have been either alive and sane at the end of the 12 years, or to have died in a state of sanity during those years.

6. If we take the three first instead of the two first years, these results are still better marked.   Thus: of the whole number of patients ever discharged as recovered (538) we have 515, or about

96 per-cent who were so discharged during the first three years, leaving only 23 to be for the first time discharged as recovered during the other 9 years, viz., 7 in 1861, 5 in 1862, 3 in 1863, 2 in 1864, 2 in 1865, and one in each of the 4 last years. Of this small group of 23 patients, who were from 3 to 12 years continuously in the asylum before being discharged as recovered, it happens that no fewer than 15 are accounted for as being among the insane still at the end of the 12 years—some being alive and insane, and others having died in a state of insanity.

7. In view of these statements, it may be asked how there can be 13 recoveries registered in the last of the 12 years. The explanation is simple: 11 of the 13 recoveries registered in 1869 refer to patients re-admitted in 1868 or 1869. So it is with the recoveries in the other years towards the close of the duodecennial period: they are almost entirely recoveries after recent re-admission.

(b) *Re-Admissions.*

1. The whole number of re-admissions is 499, but the number of patients in whose history re-admission appeared, is only 316.

2. It follows, of course, that some patients must have been often discharged and often re-admitted. So true is this, that it is found that 81 patients contributed 264, or more than one half (52·9 per-cent), of the whole re-admissions—45 patients being re-admitted twice, 11 three times, 9 four times, 7 five times, 3 six times, 3 seven times, 2 ten times, and 1 eleven times.

3. Re-admission presented itself somewhat more frequently in the history of patients discharged as recovered, than in the history of patients discharged as unrecovered—234 patients discharged as recovered giving 404 re-admissions, and 82 patients discharged as unrecovered giving 95 re-admissions.

(c) *Discharge of the Unrecovered.*

1. The number of patients discharged as unrecovered is, in the first year, 21 per-cent of the mean number resident. In no other year is it above 8 per-cent. In one year it is below 1 per-cent. The average for the last 8 years is between 2 and 3 per-cent.

(d) *Deaths.*

1. Of the 1,297 patients admitted, it is found that 412, or 31·7 per-cent had died in the asylum during the 12 years.

2. Of these 412 patients who died in the asylum, it appears that 370, or about 90 per-cent, had died in it without ever having left it after admission. In other words, of all the patients who died in the asylum 90 per-cent had never been out of it after admission. Of those who thus entered the asylum to die there, 103 died during the first year after admission, 76 during the second year, 51 during the third year, and so on—10 of the 13 who died during the twelfth year never having been out of the asylum, and all of the 14 who died in the seventh year having been 7 years in the asylum.

3. The 499 patients, in whose history re-admission appeared, yielded only 41 of the 412 deaths.

4. The rate of the mortality during the first three years was

24·8, 11·4, and 10·8 per-cent of the mean number resident. During the next three years it fell to 7·3, 8·4, and 6·4 per-cent; and during the last 6 years it fell still further, and was generally about 4·0 per-cent, only once rising to 6·0 per-cent.

5. The mean age of the whole number of patients admitted was 39·6 years. The mean age of the 103 who died during the first year was 49·5 years. The mean age of the 24 who died during the last two years was 55·8 years at the time of their death. The mean age of the 273 patients who were found remaining in the asylum at the end of the 12 years was 47·9.

6. The deaths of the first year thus occurred among the older of the patients admitted—the mean age of those who died being 10 years above the mean age of the admitted. Entering still further into details, it appears that of the 103 patients who died during the first of the 12 years, 11 were between 70 and 80 years old, 17 between 60 and 70, 17 between 50 and 60, and 25 between 40 and 50. In other words, about 70 per-cent were above the mean age of the admitted.

7. Many of the very aged are thus immediately and finally disposed of, but there occurs coincidently a final disposal of many of the younger by permanent recovery, so that the mean age of those who remain at the end of the first year does not differ much from the mean age of the admitted.

8. Those remaining at the end of the 12 years are made up largely of the younger of the patients originally admitted. Their mean age at the end of the 12 years was 47·9 years, so that their mean age at the time of admission must have been 35·9, or about 4 years below the mean age of all admitted.

9. The patients who died during the last two years had a mean age of 55·6 years, and were older than those who died during the first year, whose mean age was only 49·5 years. Their age at the time of admission was also above that of the mean. The range, however, between the ages of the oldest and youngest of those who died in the two last years was much lower than the range in the case of those who died in the first two years.

10. It thus appears that the lower death rate of the later years is not supplied by a younger, but by an older population.

IV. Concluding Remarks.

1. All these things might possibly be found to be not even substantially true of a second group of patients who were admitted for the first time during some other year, and whose history was examined in the same way. It happens, however, that we possess evidence that the study of such a second group would, in fact, yield results which would be in very close accord with all that has been disclosed regarding the 1,297 patients admitted for the first time in 1858. This evidence is furnished by the following table, which I extract from the 18th Report of the Scotch Board of Lunacy (p. xx). If it is compared with the table given in this paper (p. 428), the progressive history of the one group of patients will be found to resemble that of the other in a very striking manner.

| Year | Admitted | DISCHARGED | | | Re-admitted | Remaining at 31 December of each Year |
		Recovered	Not Recovered	Dead		
1868	1,319	305	97	107	38	848
1869	...	209	70	94	83	558
1870	...	51	23	60	40	464
1871	...	38	25	48	42	395
1872	...	26	13	28	39	367
1873	...	23	15	25	26	330
1874	...	15	11	13	23	314
1875	...	16	9	13	21	297

2. I have confined myself as much as possible in this paper to a
mere statement of the results of an enquiry, which, so far as I know,
has not previously been made, and which in some directions teaches
new lessons, and in others gives precision and certainty to opinions
already entertained.

---

*Contribution to the Study of the Death-rate of Persons in
Asylums. By* SIR ARTHUR MITCHELL, M.D., LL.D., K.C.B.,
Commissioner in Lunacy for Scotland.

In the population of Scotch Asylums, there are so few persons
below the age of 10 years that, for practical purposes, it may be
correctly said there are none.   Of the general community, on the
other hand, 25·6 per-cent are persons below that age.   It is clear,
therefore, that the death-rate of the population of asylums cannot
properly be compared with the death-rate of the general population.
To make such a comparison it is necessary to deal only with the
deaths occurring among the 74·4 per-cent of the general community
who are above the age of 10 years.   When this is done, it appears
that the mean annual death-rate for the general population is 1·7
per-cent as compared with 8·3 per-cent for the population of asylums.
These figures refer to the whole population of asylums, and to the
whole of the general population above the age of 10 years ; but in
order to show the rates at which persons of different ages die in
asylums, and the rates at which persons of corresponding ages die in
the general community, the following table has been prepared.   It is
founded on 3,800 deaths occurring during the seven years, 1870-1876,
in the Asylums of Scotland, which had a mean population of 6,421
during those years.

This table shows that the inmates of asylums, at all the
quinquennia between the ages of 10 and 50, die pretty nearly at the
same rate, with the exception of those whose ages fall in quinquen-
nium 25-30, among whom a distinctly lower death-rate occurs.

In the general population, on the other hand, the death-rates for
all the quinquennia between 10 and 50 increase in geometrical
progression as the ages rise.

Ages in Years	Mean Annual Mortality per 100 Persons resident in Asylums at different Ages	Mean Annual Mortality per 100 of the General Population at different Ages	Number of Deaths in Asylums to one Death in the Population at different Ages	One Death annually in Persons Living of the General Population at different Ages	One Death annually in Persons Living in Asylums at different Ages
From					
10 to 15	6·2	·58	10·7	172·71	16·1
15 „ 20	6·8	·79	8·6	126·74	14·7
20 „ 25	6·3	·99	6·4	101·11	16·0
25 „ 30	5·1	1·05	4·9	94·88	19·5
30 „ 35	6·2	1·09	5·7	91·32	16·1
35 „ 40	6·4	1·29	5·0	77·34	15·6
40 „ 45	6·8	1·33	5·1	75·36	14·8
45 „ 50	6·8	1·66	4·1	60·13	14·7
50 „ 55	7·9	1·97	4·0	50·84	12·6
55 „ 60	9·1	2·47	3·7	40·49	11·0
60 „ 65	11·7	3·32	3·5	30·09	8·5
65 „ 70	15·0	4·72	3·2	21·20	6·7
70 „ 75	18·7	6·91	2·7	14·47	5·3
75 „ 80	26·7	10·89	2·5	9·18	3·7
80 „ 90	39·6	20·63	1·9	5·13	2·5
90 „ 100	20·4	38·41	0·5	2·63	4·9

After the age of 50, that is, when the working period of life is over, the death-rates in asylums rise from quinquennium to quinquennium by a considerable but irregular progress.

In the general population again, after the age of 50, the rise is rapid and steady.

The third column of the table shows the proportions which the asylum death-rates at different ages bear to the death-rates at corresponding ages in the general population. For example, it shows that for every death yielded by 100 persons of the general population whose age falls between 10 and 15 years, 10·7 deaths are yielded by 100 persons in asylums whose age falls between the same years; and so in like manner with the other quinquennia. It will be seen that the figures in this column steadily fall as the ages rise: the asylum death-rate for the quinquennium 10-15 being $10\frac{1}{2}$ times the death-rate for that quinquennium in the general population, while the asylum death-rate for the quinquennium 75-80, is only $2\frac{1}{2}$ times the death-rate for that quinquennium in the general community.

The fourth and fifth columns are intended to exhibit the figures of the third column in a different aspect. The figures in these are read in the following way :—172·7 persons in the general community between the age of 10 and 15 yield one death annually, while 16·1 persons in asylums between the age of 10 and 15 yield one death annually; and so on in like manner with the other figure of the two columns.

It is scarcely necessary to point out that little reliance can be placed on the figures relating to ages from 80 to 100.

The annexed table shows—(1) the mean numbers resident in the different classes of establishments, and in all establishments, in Scotland, at different ages, for the period 1870-1876 ; (2) the mean yearly number of deaths at corresponding ages which occurred in these different classes of establishments, and in all establishments during the same period; and (3) the mean yearly deaths per 100 of the mean numbers resident, at the different ages.

Ages in Years—From:	10 to 15	15 to 20	20 to 25	25 to 30	30 to 35	35 to 40
ROYAL ASYLUMS:						
Mean numbers resident, at different ages	3·0	43·0	139·0	233·0	319·5	377·0
Mean annual number of deaths, at different ages	·429	4·286	9·143	12·143	21·714	22·857
Mean annual percentage of deaths on residents, at different ages	**14·3**	**10·0**	**6·6**	**5·2**	**6·8**	**6·1**
DISTRICT ASYLUMS:						
Mean numbers resident, at different ages	3·5	38·0	93·0	179·5	210·0	226·5
Mean annual number of deaths, at different ages	·143	2·286	6·429	9·143	12·714	13·714
Mean annual percentage of deaths on residents, at different ages	**4·1**	**6·0**	**6·9**	**5·1**	**5·3**	**6·1**
PAROCHIAL ASYLUMS:						
Mean numbers resident, at different ages	3·0	24·5	53·5	90·5	109·5	117·5
Mean annual number of deaths, at different ages	...	1·286	3·571	4·286	7·857	10·143
Mean annual percentage of deaths on residents, at different ages	...	**5·2**	**6·7**	**4·7**	**7·2**	**8·6**
PRIVATE ASYLUMS:						
Mean numbers resident, at different ages	...	3·5	10·5	18·0	21·5	28·5
Mean annual number of deaths, at different ages	...	...	·571	1·571	3·000	2·429
Mean annual percentage of deaths on residents, at different ages	...	...	**5·4**	**8·7**	**14·0**	**8·5**
LUNATIC WARDS OF POORHOUSES:						
Mean numbers resident, at different ages	2·0	8·5	30·5	39·0	59·0	66·0
Mean annual number of deaths, at different ages	·143	·143	·714	1·571	1·286	3·000
Mean annual percentage of deaths on residents, at different ages	**7·1**	**1·7**	**2·3**	**4·0**	**2·2**	**4·5**
IN ALL ESTABLISHMENTS:						
Mean numbers resident, at different ages	11·5	117·5	326·5	560·0	749·5	815·5
Mean annual number of deaths, at different ages	·715	8·000	20·429	28·714	46·571	52·143
Mean annual percentage of deaths on residents, at different ages	**6·2**	**6·8**	**6·3**	**5·1**	**6·2**	**6·4**

40 to 45	45 to 50	50 to 55	55 to 60	60 to 65	65 to 70	70 to 75	75 to 80	80 to 90	90 to 100
358·0	338·0	321·0	240·0	207·5	130·5	95·0	50·0	6·7	1·0
26·286	22·857	27·571	21·429	22·429	21·143	16·000	12·429	3·357	·286
7·3	6·8	8·5	8·9	10·8	16·2	16·8	24·9	52·3	28·6
238·5	225·0	197·0	168·0	123·5	79·5	60·5	26·0	7·0	...
13·143	15·000	12·286	13·429	14·857	11·143	10·429	8·000	2·286	·286
5·5	6·7	6·2	8·0	12·0	14·0	17·2	30·8	23·1	...
124·5	94·0	73·5	54·0	37·5	29·5	16·5	9·0	1·5	1·0
10·000	7·571	8·429	6·714	6·286	5·143	3·714	1·286	·571	·143
8·0	8·1	11·5	12·4	16·8	17·4	22·5	14·3	35·7	14·3
27·5	28·0	26·0	24·0	20·5	19·5	12·0	7·0	2·0	0·5
2·571	1·571	2·286	3·286	2·286	2·571	2·000	3·000	·571	...
9·4	5·6	8·8	13·7	11·1	13·2	16·7	42·9	28·6	...
72·0	64·0	55·0	50·5	49·5	30·0	19·5	12·5	2·7	1·0
3·429	4·000	3·000	4·000	5·429	3·429	6·000	3·286	1·143	...
4·8	6·3	5·5	7·9	11·0	11·4	30·8	26·3	52·1	...
820·5	749·0	675·5	536·5	438·5	289·0	203·5	104·5	20·0	3·5
55·429	51·000	53·571	48·857	51·286	43·429	38·143	28·000	7·923	·714
6·8	6·8	7·9	9·1	11·7	15·0	18·7	26·7	39·6	20·4

This table exhibits in detail the material which was employed in constructing the first table; but its chief use lies in this, that it shows the progress of the death-rate for different ages in asylum communities to be substantially the same in the royal, district, and parochial asylums when they are taken separately, as it is in all kinds of asylums when they are taken in mass. It will be observed that even the exceptional character of the death-rate for the quinquennium 25-30, which was seen in the figures relating to all asylums, appears also, more or less distinctly, in the figures relating to these three classes of asylums when they are dealt with separately.

The population of private asylums, and also of the lunatic wards of poorhouses, is too small to yield useful results in this matter when these establishments are dealt with apart from others.

The following table is added for the convenience of those who may be interested in this research. No explanation of it is needed:

Ages in Years	Proportion of Persons at different Ages in 1,000 of the whole Population of Scotland above 10 Years of Age	Proportion of Persons in Asylum at different Ages in 1,000 of the whole Population of Scotch Asylums
From		
10 to 15	148·9	1·8
15 „ 20	134·3	18·3
20 „ 25	117·0	50·8
25 „ 30	101·9	87·2
30 „ 35	88·1	116·7
35 „ 40	75·7	127·0
40 „ 45	70·3	127·8
45 „ 50	59·0	116·6
50 „ 55	53·9	105·2
55 „ 60	42·0	83·6
60 „ 65	39·4	68·3
65 „ 70	27·0	45·0
70 „ 75	21·8	31·7
75 „ 80	11·5	16·3
80 „ 90	8·5	3·1
90 „ 100	0·7	0·5
100 & upwards	0·03	...

The Foundation of the Institute of Actuaries.

I HAVE recently had occasion, for a special purpose, to enquire into the circumstances of the formation of the Institute of Actuaries; and it has appeared to me, especially as the history of its origin has never yet been attempted in any detail, that the results of my research, if expressed in a brief historical form, would prove of interest to the profession.

My materials consisted of the *Post Magazine* for 1848; a manuscript volume initialled as correct by Mr. W. T. Thomson,

of the Standard Life Office; and two manuscript books containing records of early meetings, which, through the kind assistance of Mr. J. C. Hopkinson, I was enabled to discover.

I have deemed it expedient to omit names, and to make no reference to any personal questions that arose.

It will be observed from this record that many of the references in the *Journal*, to the date of the establishment of the Institute and to the number in the series of some of its annual reports, are incorrect.

I.—A meeting of actuaries, and others connected with the management of life offices, was held at the Standard office in London on the 15 April 1848.

Twenty-eight gentlemen attended.

The chairman stated that, in consequence of a personal communication made to himself and others by Mr. W. T. Thomson, of the Standard office, that an Association of Managers of Life Companies in Edinburgh had been established about 15 years ago, and had accomplished valuable work, it had been deemed advisable to hold a preliminary meeting with the view of ascertaining the feelings of actuaries and officers of London companies as to the formation of a similar association in London. He then invited Mr. Thomson to give an explanation of the regulations of the Edinburgh Association, and a statement of the advantages which had resulted from its formation and work.

After Mr. Thomson had responded to the request, it was resolved:

  i. That it appears desirable that those connected with the management of life assurance institutions should have occasional opportunities of meeting together and consulting upon subjects of mutual interest; and,

  ii. That a Committee, not exceeding 20 members, be appointed, to consider the best mode of carrying out this object, and to report at a future meeting.

II.—The Committee so appointed met at the Guardian office.

Their report was dated the 19 May 1848, and stated that it was not expedient at the present time to lay down rules for the formation of a society of any description, but nevertheless, acting in the spirit of the resolution of the 15 April, they considered that occasional meetings of the kind referred to might be held with advantage if that course should be found from experience to

establish uniformity in dealing with certain points of constant
occurrence in assurance companies in respect of which there
existed considerable diversity of practice.   The report specified
some questions which demanded immediate attention, such as:
the propriety, or otherwise, of paying fees to the medical referees
of proposers; the proper persons to whom commission should be
paid, whether the persons introducing the assurance or the persons
entrusted with the payment of the premiums; the true meaning
of actual military and naval service; the extra premiums to be
charged for sea risk and foreign residence; and whether assign-
ments of policies should be recognized and Canterbury probates
required on payment of claims.

A circular, dated the 20 May 1848, was then issued by the
Committee, summoning a general meeting at the Guardian office
on the 10 June, for the purpose of receiving the report.

The circular also specified the five points of practice above
mentioned, and expressed a wish to receive the opinion of that
meeting upon them.

III.—A general meeting was accordingly held at the
Guardian office on the 10 June 1848, at which 66 gentlemen were
present.

The report of the Committee of the 19 May was then read and
received, but was not adopted; and the following resolution was
carried by a large majority, namely:

> i. That this meeting considers it desirable to establish a
>   scientific and practical association amongst actuaries,
>   secretaries, and managers of life assurance societies of
>   Great Britain.

The old Committee acquiesced in the suggestion that their
functions had terminated with the presentation of their report;
and on the proposition being made to appoint a new Committee,
consisting of the old Committee and others to be now named,
some members of the previous Committee expressed a desire to be
relieved from acting on the fresh Committee, whereupon it was
accordingly resolved:

> ii. That a new Committee of 15 be appointed, who shall
>   report on the best mode of carrying the resolution
>   now passed into effect.

The new Committee was appointed by ballot.

IV.—The first meeting of the new Committee was held on the 17 June, when a Sub-Committee was appointed for the purpose of preparing and submitting a draft scheme.

The Sub-Committee met on the 19 June, 22 June, 24 June, and 27 June—the first meeting being held at Mr. Jellicoe's, in Wimpole Street; the second at the Eagle office; and the third and fourth at the Guardian office. A report on a draft scheme was prepared, discussed, and settled for presentation to the Committee.

V.—The Committee, summoned by the Sub-Committee, then met at the Guardian office on the 27 June.

The report and scheme of the Sub-Committee was received and discussed. The scheme consisted of 35 clauses: in the first clause the Institute was proposed to be entitled "The Institute of Actuaries of Great Britain"; and the final clause suggested that the Council, when appointed, should hereafter consider the propriety of obtaining a Royal Charter of Incorporation.

It is of interest to describe in detail the objects of the proposed Institute, and accordingly I append Clauses 2 to 8, namely:

> *Clause* 2: The development and improvement of the mathematical theories on which the practice of life assurance is based, and the collection and arrangement of data connected with the subjects of the duration of life, health, and finance.

> *Clause* 3: The improvement and diffusion of knowledge, and the establishment of correct principles, relating to subjects involving monetary considerations and the doctrine of probability.

> *Clause* 4: A nearer approximation to uniformity of practice—official and professional.

> *Clause* 5: The settlement of points of professional and official usage, and protection generally to the members of the profession and the public.

> *Clause* 6: The elevation of the attainments and status of the members of the profession.

> *Clause* 7: The formation of a library, and the establishment of rooms for reading and conversation.

> *Clause* 8: The free interchange of opinion and counsel in matters of difficulty, whether theoretical, official, or professional.

The remaining clauses embody a scheme for the organization and procedure of the Institute.

The scheme was unanimously adopted, and it was decided to send a copy to every actuary and secretary whose name appeared in the *Post Magazine Almanac* for 1848.

A circular, dated from the Guardian office on the 1 July 1848, was accordingly issued by the Committee, embodying the approved draft of the scheme, under the title of "A Proposed Plan for the Establishment of an Institute of Actuaries", and summoning a general meeting for the 8 July 1848.

VI.—This general meeting was held at the Guardian office on the 8 July 1848, at which 54 gentlemen attended, together with nine gentlemen connected with Scotch companies, and representing not only their own offices, but acting also as a deputation from others in Edinburgh.

The report of the Committee, appointed on the 10 June, was read, and on the motion that the report be received, an amendment was moved that the Committee be instructed to communicate with certain members of the profession of over 15 years' standing, and with eminent mathematicians, in order to ascertain their opinion upon the expediency of forming an Institute or College of Actuaries, and the proper basis of such a society, so as to secure their support and the confidence of the public.

This amendment not being seconded, it was moved and carried unanimously—

"That this report be received."

The scheme was then discussed clause by clause: in Clause 1 the title was altered from the "Institute of Actuaries of Great Britain" to the "Institute of Actuaries of Great Britain and Ireland." Clauses 2 to 8, defining the objects of the Institute, were passed unanimously in the form above expressed, the final clause relating to a Royal Charter was also accepted, and the remaining clauses, with certain alterations, were then adopted.

It was then finally moved and unanimously carried—

"That the whole plan as laid before the meeting clause "by clause, and as amended, be approved by this "meeting."

It was then resolved that the 15 gentlemen who composed the Committee be the Enrolment Committee for the purpose of carrying out the plan now approved.

VII.—It may be mentioned in passing, that in the *Athenæum* newspaper of the 22 July an article appeared in which adverse criticism was passed on the establishment of an association as already defined, on the grounds (1) that it was not sufficiently extensive in the objects prescribed, and (2) that it was not sufficiently liberal in its constitution. The essence of the second objection consisted of the statement that the body of actuaries who were Fellows of the Society were to retain the management of the Institute without the control of the Associates who were not actuaries.

VIII.—The Enrolment Committee met—at the Guardian office—on the 13 July (when they decided to issue the resolution of the meeting of the 8 July throughout the profession); on the 27 July; 3 August; 10 August; 17 August; 24 August; 5 September; 12 September; 3 October; 5 October; and 13 October.

At these meetings, applications for admission to membership were considered: and it may be added that the greatest care appears to have been exercised in examining the qualifications of the applicants.

The receipts from fees were reported at the last meeting as £328. 13s., and the number of Members enrolled was stated to be 131. It was resolved that the President be elected by ballot. Regulations for the election of the Council and officers were also adopted; and it was decided to summon a general meeting for the 14 October, for the purpose of receiving a report, electing a President and Council, and for other business in connection with the resolution of the meeting of the 8 July.

A printed list of Fellows and Associates was also adopted.

The Committee expressed a hope that the zeal and cordiality which had existed hitherto might still be maintained, in order to carry out the great aim and objects of the Institute into complete effect.

IX.—A general meeting of the Institute was held at the Guardian office, on the 14 October 1848.

Thirty-seven gentlemen attended.

The report of the Enrolment Committee, dated the 5 October, was read and approved.

Mr. John Finlaison, the Government Actuary, was then elected President for the ensuing year.

The Council of 16 was then appointed for the ensuing year. A "Registrar of the Institute"—who should be elected annually from the Fellows, and should not be a Member of Council—was then appointed.

Four Vice-Presidents were appointed, a Treasurer, two Honorary Secretaries, and three Auditors from the Associates.

(The functions of the Registrar do not appear to have been defined.)

It was then unanimously resolved that the Council take the most effectual steps they may deem expedient for the purpose of obtaining the co-operation of those members of the profession who had not yet joined the Institute.

X.—The first meeting of Council was held on the 25 October 1848, when it was resolved that a Committee, consisting of the President, Vice-Presidents, Treasurer, and Honorary Secretaries, assisted by the Registrar, should prepare a code of bye-laws and regulations for the government of the Institute, and report thereon to a meeting of Council.

XI.—It thus appears that the true date of the formation of the Institute was 8 July 1848.

<div align="right">T. E. YOUNG.</div>

The Progress of Life Assurance Business in the United Kingdom during the last Fifty Years. By DAVID DEUCHAR, F.I.A., F.F.A., Manager and Actuary of the Caledonian Insurance Company.

NUMBER OF COMPANIES ESTABLISHED, DISCONTINUED, AND EXISTING.

OMITTING from consideration the speculative and evanescent schemes which were the precursors of life assurance companies, and taking into account only the properly-constituted life offices, we find that up to 1 January 1837 the total number of such

companies which had been established was 94, of which 20 had ceased to exist, leaving 74 then in operation. The average age of these 74 companies was 17¾ years. Not more than 18 of them had been 25 years in existence, while the remaining 56 had only attained an average age of 7 years. It at first strikes us as a very remarkable circumstance that only 20 offices out of a total of 102 had succumbed in the long period of 131 years, dating from 1706, when the Amicable was founded: but it has to be borne in mind that in 1837 the great majority of the offices had not attained an age sufficient to test their stability.

The following table shows the number of ordinary life assurance companies existing at the end of each period of five years during the last half-century, with the numbers discontinued and established during that time:

Period	Number of Ordinary Life Assurance Companies			
	Existing at commencement of Period	Discontinued during Period	Established during Period	Existing at end of Period
1 Jan. 1837 to 31 Dec. 1841	74	9	41	106
,, 1842 ,, ,, 1846	106	22	42	126
,, 1847 ,, ,, 1851	126	24	50	152
,, 1852 ,, ,, 1856	152	58	86	180
,, 1857 ,, ,, 1861	180	73	13	120
,, 1862 ,, ,, 1866	120	46	40	114
,, 1867 ,, ,, 1871	114	38	38	114
,, 1872 ,, ,, 1876	114	15	3	102
,, 1877 ,, ,, 1881	102	9	2	95
,, 1882 ,, ,, 1886	95	10	4	89
,, 1887	89	—	—	—

The highest number reached was at the commencement of the year 1856, when there were 193 companies in existence. From that time a gradual and rarely-interrupted decline set in. The decline was in great part due to the exposure of the excessively-sanguine and, in some instances, fraudulent character of the schemes of certain of the new companies. The worst of these was the Independent and West Middlesex, which, according to Mr. Walford, succeeded in obtaining from the public during its brief existence of four years a sum between £200,000 and £250,000, consisting of premiums in life assurances and prices of annuities, obtained chiefly from clergymen, half-pay officers, and ladies. The inducement offered consisted of a reduction of 30 per-cent on the terms charged by the other companies. I have

here a copy of the prospectus of the society, which I shall hand round for your inspection. You will observe that the words "Fire Insurance, Instituted 1696" are so displayed as to make it appear that the Independent and West Middlesex was founded for fire insurance business in that year. You will also notice that the device of a guardian angel, which appears on the prospectus, is copied from that of the Guardian Assurance Company. In a foot note it is stated that the company has a fire-proof safe, in which securities of all kinds and family plate-chests may be deposited by the public. The list of directors appears to have been almost wholly fictitious, well-known London names having been selected, but with the initials altered.

The distrust caused by the disclosures relating to the above-named company had not long subsided when a new cause arose to shake the public confidence in life assurance institutions. This was the publication, under the original provisions of the Joint Stock Companies Act of 1844, of the accounts of all the life assurance companies which had been established under the provisions of that Act. Much public attention was directed to these accounts, and a crushing exposure was made of the enormous expenditure of some of the companies. As the result, the number of new companies formed in succeeding years became materially diminished, and many failures of small and weak companies occurred, the five years ending with 1861 producing only 13 new companies, while they yielded no fewer than 73 failures and amalgamations. Thereafter a partial recovery took place, as during the next five years the new companies were within six of the number of discontinuances; and, during the five years immediately succeeding, the new companies were equal in number to the discontinuances. That recovery, however, was reversed by the effect of "The Life Assurance Companies Act, 1870," which, by its requirement of a deposit of £20,000, effectually put a stop to the formation of life assurance companies of a speculative and unsubstantial character.

The number of ordinary life assurance companies shown in the preceding table as existing on 1 January 1887 is 89, which agrees with the number of such life assurance companies included in the summary appended to the latest return made by the Board of Trade. This shows an increase of only 15 companies in the whole period of 50 years.

Progress of the Life Assurance Funds and Life Premium
Incomes of the Companies between 1837 and 1887.

In 1837 the existing life offices were not in the habit of publishing particulars of their funds and incomes. The most complete reticence on such subjects was then the all but invariable rule. Accordingly, in endeavouring to obtain particulars on these points, and as to the rate of interest earned by the companies, it was necessary to make special application to each office. In doing so I undertook to treat as confidential the individual figures supplied to me, and to use them solely in bringing out total results and averages. After using the figures for these purposes, I have therefore destroyed all the schedules and lists. I have to express my thanks to the chief officers of the various companies for their courtesy in replying to my enquiries, and for the considerable amount of trouble which in numerous instances they have taken to supply me with information. Through their kind assistance I am able to give the following estimate of the total life premium incomes and life assurance funds in 1837 of the 82 life assurance companies which were then in existence:

Total life assurance funds, exclusive of
paid-up capital, and of the value of
re-assurances £27,720,272
Total life premium incomes, deducting
sums paid for re-assurances   .   .   3,070,752

These figures, to the extent of £27,279,847 of life assurance funds, and £2,962,602 of premium incomes, are based on actual facts applicable to 54 companies, of which 17 are still in existence, and seven have been absorbed by companies which still exist. The remaining £440,425 of life assurance funds, and £108,150 of life premium incomes, are based on approximations applicable to 28 companies, of which three now exist, and 25 have disappeared (in most cases after a very brief existence). Of the 28 cases for which approximations were made, 18 consisted of companies which had been established for a shorter period than five years.

If we assume that the average rate of premium was 3 per-cent, then the sums assured, exclusive of bonus additions, must have amounted to £102,358,400, equal to about £4 per head of the population of the United Kingdom.

The Scottish companies then in existence were 12 in number,

but one of these had not actually commenced business. The figures applicable to the other 11 Scottish companies at that date are:

		Percentage of amount for the whole 82 offices.
Total life assurance funds, exclusive of paid-up capital and value of re-assurances . . £1,279,943		4·62
Total life premium incomes, deducting sums paid for re-assurance 272,269		8·86
Estimated sums assured, exclusive of bonus additions . . 9,075,633		8·86

It may perhaps be interesting if I give here some of the estimates which have been made at various dates between 1837 and 1872, as to the extent of business of the life offices.

In 1839 it was estimated in Chambers's *Tract on Life Assurance* that there were 80,000 life policies in force in the United Kingdom. Mr. Walford, in his *History of Life Assurance*, quoted that statement; but, while expressing the opinion that it was probably an under-estimate, he used it in bringing out the total sum assured at that date as £40,000,000, on the assumption that the average amount assured at that date was £500. From the information now before me as to the year 1837, I think it probable that the number of policies existing in 1839 was nearly 100,000, and that the average sum assured at that date was probably over £1,000, instead of £500, as assumed by Mr. Walford.

In 1843 Mr. Ansell and other actuaries estimated that there were in force in Great Britain 100,000 life policies, for the sum of £100,000,000, and that the funds of the life offices amounted to £50,000,000.

In 1847 Mr. G. R. Porter, F.R.S., in his well-known work, *The Progress of the Nation*, estimated the total sums accumulated in the hands of the various life assurance offices at £40,000,000.

In 1849 Mr. W. T. Thomson estimated the total sum assured in English offices at £150,000,000. In vol. ii of the *Journal of the Institute of Actuaries* the following approximations, applicable to the business of life assurance companies in Great Britain and Ireland in the year 1849, are given by Mr. Samuel Brown, as having been furnished to him by a friend:

Existing assurances	£150,000,000
Number of existing policies . .	300,000
Number of lives	225,000

In 1851 Mr. F. Hendriks estimated the total sum assured in Great Britain and Ireland at £150,000,000.

In 1852 Mr. W. T. Thomson estimated the sum assured in Scottish offices at £50,000,000.

In 1856 Mr. Joseph Bentley collected information from the various life offices as to their funds, income, and extent of business, and published the results of his enquiries. The following is an abstract of the information which he obtained:

Funds, including capital (127 offices)   . £53,789,431
Income, including interest (126 offices)   .   7,285,639
Existing assurances, including   bonuses
(124 offices)   179,923,328

In 1861 the Scottish life offices united in making the following return of their position:

Accumulated funds £12,807,057
Income, including interest   .   .   .   2,200,933
Sums assured and bonus additions .   .   54,692,877

Passing on to the date when exact information became obtainable as to all the existing offices, we get the following facts (applicable for the most part to the year 1870) from the first volume of the accounts and statements of life assurance companies lodged under the Act of 1870, and ordered by the House of Commons to be printed on 12 February 1872:

Total life assurance and annuity funds of
101 ordinary companies,* exclusive of
paid-up capital and value of re-assur-
ances £87,767,203
Total life assurance and annuity funds of
15 Scottish offices,† [equal to 24·2 per-
cent of the amount for the whole 101
offices]   21,256,080
Total life premium incomes of 101 or-
dinary companies   9,749,508

---

* The Blue Book gives the accounts of 111 companies. The 10 companies omitted in arriving at the above figures are accounted for thus: Industrial Offices and Friendly Societies, 5; Annuity Society, 1; Reversionary Company, 1; American Life Offices, 3.

† The English and Scottish Law Life Assurance Association was treated as an English office, its annual meetings being held in London. The North British and Mercantile and Northern companies were treated as Scottish offices, their annual meetings being held in Edinburgh and Aberdeen respectively.

Total life premium incomes of 15 Scottish
offices [equal to 25·7 per-cent of the
amount for the whole 101 offices]   .   £2,506,435

Estimated sum assured, exclusive of
bonus additions, in 101 ordinary
companies   292,557,343
[This is equal to about £9 per head of the
whole population of the United Kingdom.]

Estimated sum assured, exclusive of
bonus additions, in 16 Scottish com-
panies [equal to 25·7 per-cent of the
amount for the whole 101 offices]   .   83,547,833

Taking now the latest figures—namely, those contained in the
return which was ordered by the House of Commons to be printed
on 1 March 1887, and which for the most part apply to the
financial year ending at 31 December 1885—we get the following
particulars:

Total life assurance and annuity funds of
91 ordinary companies*   .   .   . £141,852,838

Total life assurance and annuity funds of
17 Scottish companies,† included in
the 91 ordinary companies [equal to
30·8 per-cent of the amount for the
whole 91 companies].   .   .   .   43,630,351

Total life premium incomes of 91 ordinary
companies   12,798,480

Total life premium incomes of 17 Scottish
companies, included in the 91 ordinary
companies [equal to 32·1 per-cent of
the amount for the whole 91 companies]   4,105,696

Estimated sum assured, exclusive of
bonus additions, in 91 companies   .   426,616,000
[This is equal to about £11.10s. per head of the
whole population of the United Kingdom.]

Estimated sum assured, exclusive of
bonus additions, in 17 Scottish offices
[equal to 32·1 per-cent of the amount
for the whole 91 companies]   .   .   136,856,533

* The blue book gives the accounts of 106 companies. In arriving at the
above figures 15 companies have been omitted, viz.: purely Industrial Offices,
9; Annuity Societies, 3; Reversionary Company, 1; American Life Offices, 2.
† See note † on preceding page.

Now let us glance for a moment at the department of industrial life assurance, which has been excluded in arriving at the foregoing figures. According to the summary appended to the latest return relative to life assurance companies, eight companies are classed as purely industrial, and one company is classed both as an ordinary company and as an industrial company. The London, Edinburgh and Glasgow, however, whose premiums in the ordinary branch amount to £15,782, and in the industrial branch to £25,039, has been classed in the summary solely as an ordinary company, instead of appearing in both classes; and the Methodist and General, which seems to be a purely industrial company, has been classed as an ordinary company in the summary. Adding the industrial figures of the former company, and the total figures of the latter company, to those given in the summary in the blue book, we get the following:

Industrial Business of 11 *Companies, per Accounts lodged in* 1886.

Industrial assurance funds . . say	£4,397,871
Industrial premium income . . .	3,575,346
Estimated amount of industrial assurances, assuming the average rate of premium to be £4 per-cent per annum in that class	89,383,650

This large extent of business is very remarkable, when it is considered that it has grown up within the last 20 years, and that three-fourths of the premium income and nine-tenths of the funds belong to one company, whose history affords a striking example of what may be accomplished by energy and perseverance under the guidance of high administrative skill.

Adding the figures applicable to the industrial business to those relating to ordinary business, we get the following grand totals:

Life assurance funds	£146,250,709
Life premium incomes	16,373,826
Estimated sums assured (exclusive of bonus)	515,999,650

Rate of Interest earned in 1837, and from 1865 to 1886.

I made an attempt to ascertain by special enquiries the rates of interest earned in 1837 on the life assurance funds of the offices then in existence, and I obtained replies in the case of 46

companies, among which were included all the companies which had funds of any important amount in 1837. The following abstracts show the information obtained:

I.—*Results arranged according to the Rate of Interest realized.*

ENGLISH, SCOTTISH, AND IRISH OFFICES.

Rate of Interest	No. of Offices	Total Life Assurance Funds	Total Amount of Interest
		£	£
Over 5 per-cent . . .	3	682,888	35,279
4·6 to 5 ,, . . .	7	1,012,298	48,140
4·1 ,, 4·5 ,, . . .	6	2,846,328	118,764
3·6 ,, 4·0 ,, . . .	13	4,589,017	173,858
3·1 ,, 3·5 ,, . . .	11	17,316,641	578,610
3·0 and under . . .	6	91,575	2,730
	46	26,538,747	957,381

Rate of interest per £100 invested . . 3·6 per-cent.
Average of the 46 individual rates . . 4·0 ,,

SCOTTISH OFFICES ONLY.

Rate of Interest	No. of Offices	Total Life Assurance Funds	Total Amount of Interest
		£	£
Over 5 per-cent . . .	1	99,848	5,094
4·6 to 5 ,, . . .	3	198,178	9,626
4·1 ,, 4·5 ,, . . .	3	714,978	30,707
3·6 ,, 4·0 ,, . . .	3	256,047	10,240
3·0 ,, 3·5 ,, . . .	1	10,891	327
	11	1,279,942	55,994

Rate of interest per £100 invested . . 4·4 per-cent.
Average of the 11 individual rates . . 4·3 ,,

II.—*Results arranged according to the Amounts of Funds of the Offices.*

Amount of Life Assurance Fund	No. of Offices	Total Life Assurance Funds	Total Amount of Interest	Rate per £100
		£	£	Per-cent
Over £1,000,000. . .	6	17,910,610	609,880	3·4
£600,000 to £1,000,000 .	7	5,074,507	193,084	3·8
£100,000 to £500,000 . .	12	2,732,916	120,933	4·4
Under £100,000 . . .	21	820,714	33,484	4·1
	46	26,538,747	957,381	3·6

SCOTTISH OFFICES ONLY.

Amount of Life Assurance Fund	No. of Offices	Total Life Assurance Funds	Total Amount of Interest	Rate per £100
		£	£	Per-cent
Over £100,000 . . .	3	946,152	40,791	4·3
Under £100,000 . . .	8	333,790	15,203	4·6
	11	1,279,942	55,994	4·4

It will be seen that the above information applies to $26\frac{1}{2}$ millions of funds out of a total of $27\frac{3}{4}$ millions.

The circumstance that the average rate of interest realized on the investments of life assurance companies in 1837 was only £3. 12s. per-cent, is scarcely in accordance with our preconceived notions. At present one hears so much about the fall which has taken place in the rate of interest, that it seems startling to find that the rate realized fifty years ago was lower than that now earned by the funds of life offices. Further enquiry, however, shows the low average rate in 1837 to be almost wholly due to the fact that the older and larger companies then confined their investments chiefly to consols and other Government securities, which yielded rates of 3 to $3\frac{1}{2}$ per-cent, with a small proportion of mortgages of the highest class, which yielded 4 per-cent. The information before me indicates that probably as much as 15 millions sterling were at that time invested by the companies in British Government securities. A desire for safety was, no doubt, one of the reasons which induced the companies to select consols for the investment of their funds. Another reason was, perhaps, the facility with which the existence of such investments could be verified at any moment. But probably the original cause, leading to a preference for that description of investment, dates back to the experimental stages of the older companies, when fear of the plague, or of some other epidemic, which might carry off a large proportion of their policyholders at once, was always present to the minds of the directors, and made them desirous of having their funds so invested as to be capable of being realized at once.

While the older and larger companies seem to have adhered chiefly to consols, the younger offices appear to have broken loose, to some extent, from the old traditions, as in 1837 a fair

proportion of them were earning rates which would now be considered high.

I shall now pass on to the period in regard to which information is obtainable from the returns under " The Life Assurance Companies Act, 1870." Question 9 of the 6th schedule of that Act requires a statement of the average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since last investigation. From the answers to that question recorded in the first sixteen volumes of the returns, the following statement has been drawn up; but in order to avoid giving undue weight to the experience of the smaller companies, I have omitted all those whose life assurance funds were less than £100,000.

Statement of the Average Rate of Interest earned on the Life Assurance Funds from 1865 *to* 1885 *by Companies having over* £100,000 *of such Funds, deduced from the Answers to Question* 9 *of the* 6th *Schedule of* " *The Life Assurance Companies Act,* 1870."

Year	Average Rate of Interest	Year	Average Rate of Interest
	Per-cent		Per-cent
1865	4·5	1876	4·4
1866	4·5	1877	4·4
1867	4·5	1878	4·4
1868	4·5	1879	4·4
1869	4·4	1880	4·4
1870	4·5	1881	4·3
1871	4·5	1882	4·4
1872	4·5	1883	4·3
1873	4·5	1884	4·3
1874	4·5	1885	4·2
1875	4·5		

It will be observed that although there is a tendency to a fall in the rate of interest in recent years, it is not very marked. The decline would certainly have been greater but for the movement which has been made by some of the companies towards securing enlarged powers of investment, including permission to lend in the Colonies.

I may remark, that on carefully examining the answers given by different companies as to the rates of interest realized, I find considerable want of uniformity in the mode of arriving at the replies. There can be no doubt that the question in the Act cannot be properly answered if unproductive assets be left out of

account, and I think it is also clear that what is wanted is the rate which the fund is expected to yield for the next year, not the rate which has been earned during the preceding year. The proper rate can be very conveniently and rapidly ascertained by classing together the loans and investments producing the same rates, and there can therefore be no sufficient excuse for giving a different answer from that which the question requires. On the subject of the mode of dealing with income-tax, however, some guidance is wanted, and I think the most satisfactory arrangement would be to state the interest under deduction of income-tax, both in the revenue account and in the answer to question 9 of the 6th schedule.

In these remarks as to want of uniformity in the replies, what I have had in view is the circumstance that different companies answer the question in different ways; but I have likewise observed that several offices have introduced changes in their method of dealing with reversionary transactions which have materially affected their rates of interest for successive years. These variations of treatment detract to some extent from the value for statistical purposes of the information given as to the rate of interest in the returns of life assurance companies under the Act of 1870.

Changes in Official Practice during the Last 50 Years.

To enumerate all the changes which have been introduced into the practice of life assurance companies since 1837, would occupy too much of your time. I shall therefore only refer to a few of these.

There is a considerable difference between our present mode of carrying through assurances and that which was in vogue in the early part of the century. Under the old plan, which was still acted on by some offices in 1837, an examination by the medical officer of the company was not prescribed as a necessary preliminary to the granting of a life policy, for the very good reason that in early times such a functionary was not attached to the staff of the companies. Nor was a certificate from the medical attendant always asked for in those days. The bye-laws of the Amicable Society required every applicant for membership who resided within 15 miles of London to appear in person before the court of directors, and to make oath "that he or she is in a good

" state of health, and hath no distemper which, according to the
" best of his or her knowledge, judgment, or belief, may tend to
" the shortening of his or her days." The attendance of applicants
resident more than 15 miles from London was dispensed with if
they submitted certificates by the minister and churchwardens of
their parish, and by the minister of an adjoining parish, "inserting
" therein the good state of health, age, and title of the parties
" proposed, together with an affidavit of the person to be admitted
" to the same effect, and also that he or she hath been in a good
" state of health for six months then past." In December 1837
new bye-laws were passed at a special meeting of the Amicable
Society, under which every person proposing to effect an assurance
with that corporation "is required to give in writing reference to
" two or more persons of respectability (of whom one at least
" shall be a medical practitioner), who may be enquired of
" touching the state of health of the party." The old rule
requiring personal attendance before the court of directors was,
however, left unaltered.

The original forms of proposal were very simple in character.
Several of the companies did not supply a schedule to be filled
up, but merely gave a specimen form in their prospectus. The
following is an example of a proposal given in the prospectus of
the Albion:

> " John Adams, of Park Street, Bristol, Merchant, proposes
> to insure with the Albion Fire and Life Insurance Company
> £2,000 for the whole of life [or for one year, or for seven
> years] on his own life [or on the life of Edward Johnson, of
> the Market Place, Gloucester, Woollen Draper]. The said
> John Adams [or Edward Johnson] was born at York on the
> 27th of June 1780, and will be aged therefore on his next
> birthday 26, has had the small-pox, is not in the Army or
> Navy, is not subject to gout, nor to any other disease or
> infirmity.
>
>   (Signed)   JOHN ADAMS."

Apparently, however, the company required references to some
medical or other persons; and after the acceptance of the proposal
they required the proposer to sign a declaration embodying the
statement of facts which he had made.

The tendency to make detailed enquiries into the health of
applicants for assurance, which showed itself about the year 1837,
became more fully developed in later years. By degrees the
proposal form was added to, and the questions addressed to the
medical referee or examiner became more and more numerous and

searching, until now, in 1887, a point has been reached at which
one feels inclined to pause and consider whether, after all, there is
not some danger in trying to make the forms absolutely perfect,
and in relying too much upon them; and whether the companies
might not be safer with simpler warranties, if for no other reason
than that they may keep more prominently before them the
importance of having trustworthy agents and medical officers,
and of making very careful enquiries where the applicants are
unknown to the officials or to the agent or medical officer.   While
making these remarks, I do not wish to suggest that questions
have been added to the forms without good and sufficient reason.
If we had time this evening to go over the history of life
assurance frauds, we should be able readily to trace the origin of
some of the questions which have been added to the simple
proposal forms which were in use half a century ago.   The
additions and alterations in the forms of medical report are nearly
all due to improved medical skill, and to the progress of know-
ledge on the subject of heredity of disease.

While the proposal and medical report have been increased
in stringency, the contrary has been the case with the form of
policy, the penal conditions of which have been greatly modified
and relaxed since 1837.   Take, for instance, the conditions as to
foreign travel.   The old " free limits " included permission to sail
to Ireland and the Channel Islands, and to ports on the Continent
of Europe between the Elbe and Brest, or the Texel and Brest, or
between Hamburg and Bordeaux.   If the life assured desired to
sail to Norway, Sweden, Denmark, Spain, or Portugal, it became
necessary for him to obtain the consent of the company, and to
pay such extra premiums as the directors chose to impose.
Compare this with the present conditions under which all persons,
not seafaring men, are permitted to travel to or reside in any part
of Europe, North America to the northward of 33° N. latitude,
South America to the southward of 30° S. latitude, South Africa,
Australia, New Zealand, &c., without any extra charge.

The following table shows some of the rates per-cent per
annum on the sum assured of extra premium, for foreign residence
charged by the Scottish offices in 1841, as compared with those
now usually charged for the same places:

	1841	1887
North America to north of 38° N. latitude . . .	£1 first year; 10s. thereafter.	Free.
North America, between 35° and 38° N. latitude . .	£3.	Free.
North America to south of 35° N. latitude . . .	About £5. 5s.	New Orleans, £4. Texas, £1 to £2. Other parts, generally free.
South America, between 20° and 25° S. latitude . .	£3. 3s.	10s.
South America to the south of 25° S. latitude . .	£2. 2s. first year; £1 thereafter.	Free in most cases.
East Indies	Military, £4. 4s.; Civil, £2. 2s.	Military, £1. 10s. to £2; Civil, £1 to £1. 10s.
West Indies	£3. 3s. to £9. 9s., according to age of the assured and healthiness of the island where he is located.	£1 to £2 if acclimatized; £2 to £3. 10s. for unacclimatized lives.
South Africa (Cape of Good Hope only) . . .	£1. 10s. first year; 10s. thereafter.	Free.
Australia and New Zealand .	£1. 10s. first year; 10s. thereafter.	Free.

I may remark that while in the case of extra premiums for foreign residence and travel, or for seafaring occupation, there has been a steady movement in one direction—namely, towards the reduction or abandonment of the extras; no similar movement has taken place in the case of the extra premiums imposed on account of family history or personal condition. Fifty years ago family history was scarcely recognized as an element to be taken into account in fixing the premium of a life assurance, and the ailments then chiefly feared seemed to be gout and asthma. At that date persons engaged in the retail sale of intoxicating liquor were generally accepted without addition to the ordinary rate of premium, as also were female lives (indeed, one company now existing struck out a special course for itself by advertizing in its prospectus of 1837 that it insured female lives at materially lower rates than those charged for male lives). In course of time a change took place, until a period was arrived at, 20 or 25 years ago, when cases of unfavourable family history were generally declined by the more prudent offices, or, if accepted at all, were charged at largely-increased rates. At that date much attention was directed to the condition of the heart, but as a rule all cases where heart affection of any kind was detected were declined. At

the present time the prevailing rates of extra premium for family history or personal condition are more lenient than those of 20 or 25 years ago, and many cases of heart affection are now accepted with an extra premium instead of being declined.   There is, however, a tendency to much greater strictness in enquiries into kidney ailments.   On the whole it may fairly be claimed for the present practice that it exhibits greater discrimination in dealing with points of personal health and family history than that shown at any former period.

Among the most important of the improvements introduced into life assurance business during the last half-century is the general adoption of the principle of allowing a surrender-value for discontinued policies.   In recent years there has been a movement in favour of the increase and more equitable adjustment of these values, and of the curtailment of the period during which no surrender-value is payable.   The conditions for the revival of lapsed policies have also recently been relaxed in an important degree by many offices, but it may be mentioned that 150 years ago the Amicable adopted rules permitting the revival of a lapsed policy where the premium was not more than fifteen months in arrears, without any evidence of health, and on payment of a rate of fine which bears favourable comparison with that charged at a recent date by many offices.   The Amicable at first allowed three months' grace for payment of the premium, but this was found to be too long.   It is important, however, to bear in mind, that previous to the year 1858 the offices did not hold themselves liable to pay the sum assured if death occurred during the days of grace and while the premium remained unpaid.

Between 1846 and 1860, the late Mr. W. T. Thomson, manager of the Standard and Colonial offices, who, as you are aware, was the originator and first honorary president of this Society, took a leading part in introducing improvements into life assurance business.   Chief among these were the reduction and classification of extra premiums for foreign residence, which was inaugurated by the Colonial Life Assurance Company; and the entire exemption from such extra premiums after a probationary period of five years (subsequently dispensed with) of persons who were in a position to declare that they had no prospect or intention of proceeding abroad, which was inaugurated by the Standard.

Among the first offices which published their surrender-values was the Scottish Widows' Fund; and that office, I think, divides

with the Standard the credit of having originated the plan by
which unclaimed surrender-values are carried to a separate fund
for a period of five years, during which time the parties interested
may obtain payment of the values.

In looking over some old prospectuses, many years ago, I
came across an early one of the Scottish National, in which a
plan was promulgated by Mr. M'Candlish, the present president
of the Faculty of Actuaries, whereby when three yearly premiums
had been paid, the fourth might lie over at interest without
application to the company, and, when six premiums had been
paid, the next two might lie over in a similar manner, and so on.
This plan has long ago passed out of sight, and I mention it now
merely because it seems to me to be not very far off in principle
from the system of applying the surrender-value to meet over-
due payments of premium, which was subsequently devised by
Mr. Morrice A. Black, and at a later date was introduced by him
into the practice of the Australian Mutual Provident Office, and
by means of which, in that office alone, as many as 44,995 policies
have been prevented from lapsing during the last thirteen years.

Although I have referred here chiefly to the changes intro-
duced by Scotch actuaries, I hope it will not for a moment be
thought that I undervalue the many improvements which we owe
to our English brethren.

In 1837 the number of skilled actuaries was small, and there
was no recognized standard by which the qualifications of persons
claiming to be actuaries could be tested. At that time the life
assurance companies had only a limited amount of data to go
upon. The life tables then chiefly used were the Northampton,
the Carlisle, the Equitable Experience Tables, and various hypo-
thetical tables. Professor de Morgan's essay on probabilities and
on their application to life contingencies and insurance offices was
not published until 1838; and David Jones' well-known work on
annuities and assurances did not appear until two years later.
In 1843 the first English life table was issued by the Registrar-
General, and in the same year the Seventeen Offices' Table—the
first life table deduced from the combined experience of a number
of life offices—was completed and issued.

The year 1848 was signalized by the formation of the Institute
of Actuaries, which from that time onwards has been of immense
service to life assurance companies and the public. Under its
auspices a system of examinations was established, with a view
to the granting of certificates of competency; meetings were held

for the discussion of actuarial questions; a valuable library was formed; and in 1850 the *Assurance Magazine* was started. That publication, now known as the *Journal of the Institute of Actuaries,* enjoys a world-wide reputation, and its twenty-six volumes contain many scientific and practical papers of great value in the various departments of actuarial research.   Much of the success of the *Journal* is due to its first editor, Mr. Jellicoe, who, at his own personal risk, started it, and subsequently carried it on for eighteen years, and to his immediate successor, Mr. Sprague, who conducted it with great ability for the next fifteen years.

The Faculty of Actuaries in Scotland, which was founded in 1856, has also done important service in establishing examinations, forming an actuarial library, and in encouraging, through the Actuarial Society, the reading of papers on actuarial subjects.

The most valuable additions made within the last 25 years to the life tables previously available have been the English Life Table No. 3, and the Institute of Actuaries' Table.   The former was published in 1864, and was based on the returns of two censuses, and on 6,470,720 deaths registered in 17 years.   The latter was published in two parts in 1869 and 1872, and was based on the mortality experience of 20 offices (10 English and 10 Scotch), embracing 160,426 lives, and 26,721 deaths.   The experience of the 10 Scottish offices, which was separately tabulated and exhaustively treated by Mr. Meikle, also added materials of great value on certain points which were not included in the enquiry made by the Institute of Actuaries.

Among the subjects which have occupied a large share of the attention of actuaries during the last 50 years, I may mention the following:

> The proper method of estimating the liabilities of a life assurance company.
>
> The effect of the selection of life exercised by the life assurance company, and of that exercised by the policy-holder.
>
> New plans of graduation of life tables.
>
> Improved methods of distributing profits.
>
> The magnitude of the reserves required by different mortality tables.

LEGISLATION AFFECTING LIFE ASSURANCE COMPANIES.

I shall now enumerate in their order the principal Acts of Parliament affecting life assurance companies which have been passed during the last half-century, namely:

> Joint Stock Companies Registration Act, 1844, which was the first Act enabling life assurance companies to obtain a common legal constitution.

> Companies Act, 1862, under which a form of incorporation could be had, and at the same time a limitation of the liability of shareholders could be secured.

> Policies of Assurance Act, 1867, under which the assignment of life assurance policies was simplified.

> Life Assurance Companies Act, 1870, on which I shall make a few remarks further on.

> Married Women's Property Acts of 1870 and 1882, under which a married woman may effect a policy on her own life or on the life of her husband for her separate use, the contract to be as valid as if made with an unmarried woman; and a married man may effect a policy on his life for behoof of his wife or his wife and children, or of his children, such policy, if effected while the insured was solvent, being exempt from seizure by creditors.

> Married Women's Policies of Assurance Act (Scotland), 1880, extending similar benefits to policies effected by married women and married men in Scotland.

"The Life Assurance Companies Act, 1870," was the result of the widespread alarm caused by the failure of the Albert Life Office in 1869, and by the rumours as to the condition of the European, which subsequently failed in 1872. The objects of the Act were the following: (1) To require a deposit of £20,000 from all new companies, to prevent the formation of what have been aptly termed "bubble companies"; (2) To make certain provisions relating to life assurance companies which transact also fire, marine, or accident insurance business; (3) to require the lodgment with the Board of Trade of revenue accounts and balance sheets, prepared according to certain prescribed forms, with a view to their being published; (4) to require periodical

valuations to be made, and the results to be reported, with detailed statements of the ordinary life assurances arranged according to the age of the lives, with a view to their publication by the Board of Trade; (5) to require full disclosure of the proposed terms of amalgamation or transfers, and the approval, or at least the tacit consent, of the policyholders, before such transactions are allowed to be carried out; (6) to provide for the winding-up of insolvent life assurance companies.

There can be no doubt that the Act has effectually prevented the formation of life assurance companies of the " bubble " order. Company promoters, however, still flourish; but those who devote themselves to founding insurance companies have ceased to take any interest in the department of life assurance. There has consequently been a great deal of attention paid to the formation of fire, marine, and accident insurance companies. During the 16 years ending with 1886, no fewer than 619 new insurance companies were registered, but only 11 of these were life companies. Had it not been for the passing of the Act we might to-day have had a very large number of new life assurance companies competing for business. This would have been a serious evil; but the absolute prohibition of new life assurance companies would have been an evil of scarcely less magnitude. A monopoly is seldom a good thing for the community; and, as has been pointed out by Mr. Sprague and others, there seems ground for believing that but for the energy and enterprize of new companies, and the stimulating effect which they exercise on the older companies, many of the most important improvements which have been introduced into life assurance business would never have seen the light. In that view it is very satisfactory to find that the requirement of a deposit of £20,000 has not stopped the formation of *bonâ fide* life companies, and that several of those formed since the passing of the Act have made a very good start.

As to the effect of the other leading requirements of the Act, we may, I think, without hesitation, say that the publication of accounts and valuation statements has been of great benefit to the public. It is true that the import of these statements is not always correctly understood by the public, or by journalists who make them the subject of comment, but they afford to actuaries the means of judging of the position of the companies; and in that way, when an office gets into a bad condition, the truth soon becomes known. The regulations regarding amalgamations and transfers have also been very useful in putting a stop to a class

of proceedings which at one time threw discredit on insurance business. I allude to the cases in which the directors and manager of an insolvent company received considerable sums of money by way of compensation for the loss of their appointments on the transfer of the business to another company in little better position, the interests of the policyholders in both offices being completely disregarded in the transaction.

Conclusion.

Although I have not been able to collect any statistical information as to the average rate of expenses in 1837, or as to the average standard of reserve at that date, I should like to make a few brief remarks on the subjects of expenses and reserves before concluding my paper.

Probably many of the offices which have been over 50 years in existence, may be able to show a lower rate of expenses at this date than they could have shown in 1837. This, however, must be chiefly because they were then for the most part young, with a moderate amount of existing business, on which the necessary expenses of keeping up an office and staff pressed heavily. I feel quite certain that if the rates of expense in 1837 of offices of 15 or 20 years' standing were compared with the rates of expense at this date, of offices of similar standing, the result would be to show that the expenses now materially exceed those of 50 years ago. In the early part of this century the business of life assurance was chiefly in the hands of mutual companies, which had no agents and paid no commission. Later on, when shareholding companies became more numerous, much of the business which they secured at the outset consisted of assurances obtained with little or no expense from their shareholders, each of whom was bound to introduce an amount of life assurance business proportionate to his holding of shares. The plan of charging entry-money, which was common in those days, must also have assisted in lightening the pressure of expenses on the premiums. Matters are very different now. Competition is keener, and we work at higher pressure. Expenses are incurred which would not have been thought of 50 years ago. We are compelled to put down branch offices in the leading cities, to employ a staff of inspectors or canvassers, and to advertize or send out circulars to the public. If we neglect these measures we run serious risk not merely of making no progress, but of falling behind, and gradually

losing our existing connections. The subject of expenses demands careful watching, especially in the case of companies which have weak reserves. But where a company has prescribed for itself a very high reserve, the necessity for maintaining that standard may generally be relied on as a wholesome and effectual check on any undue expenditure. I would only add here that the increase of competition and of working expenses, referred to above, is not peculiar to life assurance business. It is to be seen in every department of trade at the present day.

As to the standard of reserve, so far as I am able to judge, the average reserves now are stronger than those made 50 years ago; and the full disclosure of the valuation basis required by the Act of 1870 has a decided tendency to favour the maintenance of high reserves by inducing the offices to vie with each other in showing the safest basis rather than in declaring the largest bonuses.

A large proportion of the number of companies now existing adopt the Institute of Actuaries' Life Tables, and make their valuations at such a rate of interest as to afford an ample margin for adverse fluctuations; and I believe the financial position of the great majority of the life assurance companies of the United Kingdom to be both sound and strong at the present time. This is a highly satisfactory state of matters, and I think we must all be prepared to admit that the present position of the life assurance companies in this realm, and their steady growth and development during the last half-century, are in no small degree due to the wise laws and settled government which have prevailed during the reign of Queen Victoria.

REVIEW.

*The Assurance Risks of Warfare.**

The question of the assurance risks involved in warfare does not appear to have hitherto attracted the serious attention of British actuaries, and the pages of the *Journal* are almost free from information upon or discussion of that question. This circumstance may

---

* (1) On the Estimation of the Assurance Risks incidental to Continental Warfare, and the Rates of Extra Premium required to cover such Risks. 1st Edition, May 1888; 2nd Edition, September 1890.
  (2) On the Assurance Risks incident to Professional Military and Naval Lives; and the Rates of Extra Premiums which should be charged for such Risks. May 1890.
        By A. H. SMEE. M.R.C.S., and T. G. ACKLAND, F.I.A.
              London: C. & E. Layton.

be due to the fact that among British naval and military officers the various pension funds and schemes have, to a great extent, taken the place of assurance companies; and in the comparatively few cases where assured lives have become actually engaged in warfare, it has been deemed sufficient to assess the extra premiums upon an empirical system, the main feature of which was that of keeping well upon the side of safety. During recent years, several offices have granted assurances upon the lives of naval and military officers in time of peace, to cover all risks of war and climate, charging an annual extra premium varying from 10s. to 20s. per-cent. Other offices have charged the ordinary " with-profit" rates only, but have suspended the allocation of bonuses while the life assured remained in the service. It is to be feared, however, that in fixing these terms actuaries have had very slender data to go upon.

On the Continent of Europe, owing to the military system there prevailing, the circumstances are very different from our own; and the French, German, and Austrian actuaries and statisticians have dealt with the subject of the risks of warfare with some fulness. At the date of the publication of the first of the two papers which form the subject of this notice, more than two years ago, the strained relations between two great countries brought the question of the possibility of a European war into prominence; and as several English offices transact life assurance business on the Continent, that question became one of practical importance. Although the war clouds that were then in the horizon have now happily passed away, the possibilities of wars in the future, and their effect upon the finances of life assurance institutions, form a proper subject for enquiry.

Messrs. Smee and Ackland, the former the chief medical adviser and the latter the actuary and manager of the *Gresham Life Assurance Society*, have gathered together a number of facts bearing upon the problem of the chances of war and the risks of warfare as affecting assured lives, and have presented the results of their investigations, with tables based thereon, in the form of reports to the directors of the office they represent—the first report dealing with the risks incurred by the general population of *Continental* countries, and the second with those of professional military and naval lives at home and abroad.

A word may be said upon the fashion of collaboration between members of the medical and the actuarial professions now in vogue. The interests of the two bodies are closely allied in many respects, and there are many matters that can be made the subject of mutual investigation with great advantage to both. The reason for a medical man joining with an actuary in the investigation of the subject of the risks of warfare is not quite so apparent as in the case of the collaboration of Dr. Pollock and Mr. Chisholm in their recently-published work, entitled *The Medical Handbook of Life Assurance**, and it might perhaps have been considered more in accordance with the fitness of things if Mr. Ackland had had for his partner an experienced soldier rather than a distinguished surgeon.

* London C. & E. Layton.

However, the authors do not claim more than to have dealt with their subject upon broad and practical rather than strictly scientific lines, and indeed this is all of which the subject will admit. It is quite out of the question to attempt to express the chance of war as an algebraical quantity of the nth degree. All that can be done is to make use of the experience of past years to give a rough guide to the future, which, as a basis for premium rates, will do substantial justice between the various policyholders, and will be safe for the assurance office. As an element in favour of the office, it will probably be conceded that, with the progress of civilization, the chance of war diminishes. although perhaps the time is distant when " they shall beat their swords into plowshares, and their spears " into pruning-hooks: nation shall not lift up sword against nation, " neither shall they learn war any more."

CONTINENTAL WARFARE.

This subject is dealt with by the authors in their earlier report under six convenient sections, namely :

" I. Some details as to the past and present practice of the principal Continental companies respecting assurances against the risks of war.

" II. The estimates of the several elementary probabilities involved, with detailed and careful consideration of the available statistics, and statement of the general principles upon which our conclusions are based, as to

(1) the probability of the occurrence of war;
(2) the probability of being called to serve with the army in the field;
(3) the probability of death during the campaign;
(4) the combination of these probabilities with an ordinary mortality table, and the appropriate formulæ.

" III. Application of the formulæ and tables so deduced to the computation of premium rates, and discussion of the several modes of payment of the war premium, with the advantages and disadvantages of each method.

" IV. The principles regulating the reduction of the sum assured, in cases where the war risk is incurred without payment of extra premium.

" V. Recommendations as to the plans to be followed by the society in relation to

(1) charge of extra premium, payable in time of peace;
(2) charge of extra premium, in one sum, on the occurrence of war;
(3) reduction of sum assured or cancelment of assurance in cases where no extra premium has been paid.

" VI. Appendix, including tables, formulæ, and diagrams."

Many interesting particulars are given as to the practice of Continental assurance companies. In France it appears to be usual to suspend the assurance during the operations of war and the eight months following the cessation of hostilities, paying only the reserve value if death occur in the meantime. If the life assured survive the period the policy may be revived on payment of the premiums in arrear and interest. The risk during war may be partially covered by a special contract, upon payment of an extra premium varying from 5 per-cent upon the sum assured for officers and soldiers on active service, to 2 per-cent for administrative and civil officials. The extra premiums are carried to a special fund, and if the claims exceed the premiums received, the sums assured are reduced proportionately, the office however guaranteeing a minimum payment of one-third of the full sum assured. A Swiss company, *La Baloise Vie,* fixes the extra premium *after* the war has ended, according to the claims actually made—with a maximum of 5 per-cent upon the sum assured. We are not told what provision is made to secure payment of the war risk premiums after the war is over, in cases where the surrender-value of the policy is less than the amount of the extra premium. The method of assessing the extra premium upon the difference between the sum assured and the policy value $(1 - {}_nV_x)$ is apparently adopted by this company to some extent; but the illustration given does not make the matter clear. In the example given the amount of claims less the reserves held under the claim policies, is apportioned as a percentage upon the *total sums assured*. But it would seem that to be accurate the apportionment should be calculated upon *the total sums assured less the total reserves.* The principle of charging extra premiums upon the net sum at risk, that is to say, upon the sum assured less the reserve value, is undoubtedly the correct one, although not often adopted in practice. With regard to policies of long standing, and especially endowment-assurance policies, the point is of great consequence.

Until 1887 the German companies charged a single premium of 5 per-cent for one campaign, which might be paid by instalments during time of peace. At the close of that year, however, the *Gotha* company agreed to abolish extra premiums for military service, except in the case of professional military men, who were to pay an extra charge of 3 per mille (6s. per-cent) per annum in peace or war, and during their liability to military service.

Several other German offices followed suit, and there is not much to be said against their decision when it is remembered that nearly every policyholder either was or had been himself liable to the risks of military service.

The following are the main points of the war-risk regulations of the *Janus* of Vienna:

" Premiums covering the risks of war may be combined with ordinary assurances, the war premium to be paid with the ordinary premium for a term of six years, at the end of which time the company reserves the power to vary the rates. In the event of death during the six years, the unpaid premiums for the remainder of the

term will be deducted from the sum assured. The risks of war are classed in three categories,

" Class   I   Active army and both reserves.
   „   II   Ex-officers, doctors, chaplains, ambulance corps, and
                     first levy of " Landsturm."
   „   III   Second levy of " Landsturm."

The yearly extra premium is fixed in

" Class   I   at 6 per mille (12s. per-cent) upon the sum assured;
   „   II   at $4\frac{1}{2}$ per mille (9s. per-cent)   „   „   ;
   „   III   at 3 per mille (6s. per-cent)   „   „   ;

for assurances concluded in time of peace, or for assurances already in force and altered to war assurances.

" One-fifth per mille (say 5d. per-cent) is charged for ' policy duty ' on the issue or modification of a policy under war conditions. The premium is payable for the whole term of six years, unless the assured can prove exemption from all military service.

" The extra charge is reduced by one per-cent for each year of the existence of the policy, reckoned from the commencement, such reduction to take effect after it has been three years in force. (This is intended to give a rough approximation to the effect of applying the extra rate of premium to the difference between the sum assured and the reserve value.)

" When war is threatened or mobilization ordered before the proposal is made, assurances covering the risks of war will be granted at the following rates of single premium only:

" Class   I   5 per-cent upon the sum assured;
   „   II $3\frac{3}{4}$ per-cent   „   „   ;
   „   III $2\frac{1}{2}$ per-cent   „   „   ;

with the policy duty as before.

" When war has actually been declared, proposals to include war risk will not be accepted upon any terms. Policies effected with war risk do not participate in profits. The extra premium paid is non-returnable."

The manager of the *Janus*, Herr Rudolf Klang, has published a pamphlet dealing with the subject of war risk, and Messrs. Smee and Ackland express their obligations to him.

Herr Klang made the following assumptions in calculating the rates of extra premiums above mentioned:

1. That a war will occur once in every 6 years.
2. That 2 out of every 3 liable to serve will actually take part in the campaign.
3. That 1 in every 20 of those so taking part in the campaign will be killed.

The following table is extracted from Herr Klang's pamphlet:

Austro-Hungarian Campaigns during the last 100 *Years.*

(Extracted from Herr Klang's *Kriegs-Versicherungs Vorlagen.*)

Consecutive Number	Date of Campaign		Opposing Power, or Seat of War	Actual Duration of Campaign		
	Commencement	Conclusion		Years	Months	Days
1	1788	1791	Turkey	3	5	25
2	1790	...	Insurgents in Netherlands	...	10	29
3	1792	1797	France	5	5	27
4	1799	1801	,, ...	1	10	27
5	1805	...	,, ...	...	3	5
6	1809	...	,, ...	...	6	5
7	1812	...	Russia	...	6	...
8	1813	1814	France	...	9	18
9	1815	...	,, and Naples . .	...	7	25
9			Total for 30 years .	14	6	11
10	1821	...	Naples and Piedmont .	...	7	5
11	1831	...	Modena and Parma . .	...	1	1
12	1835	1836	Bosnia	1	...	23
13	1838	...	Montenegro . . .	...	...	3
14	1840	...	Syria	...	...	16
15	1846	...	Galicia	...	...	15
16	1848	1849	Piedmont	1	4	13
17	1854	...	Parma	...	...	...
18	1859	...	France and Italy . .	...	6	22
19	1864	...	Denmark	...	9	...
20	1866	...	Germany and Italy . .	...	3	16
21	1869	1870	South Dalmatia . .	...	2	19
22	1878	...	Bosnia and Herzegovina .	...	2	23
23	1881	1882	South Dalmatia . .	...	6	25
14			Total for 70 years .	5	10	1
23			Total for 100 years .	20	4	12

The table shows that the Austrian monarchy was engaged in 14 wars or military expeditions during the 70 years, 1816 to 1886. Deducting the 4 short expeditions, there were 10 campaigns having a total duration of 5 years 9 months. The authors have combined these results with those of an investigation by Professor Karup, of Leipzig, into the wars of Sweden, Prussia, and Denmark (the respective probabilities being estimated at ·195, ·170 and ·115 per annum), and finally agree to adopt Herr Klang's assumption that there will be one war in every 6 years.

In forming an estimate of the probability of a life having to serve in the field if war arose, the authors had to take into account the

Continental military system. They adopted, as a guide, the system of the Austro-Hungarian kingdom, which they considered to agree in the main with those of the other great Continental countries in which the *Gresham* transacts business (presumably France and Germany).

That system may be described in the authors' own words, as follows:—

"The liability to military service extends for healthy males from the 20th to the 32nd year of age, and is divided into three classes:— (i) the *first line*, including those from the 20th to the 24th year of age; (ii) the *reserve*, including those from the 24th to the 30th year; and (iii) the *second reserve* or 'Landwehr', including those from the 30th to the 32nd year of age. These three categories are destined for actual field service, and are all exposed to the same dangers in the event of war.

"The only persons above 32 years of age, and liable to be called out for service in the field, are those who are officers in the army by profession.

"By a law passed in June 1886, the liability to service in Austro-Hungary has been much extended, all male persons from the 19th to the 38th year of age, and from the 38th to the 42nd year, even if exempt under the regulations previously in force, being now liable to serve in the first or second levy respectively of (iv) the 'Landsturm' or *last reserve*.

"The two levies of the 'Landsturm' (which affect one million and a half of men) are intended for fortress and garrison duty, but the first levy is liable to serve in the line regiments should necessity arise.

"There is a further exceptional liability to military service in the case of retired or pensioned officers between the ages of 42 and 60, who may be called out as officers in the 'Landsturm' levies."

Herr Klang formed the opinion from a consideration of these regulations, and of statistical information as to the proportion of the numbers called out to those on the register, that the liability to military service may be taken as two-thirds, irrespective of age. Messrs. Smee and Ackland, recognizing the greater accuracy and convenience of assessing the liability with regard to the *age* of the life, have considered the question more closely, and the following are their conclusions:

"(a) That the liability to military service is at a maximum at age 20, and thence steadily decreases with increasing age until practical extinction at about age 42.

"(β) That, upon the declaration of war, it may be assumed that all lives assured between ages 20 and 24, and liable to serve in the first line, will be at once called out.

"(γ) That the liability to serve in the reserve, including lives from 24 to 30, and in the 'Landwehr', including lives from 30 to 32, is somewhat less than that of the first line, the proportionate liability depending upon the importance of the campaign and its duration.

" (δ) That the liability to military service on the part of the first and second 'Landsturm' levies, including lives from 19 to 38, and 38 to 42 respectively, is materially less than in the other categories of risk specified above."

" (ε) That, representing by unity the certainty of being called out during the campaign to the army in the field, the liability to service in the several risk categories may be considered to be

    (i) In the line (ages 20–24) . . . . = ·80
    (ii) In the reserves (ages 24–30) . . . = ·70
    (iii) In the ' Landwehr' (ages 30–32) . = ·60
    (iv) In the ' Landsturm '
             first levy (ages 19–38) . . = ·50
             second levy (ages 38–42) . . = ·25 "

These ratios, graduated by John Finlaison's method, have been adopted. The graduated figures for every 5th age are as follows :

Age				Estimated Liability to Service
20	.	.	.	·842
25	.	.	.	·726
30	.	.	.	·614
35	.	.	.	·467
40	.	.	.	·270
45	.	.	.	…

The final element to be determined was the probability of death during a campaign. Here again the authors have made use of Herr Klang's results, and the following table is not without interest :

Rates of War Mortality in Different European Campaigns, 1854–1878.

(Extracted from Herr Klang's *Kriegs-Versicherungs Vorlagen.*)

Campaign and Date	Rate of Mortality per-cent
France, in the Crimea, 1854–55 . .	10·3
France, in Campaign of 1859 . .	5·6
Italy ,, ,, . . .	5·5
Austria ,, ,, . .	4·7
North America, 1861–1865 . . .	5·4
Prussia, in Campaign of 1864 . .	3·3
Austria ,, ,, ,, .	4·0
Italy ,, ,, 1866 . .	4·0
Prussia ,, ,, ,, . .	3·4
Bavaria ,, ,, ,, . .	5·1
Austria ,, ,, ,, .	5·6
Germany ,, ,, 1870–71 . .	4·2
Austria ,, ,, 1878 . .	1·5
General Average (13 Campaigns) .	4·8

Reference is made to Mr. W. B. Hodge's paper on the mortality in military operations (*J.I.A.*, vii.). Mr. Hodge showed that during the years 1793-1815 (including $20\frac{1}{2}$ years of war) the total average mortality was about $5\frac{1}{2}$ per-cent. Professor Karup found the death rates in different campaigns to vary from $1\frac{1}{2}$ per-cent to 56 per-cent. and estimated the average rate at 10 per-cent. Herr Klang, however, thinks this result greatly overstated, for reasons not very clearly explained. In regard to the Franco-German war of 1870-71, the total German force engaged is said to have been 887.876, and the corresponding number reported as dead or missing 44.752, or 5·05 per-cent. Combining these results the authors think that the assumption of an *extra* war death rate of 5 per-cent (in addition to the ordinary rate of mortality) will be a safe one, and they have accordingly adopted it. The concluding chapters of the report deal with the actuarial formulas employed, the computation of the premium rates upon the basis explained, and with various modes of practically applying them. The following table shows the financial effect of the assumptions that have been made:

Net Extra Premium to cover War Risk.—Interest $3\frac{1}{2}$ per-cent.

Age	Net Single Premium per-cent	Uniform Net Annual Premium per-cent, payable		Net Premium per-cent for Current Annual Risk
		Throughout Life	Up to Age 45	
	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
20	7 3 2	0 7 3	0 9 7	0 13 7
25	5 8 0	0 5 7	0 8 2	0 11 8
30	3 12 8	0 3 11	0 6 8	0 9 10
35	2 0 2	0 2 3	0 4 11	0 7 6
40	0 14 2	0 0 10	0 3 1	0 4 4

For the full tables, with commutation columns at $3\frac{1}{2}$ per-cent interest and the office premiums proposed for adoption, reference should be made to the complete report.

Professional Military and Naval Lives.

In dealing with the assurance risks of professional military and naval lives, Messrs. Smee and Ackland have been principally guided by the statistics of the British army and navy. The points for consideration were:

" (1) The risk of death at home stations, or the *normal mortality.*
" (2) The risk of death at foreign stations in time of peace, or the *climate risk.*
" (3) The risk of death in *active military service* in the constantly recurring '*small wars.*'
" (4) The risk of death in *active military operations* in '*national wars.*'

" (5) The liability, in time of peace, to *foreign service.*

" (6) The liability to *active service* in the field in '*small wars.*'

" (7) The liability to *active service* in the field in '*national wars.*'"

The conclusions at which the authors arrived are summarized by them as follows:

" (1) That the home or normal mortality of the officers and men of the British army and navy may be taken on the basis of the Healthy Male Table of the Institute of Actuaries.

" (2) That the mortality of the army and navy on foreign service in time of peace may be considered to be equal, on the average, to an additional death rate of three-quarters per-cent per annum.

" (3) That the annual mortality from 'small wars' may be estimated at $1\frac{1}{2}$ per-cent of the total force engaged on active service.

" (4) That the mortality from 'national wars' may be estimated at 20 per-cent of the total force engaged in the campaign.

" (5) That it may be estimated that one-half of the British army and navy will be engaged on foreign service at any particular time.

" (6) That 6 per-cent of the force on foreign service will, on the average, be engaged annually in small wars.

" (7) That it may, for the purposes of this report, be assumed that the British army and navy will be engaged once in 40 years in a national war, involving the service of two-thirds of the whole force in active military operations, for a term not exceeding three years."

(The authors append the following note to this summary:—" The assumptions numbered (4) and (7) above are, as will be seen, purely matters of judgment; and it is probable that individual conclusions, based upon the available data, would show materially different results upon these points. We consider, however, that in combining the different assumptions thus made, in order to arrive at the total estimated probability of death in active service in a national war, the final result would probably not differ materially from that arrived at in this report.")

There will probably be no exception taken to the assumption that the H^{M} Table may be considered to fairly represent the home mortality of the army and navy. The official records, as published, do not give the facts in great detail, but there is sufficient to show that at the younger ages the normal mortality in the army and navy is somewhat light as compared with assured lives, the opposite feature being observable at ages above about 35.

The results given in the following table are the basis for the authors' assumption that the death rate of troops serving at foreign stations may be taken at 15 per 1,000. Finding, too, that the average death rate among troops in the United Kingdom is about $7\frac{1}{2}$ per 1,000, they are led to the conclusion that it will be fair to assume an extra rate of mortality of $7\frac{1}{2}$ per 1,000 for *climate* risk at foreign stations. Remembering that the figures in the table include some mortality in the field service, this estimate would appear to be a safe

one. Some interesting particulars obtained from the War Office as to the liability of the several regiments for foreign service gives support to the authors' conclusions that the strength of the army is about equally divided between stations at home and abroad.

Death Rate per 1,000 *of Troops serving at Foreign Stations,*
1876—1887.

Foreign Station	1887			1886			1877 to 1886
	Mean Strength	Deaths	Rate per 1,000	Mean Strength	Deaths	Rate per 1,000	Rate per 1,000
Gibraltar	4,649	29	6·29	4,307	41	9·51	7·08
Malta	5,499	36	6·54	4,736	34	7·17	9·94
,, (Fencible Artillery)	360	1	2·78	359	...	...	5·79
Cyprus (1)	472	3	6·36	636	7	11·00	17·31
Canada	1,282	7	5·46	1,283	3	2·34	5·37
Bermuda	1,183	15	12·68	1,227	15	12·22	7·90
West Indies (White)	1,121	13	11·60	1,020	15	14·70	16·68
,, (Black)	1,205	14	11·62	1,120	22	19·64	15·51
West Africa (2)	570	14	24·56	503	10	19·88	19·31
South ,, (3)	3,490	28	8·02	3,971	17	4·28	(38·03)
Mauritius	400	8	20·00	414	9	21·73	19·15
Ceylon	1,077	16	14·86	949	10	10·53	14·63
Hong Kong, &c.	2,266	13	5·74	2,269	26	11·45	9·64
India—							
Bengal	40,921	614	15·00	39,558	625	15·80	17·90
Madras	11,738	162	13·80	11,199	186	16·60	14·02
Bombay	11,283	163	14·45	11,000	147	13·36	17·13
Total India	63,942	939	14·68	61,757	958	15·51	17·02
Egypt (4)	5,272	81	15·36	11,062	408	36·88	(29·58)
On board Ship							
Going out	1,068	4	3·74	1,442	1	0·69	4·03
Coming home (5)	772	10	12·95	741	12	16·16	...
Transferring	465	1	2·15	324	1	3·09	6·77
On board Ship—							
Total	2,305	15	6·50	2,507	14	5·58	7·03
TOTALS	95,093	1,232	13·00	98,120	1,589	16·20	15·20

(1) *Cyprus.* The great decrease in the death rate arises from a comparison which includes the early years of our occupation, when the mortality was very high.

(2) *West Africa.* These are black troops entirely, with the exception of about 10 non-commissioned officers from the West Indies.

(3) *South Africa.* The high mortality from 1877 to 1886 includes the Zulu campaign (1879–81).

(4) *Egypt.* The ratio in the last column is the average of 4 years only (1883–6).

(5) *On board ship—Coming home.* The average death rate over the period 1877—1886 " differs but slightly " from that of 1886-7.

Some statistics extracted from the official reports appear to indicate that the rate of mortality in the navy is less than that of the army. The authors, however, make no reference to this point and have treated both classes alike. It seems to be the custom of British offices to undertake naval risks at a lower rate of premium than that for army men, and it would have been interesting if the point could have been closely examined: unfortunately, however, the official statistics are not so arranged as to show the rate of mortality at different ages, and the matter is left in some obscurity.

As regards the mortality arising out of "small wars", the authors show that during the 15 years, 1872—1886, the number of officers and men engaged was 92,650, of whom 1,396 or 15·10 per-cent died in action. They also show that about 6 per-cent of the total strength abroad was annually engaged in small wars. These results formed the basis of their third and sixth assumptions.

Coming now to the question of national wars, the authors reach dangerous ground, and it must be admitted that their conclusions are not altogether convincing. What bold assumptions are made, will be seen under headings (4) and (7) in the summary given above. The result of these assumptions, when combined, is to give the estimated value of the risk of death from a national war during the whole term of liability to service as 6s. 8d. per-cent per annum. One cannot help thinking that this value might, with equal show of reason, have been calculated at an amount much greater or much less than the sum mentioned, according to the individual judgment or bias of the computer. It seems clear that the life of an individual is too short to form the subject of any calculation based on the average chance of such a matter as a national war, with any approach to scientific accuracy. It can only be said that Messrs. Smee and Ackland's estimate is (as they appear frankly to recognize) no better and no worse than a thousand others that could be made. However, this is a matter to be left to the judgment of each actuary or board of directors, the authors having done their part in setting forth clearly their own opinions.

The effect upon the rate of mortality, as deduced from the official returns of the army and navy, of deaths arising among lives invalided, does not appear to have received the special attention of the authors in their present investigation. It has, however, been shown by Mr. Sprague (*J.I.A.*, xxv, 49) that while there is reason to believe that the death rate in the navy, as indicated in the official returns, would be materially increased by the inclusion of unrecorded deaths among lives invalided from the service, there exists no sufficient data upon which to form a judgment as to the precise extent or ratio of such increased mortality. It would be well if this important question could be further elucidated, and we understand that Messrs. Smee and Ackland propose giving to it their future attention.

The following scheme shows the arithmetical result of the assumptions made:

" 1. *Climate Risks.*

(*a*) The probability of serving at a foreign station $= \frac{1}{2}$.

(*b*) The increased probability, when so serving abroad, of death in time of peace $= \frac{7.5}{10000}$.

(*c*) The combined or total probability for a life now stationed at home, of death abroad in time of peace

$$= \frac{1}{2} \times \frac{7.5}{10000} = \underline{\cdot00375.}$$

" II. *War Risks—' Small Wars.'*

(*d*) The probability, when abroad, of active service in a small war $= \frac{6}{100}$.

(*e*) The probability of death during such active service $\frac{15}{1000}$.

(*f*) The combined or total probability, for a life now stationed at home, of death in active service abroad in a small war

$$= \frac{1}{2} \times \frac{6}{100} \times \frac{15}{1000} = \underline{\cdot00045.}$$

" III. *War Risks—' National Wars.'*

(*g*) The probability of the occurrence of a national war $= \frac{1}{10}$.

(*h*) The probability of active service in a national war $= \frac{2}{3}$.

(*i*) The probability of death during such active service $= \frac{20}{100} = \frac{1}{5}$.

(*j*) The combined or total probability, in respect of a life stationed at home, of death in active service in a national war

$$= \frac{1}{10} \times \frac{2}{3} \times \frac{1}{5} = \underline{\cdot00333.}"$$

" IV. *Combined War and Climate Risks.*

(*k*) The total annual probability, in respect of a life now subject to the home or normal mortality, of death from the combined effects of climate and war risks

$$= \cdot00375 + \cdot00045 + \cdot00333$$
$$= \cdot00753."$$

For the purpose of the calculation of premiums, the values of q_x according to the H^M Table for ages up to 50, were increased by ·0075, the addition from this age being gradually reduced until, for ages 56 and upwards, the H^M values are adopted, and the following table shows for every fifth age the loaded extra premiums computed upon this basis, with interest at $3\frac{1}{2}$ per cent.

Age at Entry	WHOLE-LIFE ASSURANCES			Endowment Assurances at 60, or Death (Extra Premium payable until Policy matures)
	Extra Premium for Climate and War Risks payable			
	Throughout Life	During 20 Years	Until Retirement	
	s. d.	s. d.	s. d.	s. d.
20	10 3	12 5	12 6	9 9
25	9 8	11 4	12 6	9 3
30	8 8	10 2	12 6	8 8
35	7 9	8 11	12 6	8 0
40	6 1	7 1	12 6	7 0
45	3 9	4 4	12 6	5 4

It will be seen from the table that the effect of adding ·0075 to the value of q_x is to increase the office annual premium by ·00625. This is not an exact result, but the authors probably found that as the values varied very slightly at the different ages at entry, it would be convenient to adopt an average value. This result fairly accords with that arrived at by Mr. G. H. Ryan in his paper on "Extra Risks" (*J.I.A.*, xxiv, 19), wherein it is shown that an increase of ·01 in the values of P_x is equivalent to an addition of about ·012 or ·013 to the corresponding values of q_x, as regards the greater period of life.

The rates of extra premium appear to be reasonable and not inconsistent with the existing practice of several assurance companies. In an appendix mention is made of the methods adopted by a number of companies, and it is noticeable that three offices charge no extra rate for the army and navy, but stipulate that the allocation of bonus shall be suspended until retirement from the service.

In bringing this notice to a conclusion, it is only necessary to mention that tables of office premium rates calculated for the *Gresham* office are given, as well as several statistical tables compiled from official sources. The thanks of the Institute of Actuaries are due to Messrs. Smee and Ackland for their laborious investigation of a very difficult question, and it will be agreed that their two reports, although by no means completely solving the problem, yet throw a great deal of light upon it. The results they have obtained will undoubtedly be of practical use to assurance companies, as well as to future investigators of the subject of the risks of warfare.

H. J. ROTHERY.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.
[Extracted from the Parliamentary Return for 1889, published in 1890.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	155,208,202	6,202,470	161,410,672
Adjustments for two Ordinary (£54,342 and £500) and two Industrial (Deficiencies of £240 and £3,514) Returns discontinued, and for a Balance (£10,204) transferred from the Industrial to the Ordinary Branch	− 44,638	− 6,450	− 51,088
	155,163,564	6,196,020	161,359,584
Premiums	13,928,001	4,360,438	18,288,439
Consideration for Annuities . .	1,107,787	175	1,107,962
Interest and Dividends (less Tax)	6,325,678	219,419	6,545,097
Increase in value of Investments .	239,580	...	239,580
Fines, Fees, &c.	8,100	297	8,397
Capital Paid-up	9,782	3,532	13,314
Customs Timber Measuring, &c. .	2,548	...	2,548
Donations (Itinerant Methodists) .	3,135	...	3,135
Transfers from other Accounts .	18,979	...	18,979
Miscellaneous	1,229	...	1,229
	176,808,383	10,779,881	187,588,264

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	11,001,581	1,663,661	12,665,242
Cash Bonuses and Reduction of Premiums	1,063,444	46	1,063,490
Annuities	743,326	56	743,382
Surrenders	888,728	9,495	898,223
Commission	709,247	1,179,055	1,888,302
Expenses of Management . .	1,424,505	704,241	2,128,746
Bad Debts	1,315	576	1,891
Decrease in value of Investments .	68,252	191	68,443
Interest on Capital and Dividends and Bonuses to Shareholders .	601,815	54,448	656,263
Transfers to other Accounts . .	40,897	243	41,140
Capital repaid (Argus) . . .	84,000	...	84,000
Miscellaneous (including defalcations, £3,350)	8,668	...	8,668
Balance* at the end of the Year .	160,172,605	7,167,869	167,340,474
	176,808,383	10,779,881	187,588,264

* This Balance includes the whole of the Life and Annuity Funds (£161,821,078), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1889).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . .	11,343,334	290,916	11,634,250
Life and Annuity Funds . .	154,942,559	6,878,519	161,821,078
Fire Funds of Companies transacting Life Business . . .	9,627,494	...	9,627,494
Marine Funds of Companies transacting Life Business . . .	588,169	...	588,169
Reserve Funds	3,625,646	...	3,625,646
Other Funds	628,854	6,574	635,428
Profit and Loss Balances . .	2,603,110	...	2,603,110
Depreciation and Investment Balances	567,146	...	567,146
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims . . .	3,277,869	9,860	3,287,729
Outstanding Accounts . . .	442,007	1,112	443,119
Temporary Loans	81,750	95	81,845
Sundries	12,562	...	12,562
	188,843,300	7,187,076	196,030,376

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	78,078,045	152,795	78,230,840
Loans on Policies	8,804,757	10,547	8,815,304
,, Rates (and Rent-charges)	21,701,631	1,917,159	23,618,790
British Government Securities .	5,526,669	699,868	6,226,537
Indian and Colonial Government Securities	12,527,740	68,496	12,596,236
Foreign Government Securities .	3,588,248	...	3,588,248
Debentures	16,809,493	1,327,379	18,136,872
Shares and Stocks . . .	12,470,045	14,650	12,484,695
Companies' own Shares . .	461,133	..	461,133
Land and House Property and Ground Rents	11,830,769	2,488,344	14,319,113
Life Interests and Reversions .	3,268,031	...	3,268,031
Loans on Personal Security . .	1,208,959	12,209	1,221,168
Agents' Balances and Outstanding Premiums	3,888,996	330,501	4,219,497
Outstanding Interest . . .	1,836,215	68,321	1,904,536
Cash, Deposits, Stamps, &c. . .	6,479,706	72,839	6,552,545
Customs Timber Measuring Balances, &c.	2,086	...	2,086
Book-Room Grant (Itinerant Methodists)	75,000	...	75,000
Deficiencies, Preliminary Expenses, &c.	285,777	23,968	309,745
	188,843,300	7,187,076	196,030,376

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's*
SUMMARY (1889), *when compared with the corresponding Items
for the previous Year.*

	Ordinary Companies	Industrial Companies
	£	£
INCOME.		
Premiums	+ 341,676	+ 351,526
Consideration for Annuities . . .	+ 29,127	− 200
Interest and Dividends (less Tax) . .	+ 155,507	+ 22,990
Net Increase in Value of Investments .	+ 62,430	− 90
OUTGO.		
Claims	− 709,114	+ 75,487
Annuities	+ 38,337	+ 29
Surrenders	+ 17,765	+ 1,102
Commission	+ 37,272	+ 76,948
Expenses of Management	+ 48,092	+ 35,582
LIABILITIES.		
Paid-up Capital (including sundry Shareholders' Balances)	− 114,572	− 4,005
Life and Annuity Funds	+ 5,027,282	+ 959,680
ASSETS.		
Mortgages (including Loans on Rates* and Rent-charges)	+ 303,312	+ 222,070
Life Interests and Reversions . . .	+ 9,406	− 19,262
Loans on Policies	+ 175,669	+ 2,595
British Government Securities . . .	+ 185,472	+ 7,300
Indian and Colonial Government Securities*	+ 613,825	+ 41,322
Foreign Government Securities*. . .	− 104,275	...
Debentures	+ 1,947,272	+ 416,615
Shares and Stocks	+ 429,506	+ 1,950
Companies' own Shares	− 104,322	...
Land and House Property and Ground Rents	+ 624,223	+ 316,403
Loans on Personal Security . . .	− 28,121	− 1,338

* One of the Companies states that £124,927, described last year as Colonial Municipal Securities, and £15,713, described as Foreign Municipal Securities, should have been described respectively as Colonial and Foreign Provincial Securities.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 104, of which 93 have been classed as Ordinary, 9 as Industrial, and 2 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately.

During the year five names have been removed from the official List of Companies, namely, Liverpool, Manchester, and Birmingham Industrial (Limited), London Annuity, Monthly (Limited), Preserver, which have all ceased business, and the Royal Farmers' and General, the business of which has been transferred. And one new name has been added, namely, the National Pension Fund for Nurses.

Summary of the Assurances in Force, *as shown by the last Returns of the Companie*
ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assur-ances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	670,142	340,194,086	79,819	54,989,920	749,961	395,184,006	18,187,391	376,996,6
Limited number of Premiums . .	25,529	14,432,536	4,276	1,972,068	29,805	16,404,604	562,594	15,842,0
	695,671	354,626,622	84,095	56,961,988	779,766	411,588,610	18,749,985	392,838,6
Endowments . .	2,808	512,068	5,874	929,154	8,682	1,441,222	4,035	1,437,1
Endowment Assur-ances . . .	129,661	27,422,363	25,643	7,329,332	155,304	34,751,695	444,115	34,307,5
Joint Lives . .	7,801	2,111,639	2,026	935,712	9,827	3,047,351	320,741	2,726,6
Last Survivor . .	1,258	997,726	1,189	1,126,122	2,447	2,123,848	152,291	1,971,5
Contingent . .	54	61,464	2,800	4,737,116	2,854	4,798,580	1,066,409	3,732,1
Issue . . .	5	6,150	667	2,826,602	672	2,832,752	784,999	2,047,7
Miscellaneous .	291	123,512	3,679	4,151,448	3,970	4,274,960	900,177	3,374,7
	837,549	385,861,544	125,973	78,997,474	963,522	464,859,018	22,422,752	442,436,2
Annuities.						Per Annum		
Immediate . .	...	...	...	...	16,555	737,920	3,889	734,0
Deferred . .	...	...	...	...	4,453	138,731	1,760	136,9
	...	...	...	...	21,008	876,651	5,649	871,0

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assur-ances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.						£		£
Whole Term of Life	...	...	...	...	9,215,992	83,005,078	...	83,005,07
Limited number of Premiums . .	...	...	...	...	3	7	...	
	...	...	...	...	9,215,995	83,005,085	...	83,005,08
Endowments . .	...	...	...	...	25,864	348,006	...	348,00
Endowment Assur-ances . . .	...	...	...	...	70,650	1,099,562	...	1,099,5
Joint Lives . .	...	...	...	...	100,482	1,467,986	...	1,467,9
	...	...	...	...	9,412,991	85,920,639	...	85,920,6
Annuities.						Per Annum		
Immediate . .	...	...	...	...	1	15	...	

The above figures are based on Returns deposited for the most part during the past five years, a are, therefore, merely an approximation to the amount of contracts in force at the present time. In case of six Companies, namely, the Co-operative, Customs' Fund, Industrial of Great Britain, Northe Provincial, and Royal, the amount of business at a more recent date has been included.

ACTUARIAL NOTES.

On Makeham's Extensions of Gompertz's Law.   *By*
W. S. B. Woolhouse, F.I.A., &c.

As a preliminary it may be well, in the first place, to give a succinct statement of Gompertz's law, and of the important developments Mr. Makeham has, so far successfully, engrafted upon it. This I cannot do better than as comprehensively stated by Mr. Makeham himself (*J.I.A.*, xxviii, 191).

Thus, the form of function—

$$\left.\begin{array}{l}\text{1. Whose logs} \\ \text{2. Whose } \Delta_1 \text{ logs} \\ \text{3. Whose } \Delta_2 \text{ logs} \\ \text{4. Whose } \Delta_3 \text{ logs}\end{array}\right\} \begin{array}{c}\text{are in} \\ \text{geometrical} \\ \text{progression} \\ \text{is}\end{array} \left\{\begin{array}{l} g^{q^x} \\ dg^{q^x} \\ ds^x g^{q^x} \\ ds^x w^{x^2} g^{q^x}\end{array}\right.$$

and so on (*d, g, q, s, w* denoting constants).

Otherwise, these relations may be expressed in another way which, by Mr. Makeham, is considered to be preferable, namely,

$$\left.\begin{array}{l}\text{1. Gompertz's law} \\ \text{2. Makeham's 1st development} \\ \text{3. \quad\quad ,, \quad\quad 2nd \quad\quad ,,}\end{array}\right\} cl_x = \left\{\begin{array}{l} g^{q^x} \\ g^{q^x} . s^z \\ g^{q^x} . s^x . w^{x^2}\end{array}\right.$$

c^{-1} being $= d$ of Gompertz's formula.

The great value of Mr. Makeham's first development is the beautiful maintenance of the law of seniority under a somewhat different but equally convenient form to that belonging to Gompertz's law. When we come to Mr. Makeham's second development, there is found to exist a similar law of seniority, but it is unfortunately accompanied by an inconvenient imposition of a new rate of interest, having a determinate value depending on the relative ages of the set of lives which may enter any special calculation.

After reading Mr. Makeham's valuable paper (*J.I.A.*, xxviii, 152, 185, 316), I felt so much interested in what related to the further extension of his developments, that I was induced spontaneously to put down some short notes for myself, so as to be enabled to conceive the subject matter independently, from my

own point of view. These notes, which necessarily confirmed Mr. Makeham's results, I recently sent to him, and it is at his suggestion that I send them to the Editor of the *Journal* for insertion, if considered sufficiently useful or interesting. The following are these notes, which, it will be seen, are briefly stated: but I trust that no important step will be found wanting.

NOTES.

Gompertz: $\log l_x = A + Bq^x$

Makeham (1): $\log l_x = A + bx + Bq^x$

   ,,   (2): $\log l_x = A + bx + cx^2 + Bq^x$

Gompertz: $\log p_{x,t} = Bq^{x+t} - Bq^x = B(q^t - 1)q^x$

Therefore, if $q^x + q^y + q^z \ldots = q^s$; then $\log p_{xyz\ldots,t} = \log p_{s,t}$

and $a_{xyz\ldots} = a_s$

Makeham (1): $\log p_{x,t} = bt + B(q^t - 1)q^x$

$\log p_{xyz\ldots,t} = nbt + B(q^t - 1)(q^x + q^y + q^z \ldots)$

$\log p_{sss\ldots,t} = nbt + B(q^t - 1).nq^s$

Therefore, if $q^x + q^y + q^z \ldots = nq^s$; then $a_{xyz\ldots} = a_{sss\ldots}$

Makeham (2): $\log p_{x,t} = bt + c(2tx + t^2) + B(q^t - 1)q^x$

$\log p_{xyz\ldots,t} = n(bt + ct^2) + 2ct(x + y + z \ldots)$
$\qquad\qquad\qquad\qquad\qquad + B(q^t - 1)(q^x + q^y + q^z \ldots)$

$\log p_{sss\ldots,t} = n(bt + ct^2) + 2ct.ns + B(q^t - 1).nq^s$

Therefore, if $q^x + q^y + q^z \ldots = nq^s$ . . . . . (1)

and $\log (p_{xyz\ldots,t}.v^t) = \log (p_{sss\ldots,t}.v^{1t})$ . . . (2)

then $\log \dfrac{v}{v^1} = \dfrac{1}{t}(\log p_{sss\ldots,t} - \log p_{xyz\ldots,t})$

$\qquad\qquad = 2c\{ns - (x + y + z \ldots)\}$ . . . . (3)

which may be readily shown to be constant, and independent of t; thus, multiply (1) by q^t, and

$$q^{x+t} + q^{y+t} + q^{z+t} \ldots = nq^{s+t};$$

therefore, when $x, y, z, \ldots$ become $x+t, y+t, z+t \ldots$, s becomes $s+t$; and by substitution the value of (3 remains unaltered.

Mr. Makeham's first extension, so well known and identified with his name, is, by far, the most useful and important. The second extension is less practical, as it involves changes in the rate of interest, but it may be better for the construction of mortality tables. If the foregoing notes be conceived to be advanced another step, it will be quite evident that no further extension of Gompertz's law is possible, at least in this direction, without abandoning altogether the very convenient law of seniority. But as regards the construction of mortality tables, alone considered, it is obvious that the extensions accomplished by Mr. Makeham may be carried on indefinitely.

Fines for Deferred Assurances with Deferred Premiums. By H. J. Rothery, F.I.A.

It occasionally happens, chiefly in connection with advances upon or purchases of reversionary interests, that a policy is effected upon a life for a certain amount, provision being made that, in the event of a further stated amount of assurance being required at the end of a stated period, the office shall, if required, grant such further assurance at the ordinary rate of premium for the then age, but without medical examination or enquiry as to health, &c. The consideration for this deferred assurance usually takes the form of a single premium, or fine.

Mr. Sprague's "select" tables, founded upon the H^M experience, afford a convenient means of obtaining the value of the deferred assurance; and the tables appended to this note, which are calculated for interval ages and for various deferred terms, will, it is hoped, prove to be of practical value, as well as of some interest, to the readers of the *Journal.*

Mr. Sprague, in his paper " On the Construction and Use of a Series of Select Mortality Tables" (*J.I.A.,* xxii, 433), has given a formula adapted to the purpose; but I will first explain a formula which I arrived at independently, and which seems to be

simpler: and I will afterwards show that both formulas are identical in value.

Let it be supposed that an assurance of 1 is to be granted upon a life now "select" and of the age x, the risk to commence at the expiration of n years if (x) be then living, the annual premium for a select life of age $x+n$ to become payable at the same time. What sum should be paid to the office in consideration of the granting of the deferred assurance?

Using Mr. Sprague's well-known notation, the value of the benefit to the assured may be denoted by

$$\frac{M_{[x]+n}}{D_{[x]}},$$

and the value of his contributions by

$$P_{[x+n]} \cdot \frac{N_{[x]+n}}{D_{[x]}}.$$

The amount of the single premium or fine should be the difference between the value of the benefit and the value of the contributions, and is therefore

$$\frac{M_{[x]+n} - P_{[x+n]} \cdot N_{[x]+n}}{D_{[x]}} \quad \ldots \quad (1)$$

This expression may be written

$$\frac{D_{[x]+n}}{D_{[x]}} \left\{ A_{[x]+n} - P_{[x+n]}(1 + a_{[x]+n}) \right\} \quad \ldots \quad (2)$$

From its form, it is evident that the expression within the brackets in (2) represents the value of a policy on a life select n years ago at age x, the premium being for a select life aged $x+n$. It would therefore appear that expression (2) will reduce to

$$\frac{D_{[x]+n}}{D_{[x]}} \left(1 - \frac{a_{[x]+n}}{a_{[x+n]}}\right) \quad \ldots \quad (3)$$

which is the formula I have adopted. A simple algebraical transformation, unnecessary to be given here, will show formulas (2) and (3) to be identical.

It may be worth while pointing out, as mentioned by Mr. Sprague, that the fine cannot safely be allowed to be paid annually during the n years unless the payments are guaranteed.

Nor can the fine be waived in consideration of an increase in the annual premium to be payable at age $x + n$, unless the payment of that premium is secured.

The formula deduced by Mr. Sprague is

$$\frac{(u\mathrm{D})_{x+n}}{\mathrm{D}_{[x]}}\left(1 - \frac{(u\mathbf{a})_{x+n}}{\mathbf{a}_{[x+n]}}\right),$$

the prefix u denoting *unhealthy* or damaged lives, as explained in his paper.

This formula, as it stands, is only strictly applicable to cases where the deferred term is 5 years or more. To apply to all cases, it should be written

$$\frac{(u\mathrm{D})_{[x]+n}}{\mathrm{D}_{[x]}}\left(1 - \frac{(u\mathbf{a})_{[x]+n}}{\mathbf{a}_{[x+n]}}\right) \quad . \quad . \quad . \quad . \quad (4)$$

Mr. Sprague has tabulated the values of $(u\mathrm{D})_x$, $(u\mathbf{a})_x$, &c., at 4 per-cent interest (*J.I.A.*, xxii, 430), and these values may be applied to formula (4) if n is not less than 5, $x + n$ being written for $[x] + n$. The values of $(u\mathrm{D})_{[x]+n}$, &c., where n is less than 5 are not tabulated, and, although they can be obtained from the relations $(u\mathrm{D})_{[x]+n} = \mathrm{D}_{[x]+n} - \mathrm{D}_{[x+n]}$, &c., it is clear that in such cases formula (3) is easier of application than formula (4).

Mr. Sprague has pointed out that the quantity $1 - \dfrac{(u\mathbf{a})_{[x]+n}}{\mathbf{a}_{[x+n]}}$ represents the value of a policy on a damaged life at the date of its issue, the age at the issue of the policy being $x + n$, and the life having been select at age x. Similarly, in formula (3), $1 - \dfrac{\mathbf{a}_{[x]+n}}{\mathbf{a}_{[x+n]}}$ is the value of a policy just effected on a life select n years previously, but who may now be healthy or damaged—it is not known which. In both formulas (3) and (4), the policy-value is suitably discounted to the point of time represented by age $[x]$.

I now proceed to show that the expressions (3) and (4) are identical, or that

$$\frac{(u\mathrm{D})_{[x]+n}}{\mathrm{D}_{[x]}}\left(1 - \frac{(u\mathbf{a})_{[x]+n}}{\mathbf{a}_{[x+n]}}\right) = \frac{\mathrm{D}_{[x]+n}}{\mathrm{D}_{[x]}}\left(1 - \frac{\mathbf{a}_{[x]+n}}{\mathbf{a}_{[x+n]}}\right).$$

We have $(u\mathrm{D})_{[x]+n} = \mathrm{D}_{[x]+n} - \mathrm{D}_{[x+n]},$

and $(u\mathbf{a})_{[x]+n} = \dfrac{(u\mathrm{N})_{[x]+n}}{(u\mathrm{D})_{[x]+n}} = \dfrac{\mathrm{N}_{[x]+n} - \mathrm{N}_{[x+n]}}{\mathrm{D}_{[x]+n} - \mathrm{D}_{[x+n]}}.$

Hence $(u\mathrm{D})_{[x]+n}\left\{1 - \dfrac{(u\mathbf{a})_{[x]+n}}{\mathbf{a}_{[x+n]}}\right\} = \mathrm{D}_{[x]+n} - \mathrm{D}_{[x+n]}$

$$- (\mathrm{N}_{[x]+n} - \mathrm{N}_{[x+n]})\dfrac{\mathrm{D}_{[x+n]}}{\mathrm{N}_{[x+n]}}$$

$$= \mathrm{D}_{[x]+n} - \mathrm{N}_{[x]+n} \cdot \dfrac{\mathrm{D}_{[x+n]}}{\mathrm{N}_{[x+n]}}$$

$$= \mathrm{D}_{[x]+n}\left(1 - \dfrac{\mathbf{a}_{[x]+n}}{\mathbf{a}_{[x+n]}}\right),$$

which proves the identity.

It will be seen from the table that the values of the single premiums or fines are not very regular in progression, this irregularity being no doubt due—to some extent, at any rate—to the defective graduation of the H$^\mathrm{M}$ Table. The figures will, however, give a sufficient indication of the nature of the risk incurred in deferred assurances of the kind under consideration. In practice there are sometimes margins in favour of the office: for instance, where the policy is to protect a reversionary life interest, it is often provided that the assurance shall not be called for if the life tenant, generally much older than (x), shall not be living at the end of the n years. Further, in practice, the taking up of the deferred policy does not entirely depend upon the state of health of the life (x), but to a greater extent upon financial considerations. Notwithstanding these circumstances, however, actuaries will probably think it well to add a substantial loading to the net rates of the table when quoting office premiums for these risks.

I have not included in the table any values for a longer deferred term than 10 years. The H$^\mathrm{M}$ Select Tables only trace the effect of selection through the first 5 years of assurance, and it seems to me that values of assurances deferred for a longer period, and calculated upon the basis of these tables, are therefore likely to be misleading. Moreover, the tables were intended for practical use, and I believe it is very unusual for the term of deferment to be greater than 5 years.

The present note does not by any means exhaust the subject, and one important example, namely, the fine to be paid when the assurance is upon one life against the other, is not touched upon. Some further developments may be dealt with upon a future occasion.

Single Premium or Fine for an Assurance of 100, *to commence at Age x + n, upon a Life Select at Age* [x], *the Annual Premium to be* $100P_{[x+n]}$.

H^M *Seleet.* $3\frac{1}{2}$ *per-cent.*

Age [x]	n = 1	n = 3	n = 5	n = 10	Age [x]
20	·590	·833	·728	·445	20
25	·488	·664	·559	·625	25
30	·408	·699	·778	·630	30
35	·481	·809	·787	·833	35
40	·504	·896	1·041	1·142	40
45	·671	1·305	1·444	1·271	45

H^M *Select.*   4 *per-cent.*

Age [x]	n = 1	n = 3	n = 5	n = 10	Age [x]
20	·583	·816	·707	·422	20
25	·483	·651	·542	·592	25
30	·403	·685	·755	·598	30
35	·474	·792	·764	·789	35
40	·497	·877	1·012	1·081	40
45	·663	1·276	1·401	1·204	45

CORRESPONDENCE.

HYPOTHETICAL AND NET POLICY-VALUES.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—Will you permit me to point out that the demonstration given in the the *Text-Book*, Part II, p. 330, Art. 41, of the relative magnitudes of the hypothetical and net values of a policy, and of the manner in which those relative magnitudes depend on the loading of

the premiums, may be more shortly and conveniently arranged in the following manner?

The office premium P' being obtained by loading the net premium P with a percentage κ and a constant c, we at once have

$$_nV'_x = \frac{P'_{x+n} - P'_x}{P'_{x+n} + d}$$

$$= \frac{P_{x+n}(1+\kappa) + c - P_x(1+\kappa) - c}{P_{x+n}(1+\kappa) + c + d}$$

$$= \frac{_nV_x(P_{x+n} + d)(1+\kappa)}{(P_{x+n} + d)(1+\kappa) + c - \kappa d}$$

$$= \frac{_nV_x}{1 + \dfrac{c - \kappa d}{(P_{x+n} + d)(1+\kappa)}} \ .$$

Not only is this process much shorter, but the final form is simpler than that given in the *Text-Book*, and consequently the relation between the two values is shown rather more clearly, $_nV'_x$ being obviously $> = <{_nV_x}$ according as $c < = > \kappa d$. These results are, indeed, equally obvious from the third step, $_nV'_x$ being clearly $> = < {_nV_x}$ according as $c - \kappa d$ is negative, zero, or positive.

Any difficulty which may be experienced in passing from the second to the third step will be at once removed by the consideration that the latter is obtained by a simple application of the formula contained in the first step, unaccented symbols being, of course, substituted for accented.

<div style="text-align:center">

I am, Sir,

Your obedient servant,

</div>

19 & 20, *Cornhill, E.C.*   H. C. THISELTON.
    10 *June* 1890.

THE REV. JOHN HODGSON'S CLERGY MORTALITY TABLES.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—On examining the manuscript of the late Rev. John Hodgson's work, *Observations in reference to Duration of Life amongst the Clergy of England and Wales*, London, 1865, I find that the number of deaths at ages 55–59 among " entrants in this century " was 100, and not 90, as appears on page 29.   The rate of mortality per-cent is correctly stated at 1·655.

I am, Sir,

Your obedient servant,

2 & 3, *The Sanctuary,*   FRANK B. WYATT.
         Westminster, S.W.
            30 *June* 1890.

THE INSTITUTE OF ACTUARIES.

NEW BYE-LAWS.

At a Special General Meeting of the Fellows and Associates of the Institute of Actuaries, held on 17 February 1890, it was moved, with the approval and on behalf of the Council of the Institute, that certain alterations be made in the Bye-Laws. The meeting adopted the motion, and the resolution was duly confirmed at a further Special General Meeting, held on 14 March 1890.

The proposed amendments, as under, have now received the sanction of the Privy Council, and the existing copies of the Bye-Laws should be altered accordingly.

Ed. *J.I.A.*

*Official
Seal.*

AT THE

COUNCIL CHAMBER, WHITEHALL.

The 20th day of May, 1890.

By a Committee of the Lords of Her Majesty's Most
Honourable Privy Council.

Present:

Lord President,
Duke of Richmond and Gordon,
Marquess of Lothian,
Sir Francis Sandford,
Mr. Robertson.

Whereas there was this day read at the Board a letter dated
19th May, 1890, from Messrs. Markby, Stewart & Co., submitting
certain Bye-laws of the Institute of Actuaries, in substitution of
Bye-laws 40 and 53, of the Institute;

And Whereas, by the 45th Article of the Charter of Incorpo-
ration of the Institute, it is provided that Bye-laws made by the
Institute shall not commence or have effect until they have been
submitted to and allowed by the Lords of the Council;

NOW THEREFORE, their Lordships having taken the said
Bye-laws into consideration (a copy of which is hereunto annexed)
are pleased to allow the same.

C. L. PEEL.

BYE-LAWS REFERRED TO IN THE FOREGOING ORDER.

At a Special General Meeting of the Fellows and Associates of
the Institute of Actuaries, duly convened and held at Staple Inn
Hall, on Monday, 17th day of February, 1890, and at a subsequent

Special General Meeting of the said Fellows and Associates, also duly convened and held at the same place, on Friday, the 14th day of March, 1890, Bye-laws 40 and 53 were rescinded, and in substitution thereof the following Bye-laws, to be numbered respectively 40 and 53, were duly made, passed and confirmed.

40. "After the expiration of five years from the date of the "Charter, no person, whether already an Associate or Student "or not, shall be eligible as a Fellow unless he shall have passed "the Examination or Examinations declared by the Rules of "Examination to appertain to the Class of Fellows. Never- "theless, it shall be in the power of the Council to dispense "with the aforesaid Examination or Examinations in the case "of a Candidate possessing one or other of the qualifications "hereinafter mentioned, provided that not less than two-thirds "of the whole Council shall vote, in person or by proxy, in favor "of such dispensation:—

"(*a*) That he is the Actuary, or the principal Officer if "responsible for the actuarial work of some Life "Assurance Society or Company having its Head "Office in the United Kingdom of Great Britain "and Ireland, or in one or other of its Colonies or "Dependencies.

"(*b*) That he is in the service of the Crown in the capacity "of Actuary of some department of the Government.

"(*c*) That he is the Actuary of some Annuity or Rever- "sionary Interest Society, or analogous Institution "having its Head Office as aforesaid, whose experience "in matters relative to the profession of Actuary is "such as to render him in the opinion of the Council "worthy of election to the Class of Fellows.

53. "Except in the cases hereinafter mentioned in Sections "58 and 59, Fellows and Associates shall be elected by ballot at "Ordinary General Meetings, provided always that only Fellows "shall vote in the election of Fellows."

L. S.

ERRATA.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

On pp. 239–246 the summaries of the Returns to the Board of Trade for the years 1887 and 1888 are given in reverse order, and are incorrectly described. Pages 243–246 are extracted from the Parliamentary Return for 1887 (published in 1888), and pp. 239–242 from the similar Return for 1888 (published in 1889).

The corresponding summary for 1889 (published in 1890) is given on p. 477.

MR. SPRAGUE'S PRIZE ESSAYS.

The motto of Mr. P. L. Newman's essay, which gained one of Mr. Sprague's prizes, is "*Fide et Fortitudine*", and not "*Fide Fortitude*", as printed on p. 414 of the present volume.

INDEX TO VOL. XXVIII.

END OF VOL. XXVIII.